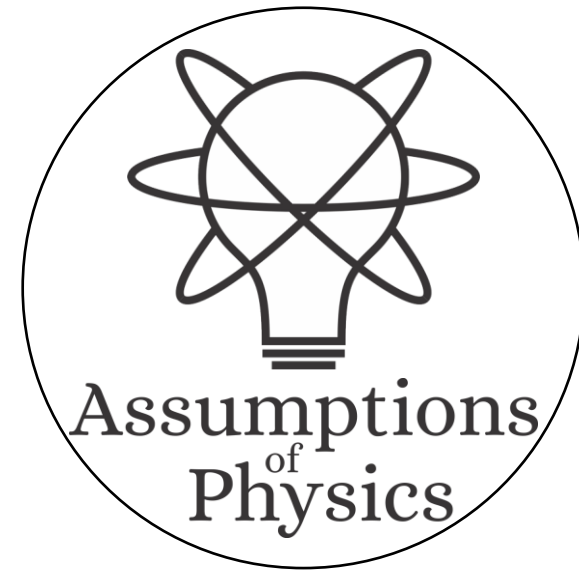


Assumptions of Physics
Summer School 2026

Introduction to Physical Mathematics

Gabriele Carcassi, Christine A. Aidala, and **Tobias Thrien**

Physics Department
University of Michigan

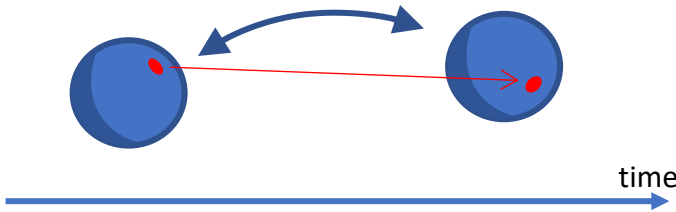


Main goal of the program

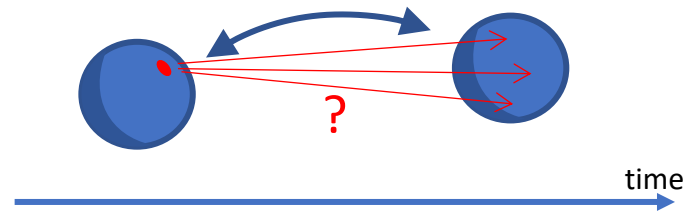
Identify a handful of physical starting points from which the basic laws can be rigorously derived

For example:

Infinitesimal reducibility $\Rightarrow$ Classical state



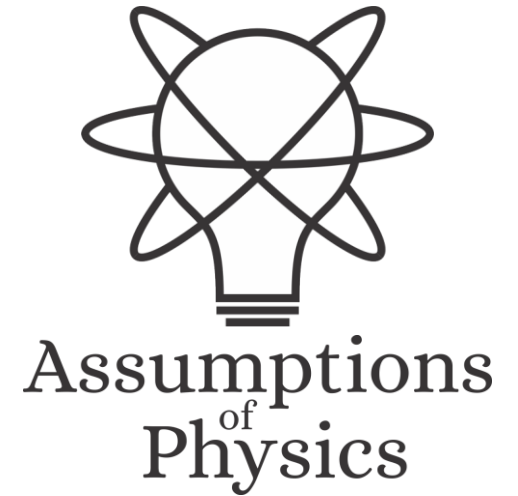
Irreducibility $\Rightarrow$ Quantum state



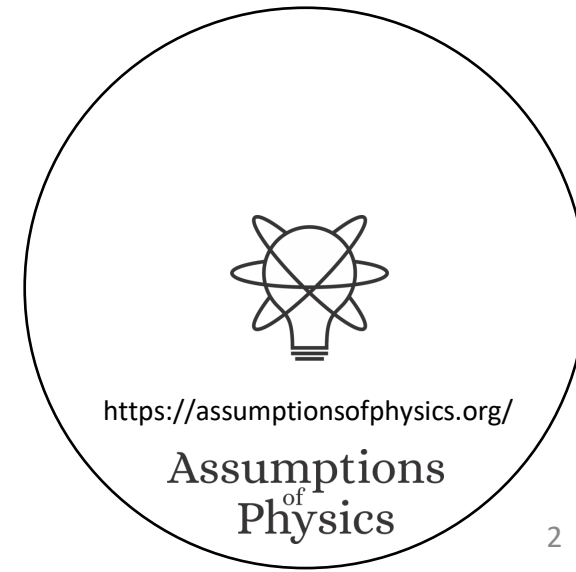
This also requires rederiving all mathematical structures from physical requirements

For example:

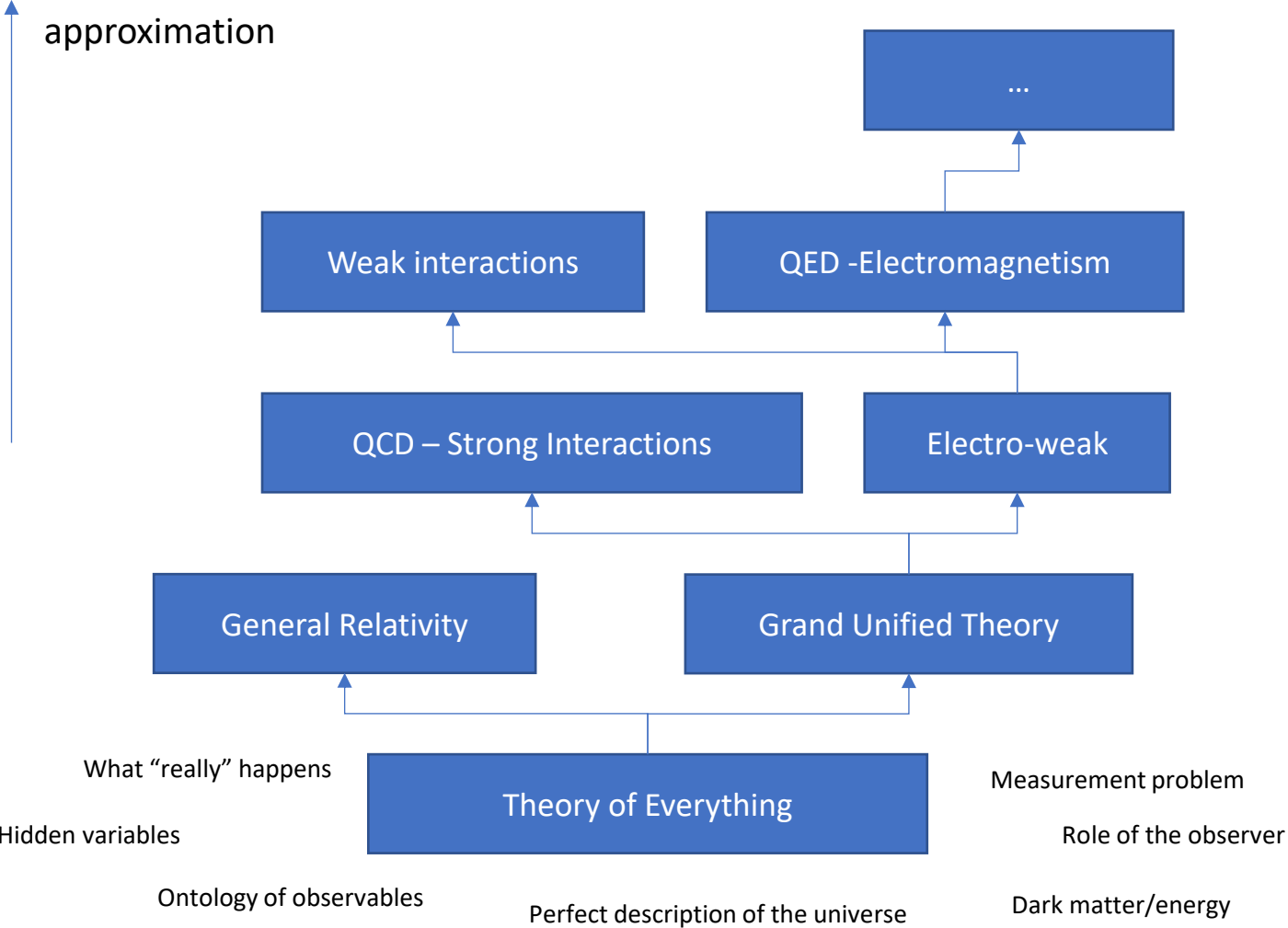
Science is evidence based $\Rightarrow$ scientific theory must be characterized by experimentally verifiable statements $\Rightarrow$ topologies and σ -algebras



<https://assumptionsofphysics.org>



Standard view of the foundations of physics

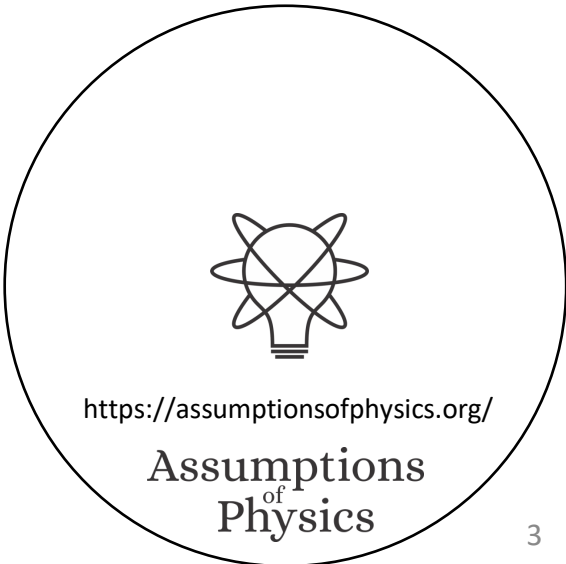


Goal of physics is to find the true laws of the universe!

The “real” physics!

The foundations of physics!

Everything else is an approximation



We found:

Experimental verifiability $\Rightarrow$ topologies and σ -algebras

Geometric structures $\Leftrightarrow$ Entropic structures

Hamiltonian evolution $\Leftrightarrow$ det-rev/isolation + DOF independence

Massive particles and potential forces $\Leftrightarrow$  + Kinematic eq

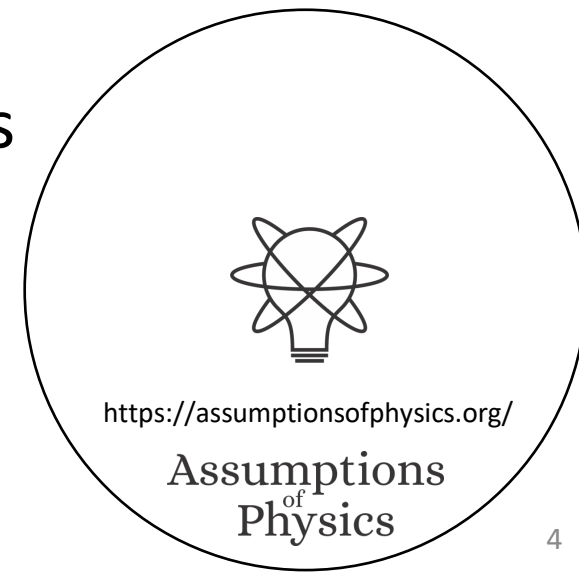
Physical requirements and assumptions drive most of the theoretical apparatus

~~Goal of physics is to find the
true laws of the universe!~~

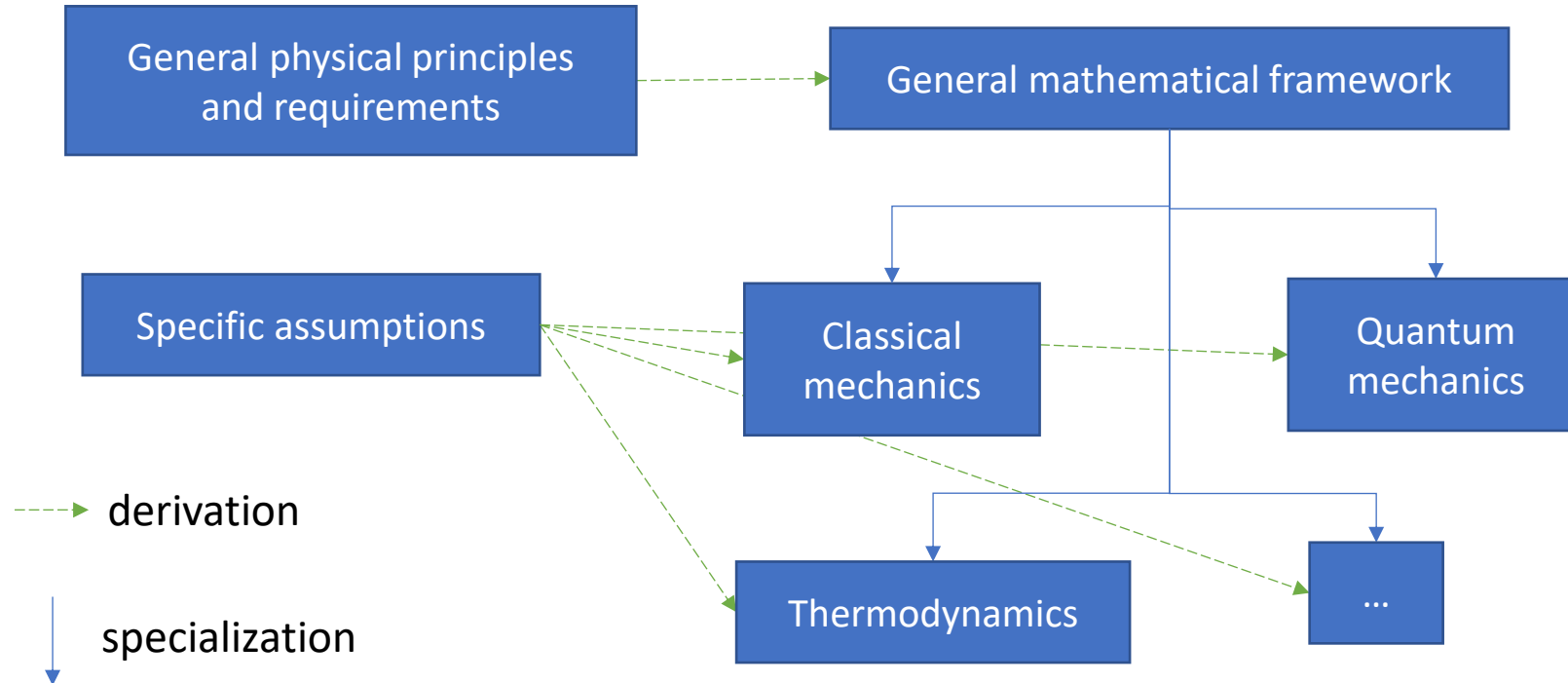
Less productive point of view

Goal of physics is to find models
that can be empirically tested

More productive point of view



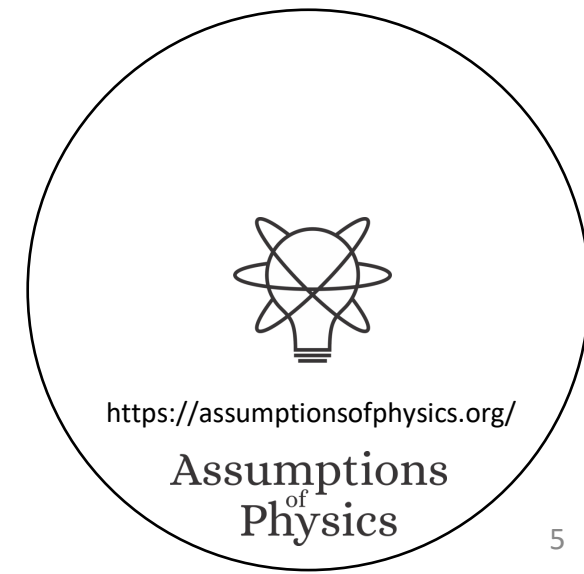
Our view of the foundations of physics



Foundations of
physics



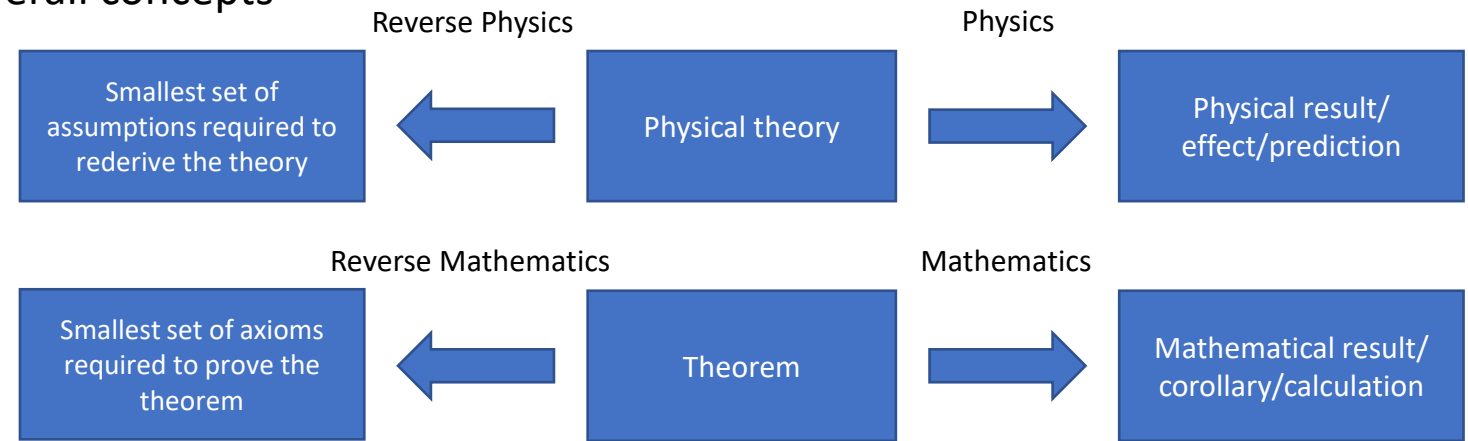
The theory of
physical models



Find the right overall concepts

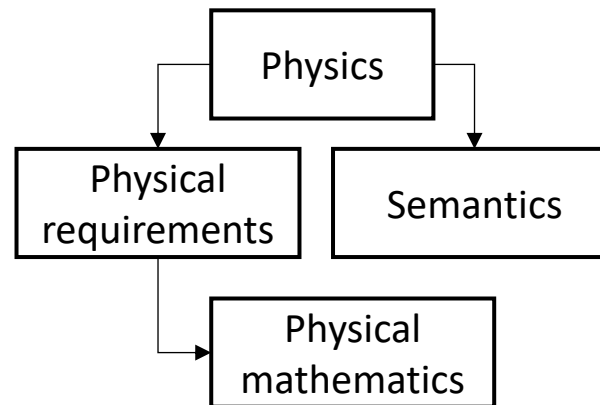
Reverse physics:
Start with the equations,
reverse engineer physical
assumptions/principles

Found Phys **52**, 40 (2022)

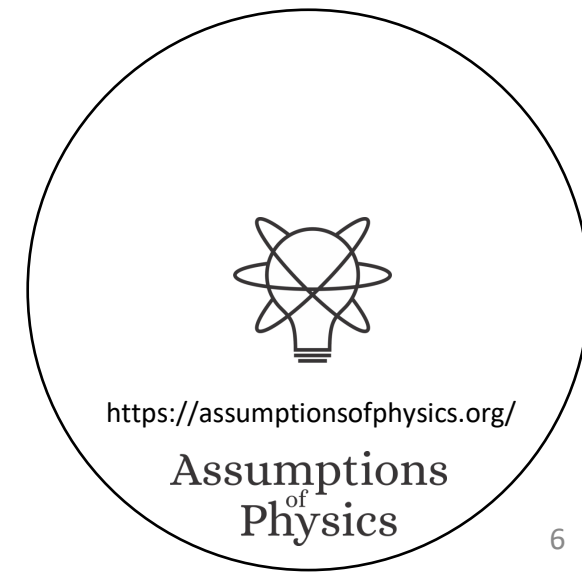


Goal: find the right overall physical concepts, “elevate” the discussion from mathematical constructs to physical principles

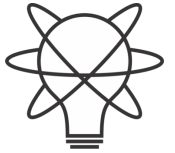
Physical mathematics:
Start from scratch and rederive
all mathematical structures from
physical requirements



Goal: get the details right, perfect one-to-one map between mathematical and physical objects



The fundamental misunderstanding in the foundations of physics



<https://assumptionsofphysics.org/>

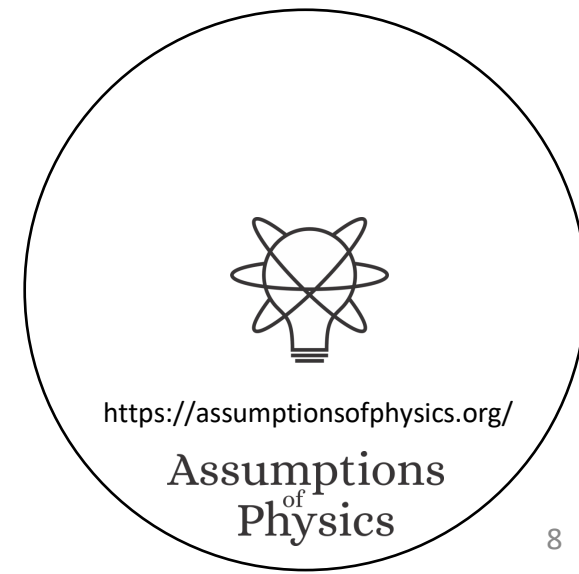
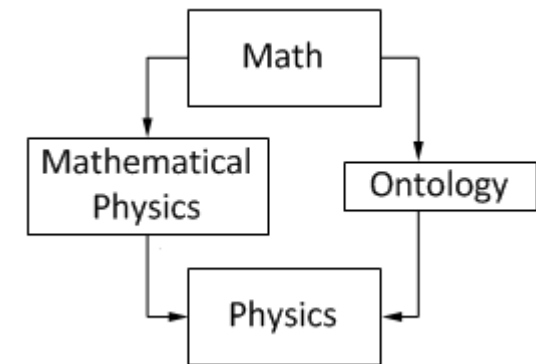
Assumptions
of
Physics

Math is a just tool for calculation, whose technical details are better left to mathematicians

But we don't develop theories by writing down assumptions and then derive observable consequences in a sequence of theorems and proofs. In physics, theories almost always start out as loose patchworks of ideas. Cleaning up the mess that physicists generate in theory development, and finding a neat set of assumptions from which the whole theory can be derived, is often left to our colleagues in mathematical physics—a branch of mathematics, not of physics.

Sabine Hossenfelder – Lost in Math

Even those that work on the math,
they work on it as mathematicians

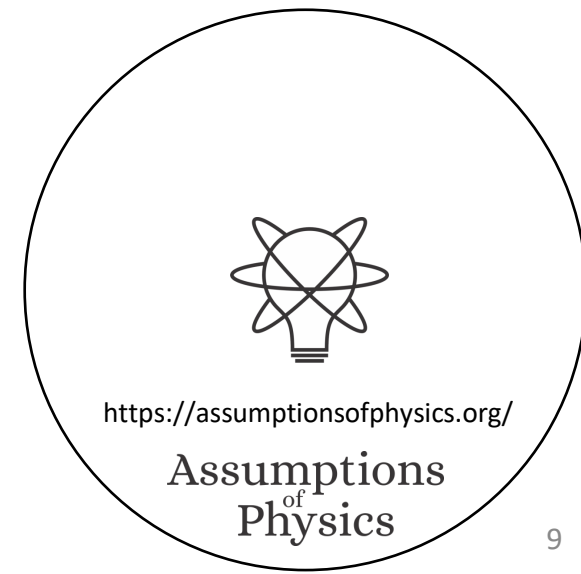


We use mathematics to specify our models, not just calculations, and specifying physical models is the whole point of physics

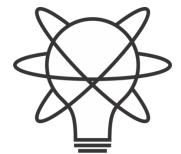
Also, there is no single way to “clean up the mess”: each axiom and definition represents a choice in mathematical modeling

Those are physical choices, which mathematicians are ill-equipped to make

So we end up with **THE WRONG MATH**



Examples of unphysical mathematics



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Assumptions
of
Physics

In differential geometry, tangent vectors are derivations

$$v: C^\infty(X) \rightarrow C^\infty(X)$$

$$v = v^i \partial_i$$

component basis

In polar coordinates

$$\partial_r + \partial_\theta = ???$$

[m] [rad]

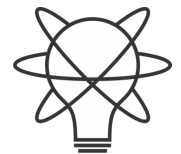
In phase space

$$\partial_q + \partial_p = ???$$

[m] [Kg m s⁻¹]

Doesn't work with units

Mathematically precise $\nRightarrow$ physically precise



<https://assumptionsofphysics.org/>

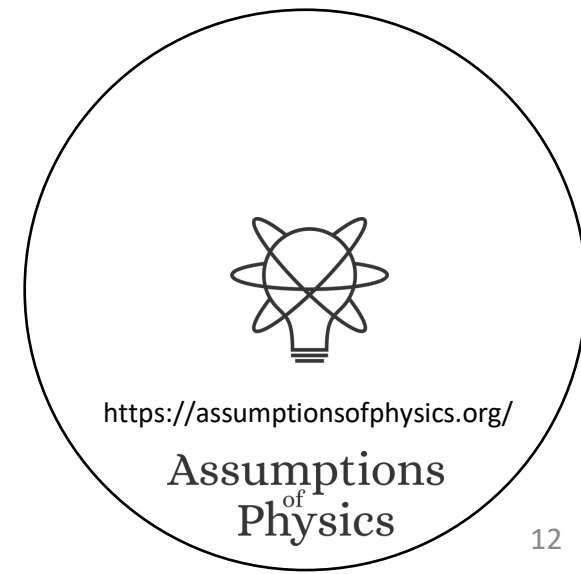
Assumptions
of
Physics

Inconsistent convergence in Hilbert spaces

Consider the sequence of states given by

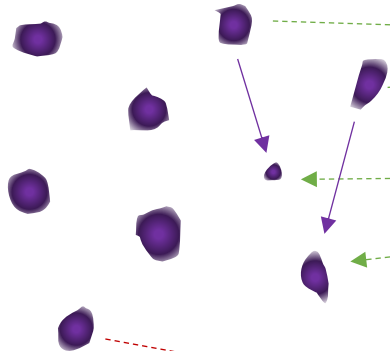
$$|\psi_i\rangle = \sqrt{\frac{i-1}{i}}|0\rangle + \sqrt{\frac{1}{i}}|i\rangle$$
$$\langle\psi_i|N|\psi_i\rangle = \frac{i-1}{i}\langle 0|N|0\rangle + \frac{1}{i}\langle i|N|i\rangle \rightarrow 1$$
$$|\psi_i\rangle \rightarrow |0\rangle$$

The state converges to the ground state
while the energy level converges to the first level



Physical world (informal system)

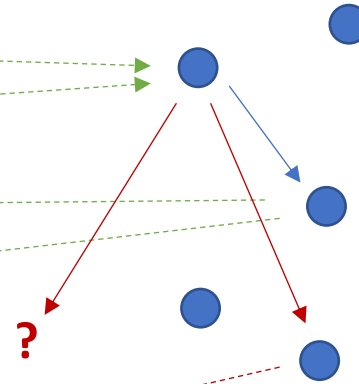
well-defined
physical
objects



ill-defined

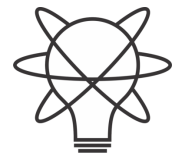
Mathematical representation (formal system)

well-defined
mathematical
objects



ill-defined

Current state of the art in theoretical physics

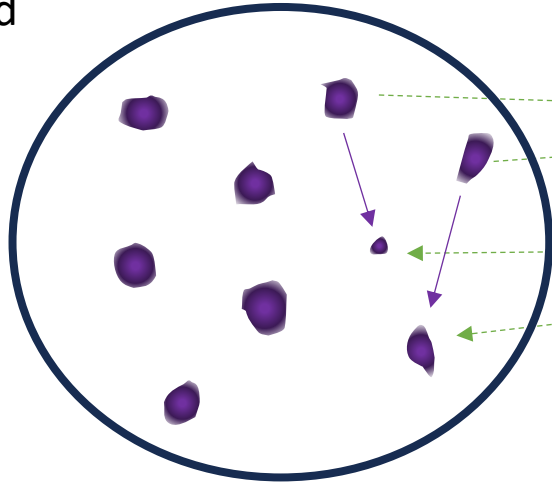


<https://assumptionsofphysics.org/>

Assumptions
of
Physics

Physical world (informal system)

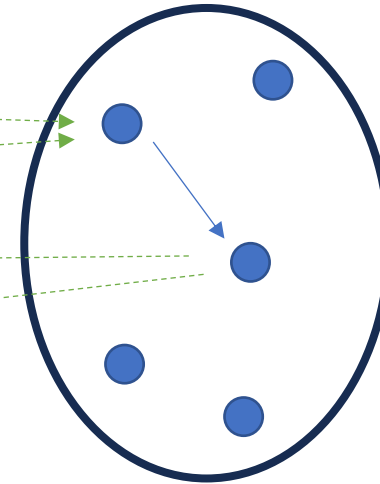
well-defined
physical
objects



Physical specifications

Mathematical representation (formal system)

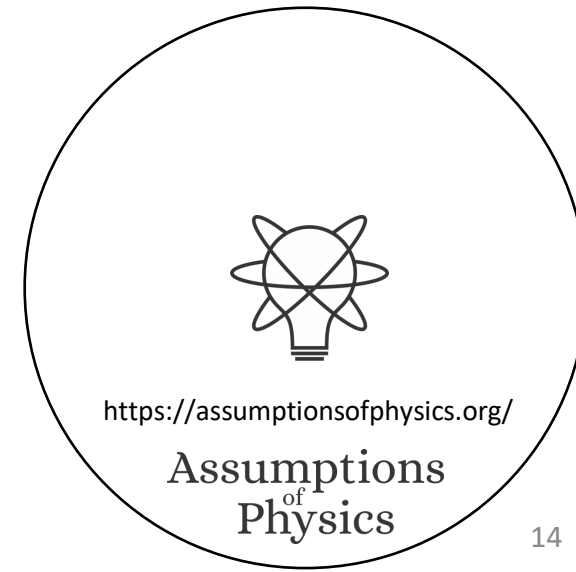
well-defined
mathematical
objects



Mathematical definition



When physical objects are mapped to the right mathematical objects,
the physical specification maps to the mathematical definitions

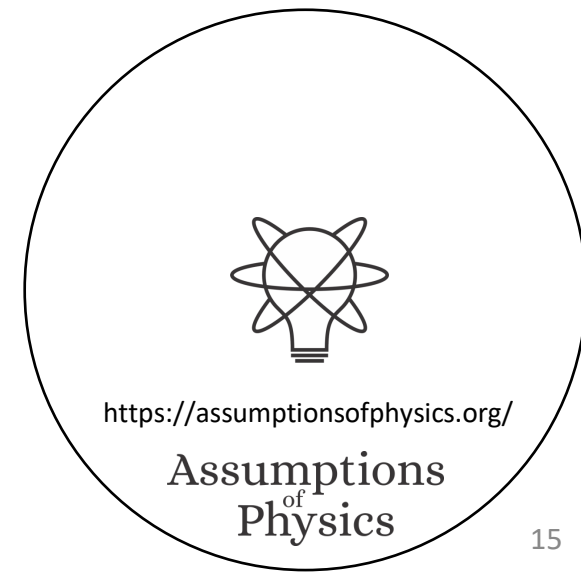


Mathematicians have developed
standards of rigor for their discipline

What standard of rigor should
we have for physical mathematics?

For the math part, the same as mathematics

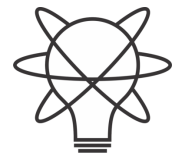
What should we do for the physics part?



Axiom 1.7 (Axiom of mixture). *The statistical mixture of two ensembles is an ensemble.*

Informal intuitive requirement

(something that makes sense to a physicist or an engineer)



<https://assumptionsofphysics.org/>

Assumptions
of
Physics

Axiom 1.7 (Axiom of mixture). *The statistical mixture of two ensembles is an ensemble. Formally, an ensemble space $\mathcal{E}$ is equipped with an operation $+: [0, 1] \times \mathcal{E} \times \mathcal{E} \rightarrow \mathcal{E}$ called **mixing**, noted with the infix notation $pa + \bar{p}b$, with the following properties:*

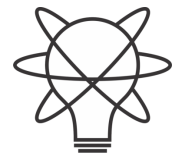
- **Continuity:** the map $+(p, a, b) \rightarrow pa + \bar{p}b$ is continuous (with respect to the product topology of $[0, 1] \times \mathcal{E} \times \mathcal{E}$)
- **Identity:** $1a + 0b = a$
- **Idempotence:** $pa + \bar{p}a = a$ for all $p \in [0, 1]$
- **Commutativity:** $pa + \bar{p}b = \bar{p}b + pa$ for all $p \in [0, 1]$
- **Associativity:** $p_1e_1 + \bar{p}_1 \left(\left(\frac{p_3}{\bar{p}_1} \right) e_2 + \frac{p_3}{\bar{p}_1} e_3 \right) = \bar{p}_3 \left(\frac{p_1}{\bar{p}_3} e_1 + \left(\frac{p_1}{\bar{p}_3} \right) e_2 \right) + p_3e_3$ where $p_1 + p_3 \leq 1$

Informal intuitive requirement

(something that makes sense to a physicist or an engineer)

Formal definition

(something a mathematician will find precise)



<https://assumptionsofphysics.org/>

Assumptions
of
Physics

Axiom 1.7 (Axiom of mixture). *The statistical mixture of two ensembles is an ensemble. Formally, an ensemble space $\mathcal{E}$ is equipped with an operation $+$: $[0, 1] \times \mathcal{E} \times \mathcal{E} \rightarrow \mathcal{E}$ called **mixing**, noted with the infix notation $pa + \bar{p}b$, with the following properties:*

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Justification. This axiom captures the ability to create a mixture merely by selecting between the output of different processes. Let e_1 and e_2 be two ensembles that represent the output of two different processes P_1 and P_2 . Let a selector S_p be a process that outputs two symbols, the first with probability p and the second with probability $\bar{p}$. Then we can create another process P that, depending on the selector, outputs either the output of P_1 or P_2 . All possible preparations of such a procedure will form an ensemble. Therefore we are justified in equipping an ensemble space with a mixing operation that takes a real number from zero to one, and two ensembles.

Given that mixing represents an experimental relationship, and all experimental rela-

Informal intuitive requirement

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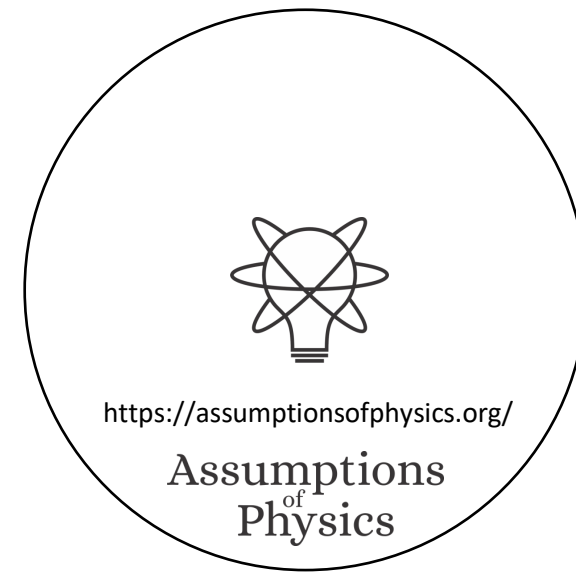
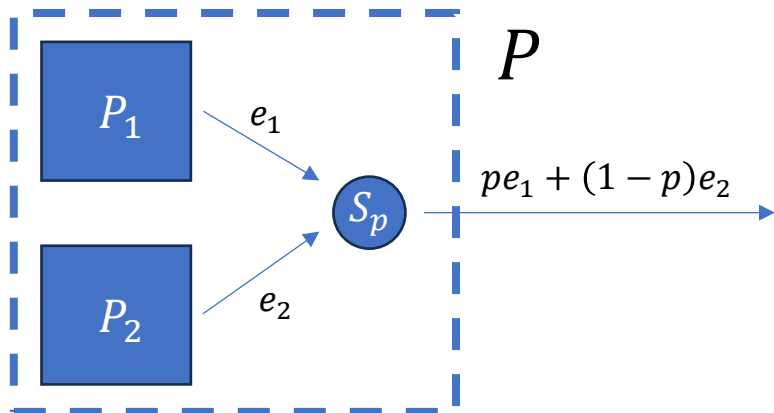
Formal definition

(something a mathematician will find precise)

Show that the formal definition

follows from the intuitive requirement

Clear idea of what is being modelled



Axiom 1.7 (Axiom of mixture). *The statistical mixture of two ensembles is an ensemble. Formally, an ensemble space $\mathcal{E}$ is equipped with an operation $+: [0, 1] \times \mathcal{E} \times \mathcal{E} \rightarrow \mathcal{E}$ called mixing, noted with the infix notation $pa + \bar{p}b$, with the following properties:*

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If $p = 1$, the output of P will always be the output of P_1 . This justifies the identity property. If P_1 and P_2 are the same process, then the output of P will always be the output of P_1 . This justifies the idempotence property. The order in which the processes are given does not matter as long as the same probability is matched to the same process. The process P is identical under permutation of P_1 and P_2 . This justifies commutativity. If we are mixing three processes P_1 , P_2 and P_3 , as long as the final probabilities are the same, it does not matter if we mix P_1 and P_2 first or P_2 and P_3 . This justifies associativity. $\square$

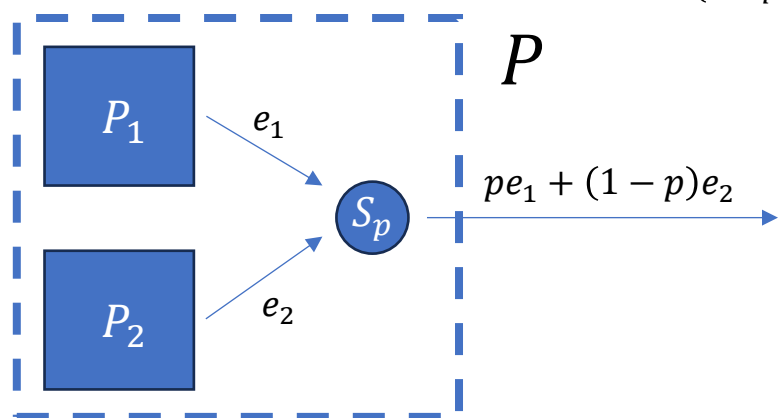
Informal intuitive requirement

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Formal definition

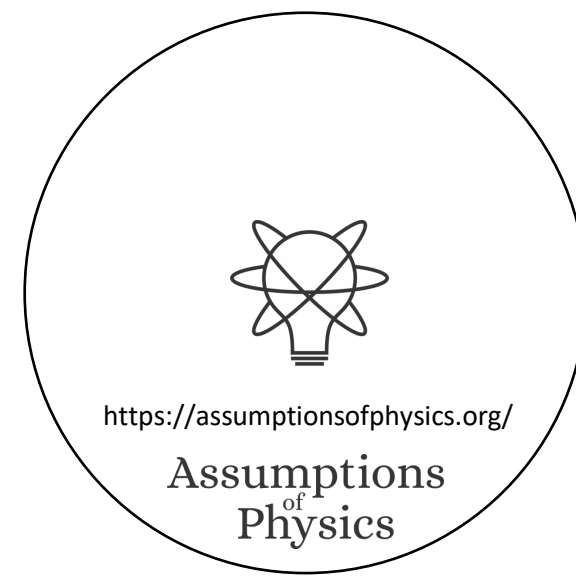
(something a mathematician will find precise)

Show that the formal definition follows from the intuitive requirement



$$(1 - p_1) \left(1 - \frac{p_3}{1 - p_1} \right) = 1 - p_1 - p_3 = (1 - p_3) \left(1 - \frac{p_1}{1 - p_3} \right)$$

Properties justified by understanding the model



Axiom 1.7 (Axiom of mixture). *The statistical mixture of two ensembles is an ensemble. Formally, an ensemble space $\mathcal{E}$ is equipped with an operation $+: [0, 1] \times \mathcal{E} \times \mathcal{E} \rightarrow \mathcal{E}$ called **mixing**, noted with the infix notation $pa + \bar{p}b$, with the following properties:*

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Informal intuitive requirement

(something that makes sense to a physicist or an engineer)

Formal definition

(something a mathematician will find precise)

Show that the formal definition

follows from the intuitive requirement

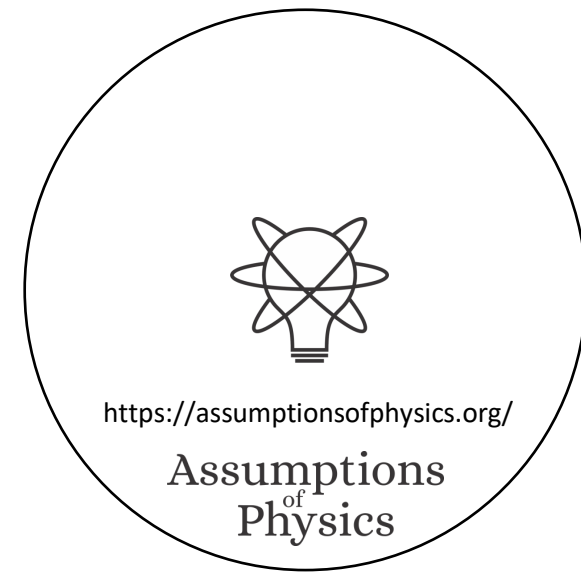
There is no question as to what the math describes

The properties are justified by, are a consequence of, what the model describes

Every math proof can be understood physically

$\Rightarrow$ The math describes and only describes physically meaningful concepts

It's physical mathematics

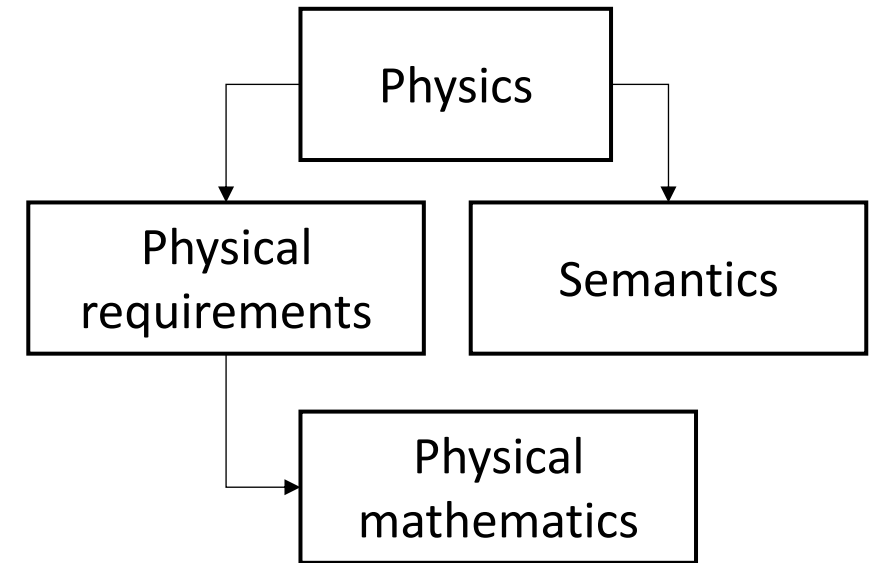


The goal of physical mathematics is to recover ALL mathematical structures used in physics from clear physical requirements

Clarify realm of applicability of each mathematical structure

Perfect map between math and physics

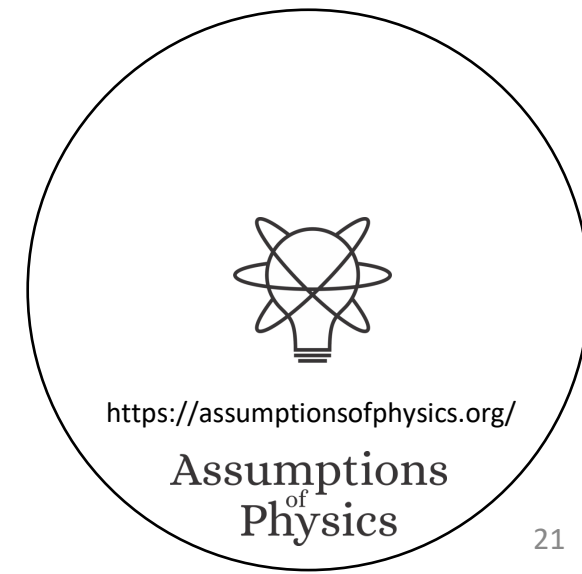
Provide a generalized structure for all physical theories



It's a better way to do physics

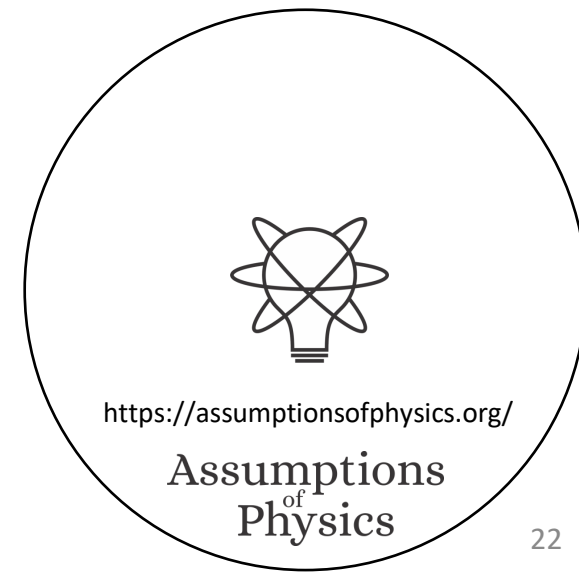
It forces you to think a lot deeper about physics, what it means to have an experimentally based theory, what it means to define a state, what is entropy or energy, ...

It's not just a "math thing"

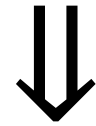


Takeaway

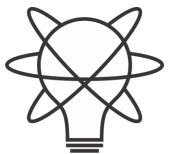
- It is possible to define the starting points of our physical theories so that they are both mathematically precise and physically meaningful (and philosophically consistent)
- Physical mathematics: mathematical structures justified by the physics
 - Justifications provide a new standard of rigor for physical theories
- Only mathematical structures that are justified by unavoidable physical requirements can serve as truly foundational structures
 - **All** physical theories must satisfy those requirements
- $\Rightarrow$ Foundations of physics is not “guessing” what the physical world is “made of,” but articulating in a precise way what physical theories are



Logic of experimental verifiability



topologies and σ -algebras



<https://assumptionsofphysics.org/>

Assumptions
of
Physics

Principle of scientific objectivity. Science is universal, non-contradictory and evidence based.

⇒ Science is about statements that are associated to experimental tests

Statements must be either true or false for everybody

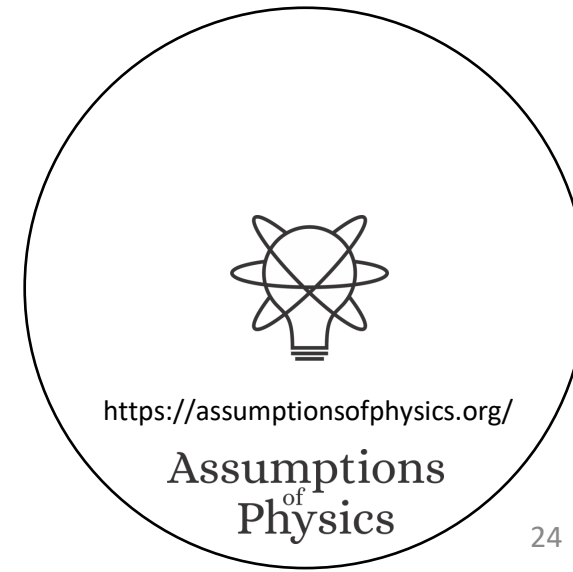
two-valued logic

Statement	Test Result
T	SUCCESS (in finite time)
	UNDEFINED
F	UNDEFINED
	FAILURE (in finite time)

Tests may or may not terminate (i.e. may be inconclusive)

three-valued logic

Verifiable statement	Test Result
T	SUCCESS (in finite time)
	UNDEFINED
F	FAILURE (in finite time)



Axioms of logic

Axiom 1.2 (Axiom of context). A **statement** s is an assertion that is either true or false. A **logical context** $\mathcal{S}$ is a collection of statements with well defined logical relationships. Formally, a logical context $\mathcal{S}$ is a collection of elements called statements upon which is defined a function $\text{truth} : \mathcal{S} \rightarrow \mathbb{B}$.

Axiom 1.4 (Axiom of possibility). A **possible assignment** for a logical context $\mathcal{S}$ is a map $a : \mathcal{S} \rightarrow \mathbb{B}$ that assigns a truth value to each statement in a way consistent with the content of the statements. Formally, each logical context comes equipped with a set $\mathcal{A}_{\mathcal{S}} \subseteq \mathbb{B}^{\mathcal{S}}$ such that $\text{truth} \in \mathcal{A}_{\mathcal{S}}$. A map $a : \mathcal{S} \rightarrow \mathbb{B}$ is a possible assignment for $\mathcal{S}$ if $a \in \mathcal{A}_{\mathcal{S}}$.

Axiom 1.9 (Axiom of closure). We can always find a statement whose truth value arbitrarily depends on an arbitrary set of statements. Formally, let $S \subseteq \mathcal{S}$ be a set of statements and $f_{\mathbb{B}} : \mathbb{B}^S \rightarrow \mathbb{B}$ an arbitrary function from an assignment of S to a truth value. Then we can always find a statement $\bar{s} \in \mathcal{S}$ that depends on S through $f_{\mathbb{B}}$.

Lead to standard logic
(i.e. Boolean algebra)

two-valued logic

Axioms of verifiability

Axiom 1.27 (Axiom of verifiability). A **verifiable statement** is a statement that, if true, can be shown to be so experimentally. Formally, each logical context $\mathcal{S}$ contains a set of statements $\mathcal{S}_v \subseteq \mathcal{S}$ whose elements are said to be verifiable. Moreover, we have the following properties:

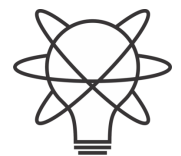
- every certainty $\top \in \mathcal{S}$ is verifiable
- every impossibility $\perp \in \mathcal{S}$ is verifiable
- a statement equivalent to a verifiable statement is verifiable

Axiom 1.31 (Axiom of finite conjunction verifiability). The conjunction of a finite collection of verifiable statements is a verifiable statement. Formally, let $\{s_i\}_{i=1}^n \subseteq \mathcal{S}_v$ be a finite collection of verifiable statements. Then the conjunction $\bigwedge_{i=1}^n s_i \in \mathcal{S}_v$ is a verifiable statement.

Axiom 1.32 (Axiom of countable disjunction verifiability). The disjunction of a countable collection of verifiable statements is a verifiable statement. Formally, let $\{s_i\}_{i=1}^{\infty} \subseteq \mathcal{S}_v$ be a countable collection of verifiable statements. Then the disjunction $\bigvee_{i=1}^{\infty} s_i \in \mathcal{S}_v$ is a verifiable statement.

Lead to intuitionist logic
(i.e. Heyting algebra)

three-valued logic



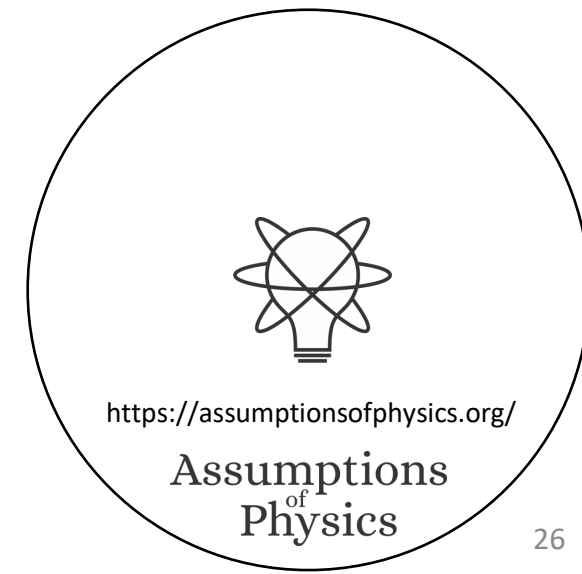
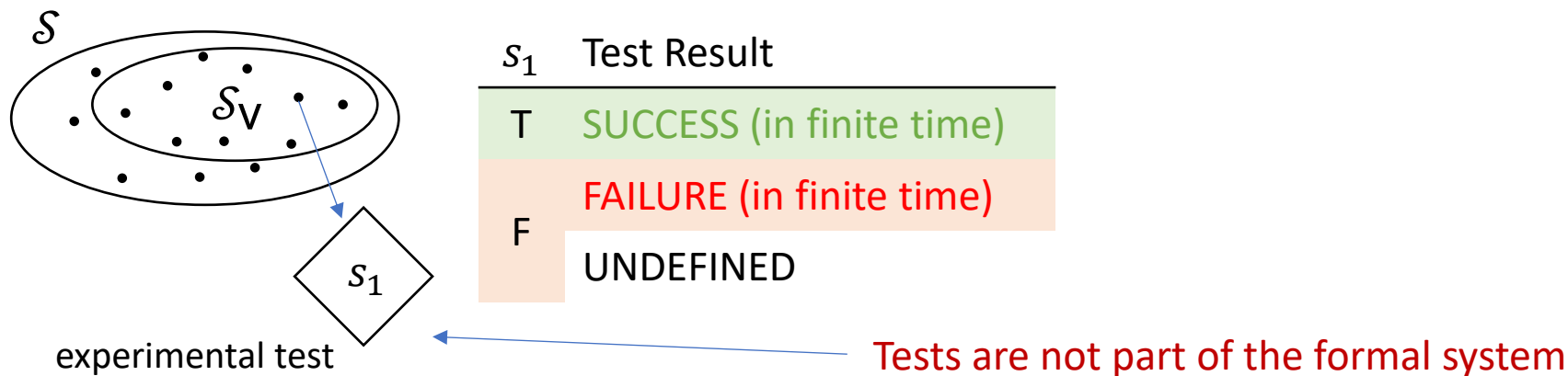
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Assumptions
of
Physics

Axiom 1.27 (Axiom of verifiability). A *verifiable statement* is a statement that, if true, can be shown to be so experimentally. Formally, each logical context $\mathcal{S}$ contains a set of statements $\mathcal{S}_v \subseteq \mathcal{S}$ whose elements are said to be verifiable. Moreover, we have the following properties:

- every certainty $\top \in \mathcal{S}$ is verifiable
- every impossibility $\perp \in \mathcal{S}$ is verifiable
- a statement equivalent to a verifiable statement is verifiable

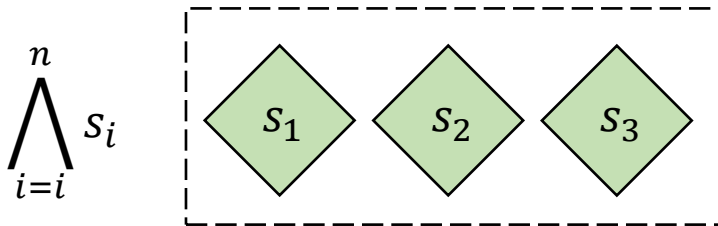
Remark. The **negation or logical NOT** of a verifiable statement is not necessarily a verifiable statement.



Axiom 1.31 (Axiom of finite conjunction verifiability). *The conjunction of a finite collection of verifiable statements is a verifiable statement. Formally, let $\{s_i\}_{i=1}^n \subseteq \mathcal{S}_v$ be a finite collection of verifiable statements. Then the conjunction $\bigwedge_{i=1}^n s_i \in \mathcal{S}_v$ is a verifiable statement.*

Conjunction (AND) of verifiable statements:
check that all tests terminate successfully

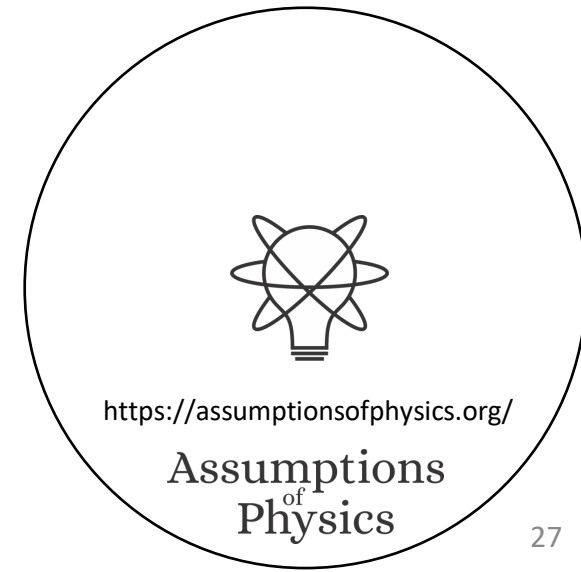
⇒ Only finite conjunction is guaranteed to terminate



All tests must succeed

$\wedge (e_i)$:

1. Run all e_i
2. If all succeed, return SUCCESS
3. Return FAILURE

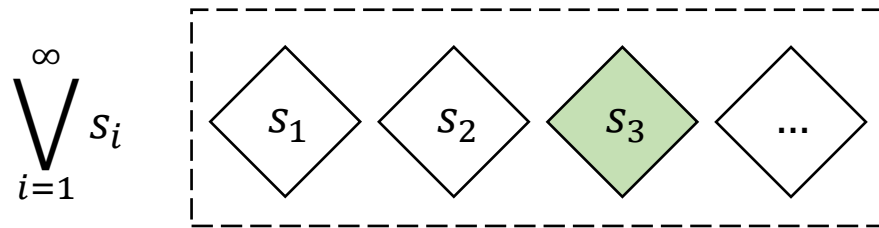


Axiom 1.32 (Axiom of countable disjunction verifiability). *The disjunction of a countable collection of verifiable statements is a verifiable statement. Formally, let $\{s_i\}_{i=1}^{\infty} \subseteq \mathcal{S}_v$ be a countable collection of verifiable statements. Then the disjunction $\bigvee_{i=1}^{\infty} s_i \in \mathcal{S}_v$ is a verifiable statement.*

Disjunction (OR) of verifiable statements:
check that ONE test terminates successfully

watch out for non-termination!

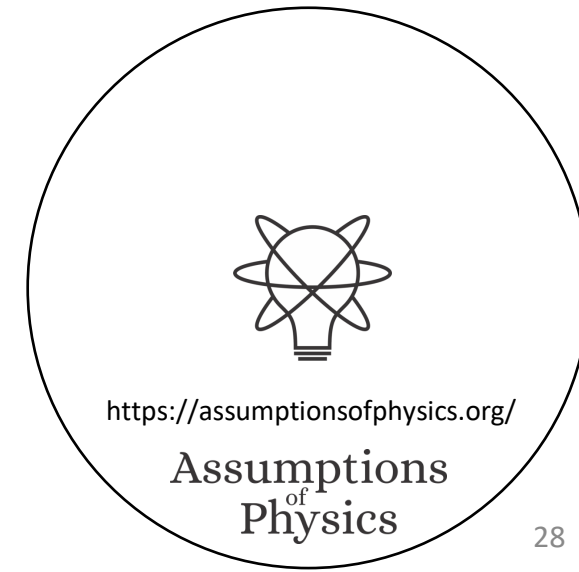
⇒ Only countable disjunction can reach all tests



One successful test is sufficient

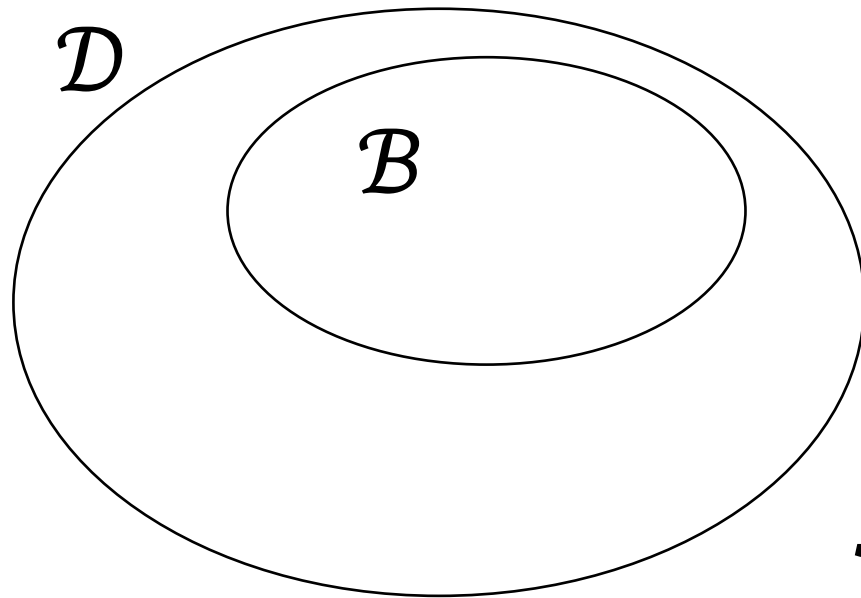
$\vee (e_i)$:

1. Initialize n to 1
2. For each $i = 1 \dots n$
 - a) Run e_i for n seconds
 - b) If e_i succeeds, return SUCCESS
3. Increment n and go to 2



Definition 1.34. Given a set $\mathcal{D}$ of verifiable statements, $\mathcal{B} \subseteq \mathcal{D}$ is a **basis** if the truth values of $\mathcal{B}$ are enough to deduce the truth values of the set. Formally, all elements of $\mathcal{D}$ can be generated from $\mathcal{B}$ using finite conjunction and countable disjunction.

Definition 1.35. An **experimental domain** $\mathcal{D}$ represents a set of verifiable statements that can be tested and possibly verified in an indefinite amount of time. Formally, it is a set of statements, closed under finite conjunction and countable disjunction, that includes precisely the certainty, the impossibility, and a set of verifiable statements that can be generated from a countable basis.

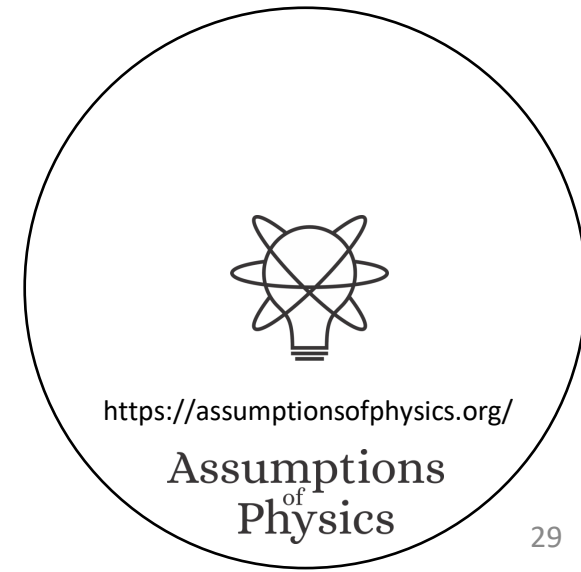


$$\mathcal{B} = \{e_1, e_2, e_3, \dots\}$$

Countable basis

Only finite conjunction and countable disjunction

$$s_1 = (e_1 \vee e_3) \wedge e_2 \dots$$

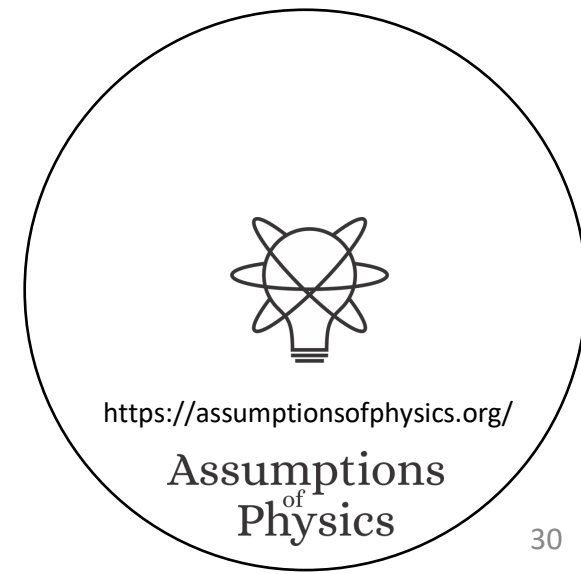


Definition 1.36. *The **theoretical domain** $\bar{\mathcal{D}}$ of an experimental domain $\mathcal{D}$ is the set of statements constructed from $\mathcal{D}$ to which we can associate a test regardless of termination. We call **theoretical statement** a statement that is part of a theoretical domain. More formally, $\bar{\mathcal{D}}$ is the set of all statements generated from $\mathcal{D}$ using negation, finite conjunction and countable disjunction.*

Extend the domain to include all statements that are associated with a test, regardless of termination.

All statements depend on the verifiable statements
(which depend on the basis)

No new information is captured



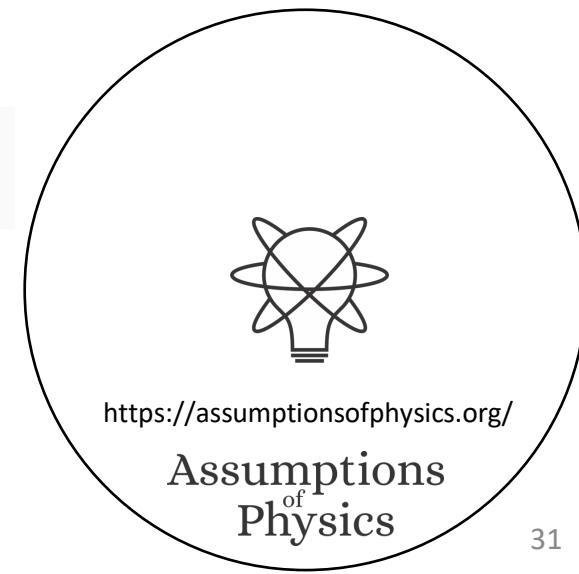
Definition 1.47. A *possibility* for an experimental domain $\mathcal{D}$ is a statement $x \in \bar{\mathcal{D}}$ that, when true, determines the truth value for all statements in the theoretical domain. Formally, $x \neq \perp$ and for each $s \in \bar{\mathcal{D}}$, either $x \leq s$ or $x \not\leq s$. The **full possibilities**, or simply the **possibilities**, X for $\mathcal{D}$ are the collection of all possibilities.

A possibility of a domain is a statement that picks one assignment

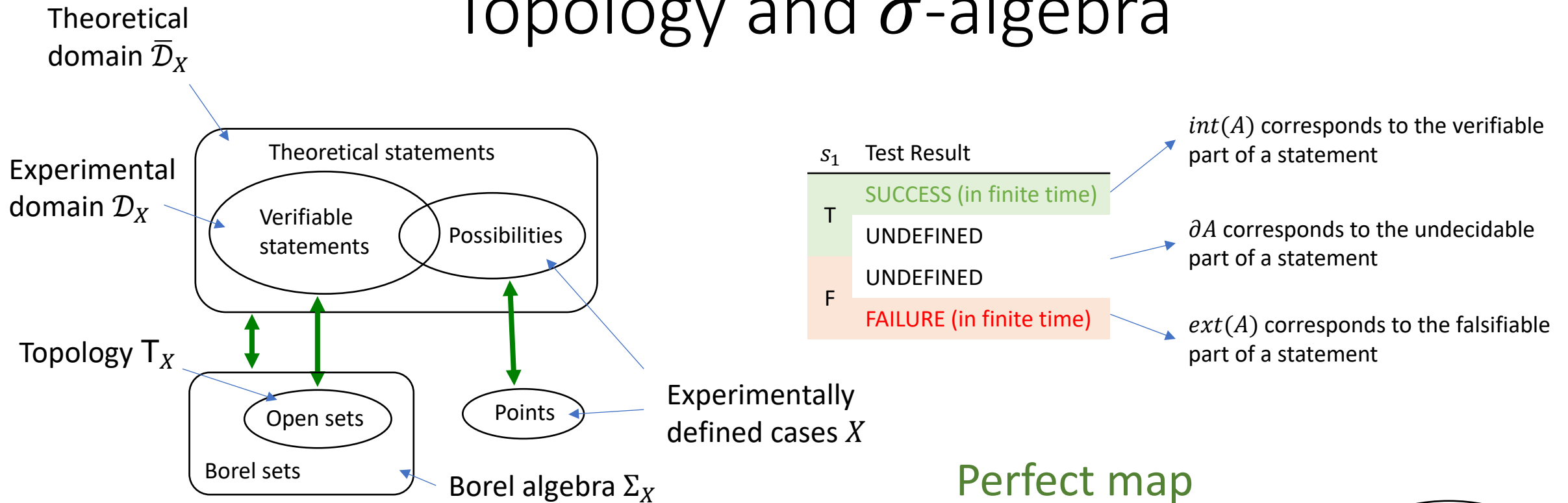
Possibilities: experimentally defined alternative cases defined by the verifiable cases

s_1	s_2	s_3	...	x_1	x_2	x_3	x_4
T	T	F	T	T	F	F	F
F	F	T	T	F	T	F	F
F	T	T	F	F	F	T	F
T	F	F	T	F	F	F	T

Proposition 1.48. Let $\mathcal{D}$ be an experimental domain. A possibility for $\mathcal{D}$ is any minterm of a basis that is not impossible.



Topology and σ -algebra

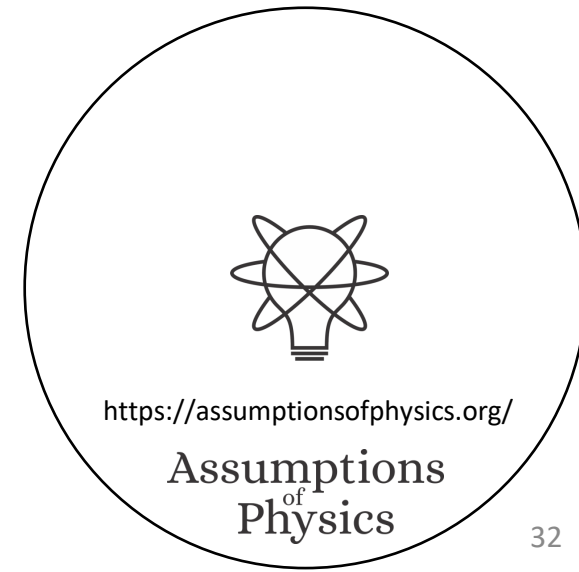


Perfect map
between math
and physics

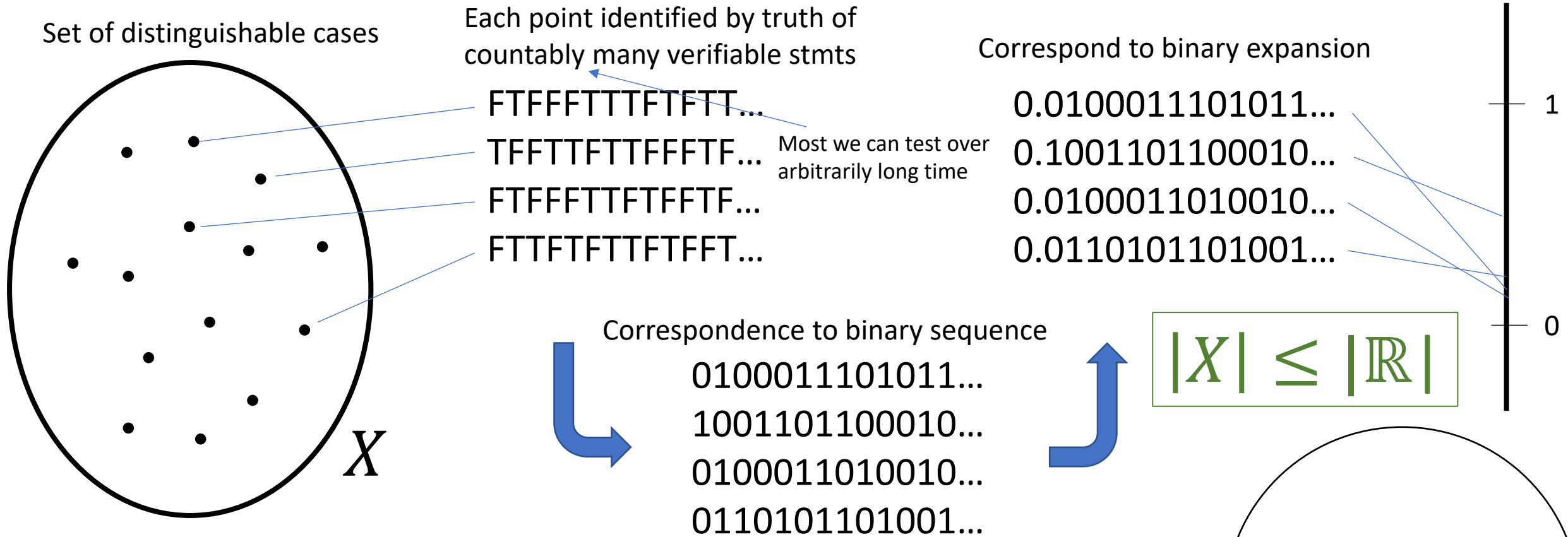
Open set $(509.5, 510.5) \Leftrightarrow$ Verifiable “the mass of the electron is 510 ± 0.5 KeV”

Closed set $[510] \Leftrightarrow$ Falsifiable “the mass of the electron is exactly 510 KeV”

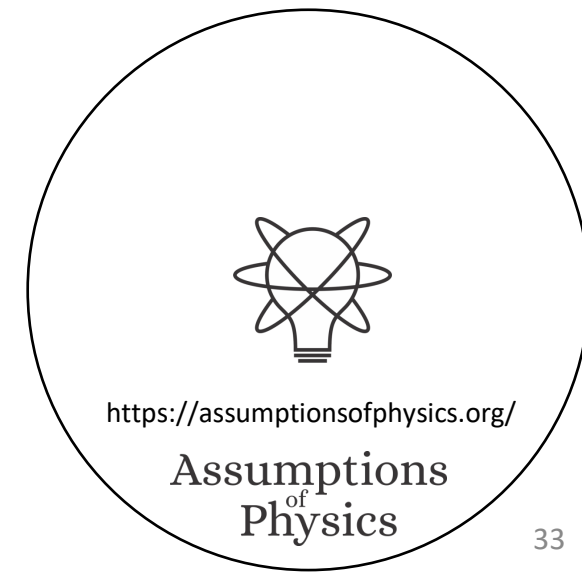
Borel set $\mathbb{Q}$ ($int(\mathbb{Q}) \cup ext(\mathbb{Q}) = \emptyset$) $\Leftrightarrow$ Theoretical “the mass of the electron in KeV is a rational number” (undecidable)



Maximum cardinality of distinguishable cases $\mathbb{R}$



- Sets with greater cardinality (e.g. the set of all discontinuous functions from $\mathbb{R}$ to $\mathbb{R}$) cannot represent physical objects
- Issues about higher infinities (e.g. large cardinals) are not relevant, but those surrounding the continuum hypothesis may be



Power set vs Borel algebra

Using the set $2^{\mathbb{R}^n}$ of all possible subsets for $\mathbb{R}^n$ is problematic

Notion of size (i.e. measure)
cannot be defined on all sets

Using non-measurable sets leads
to the Banach-Tarski paradox

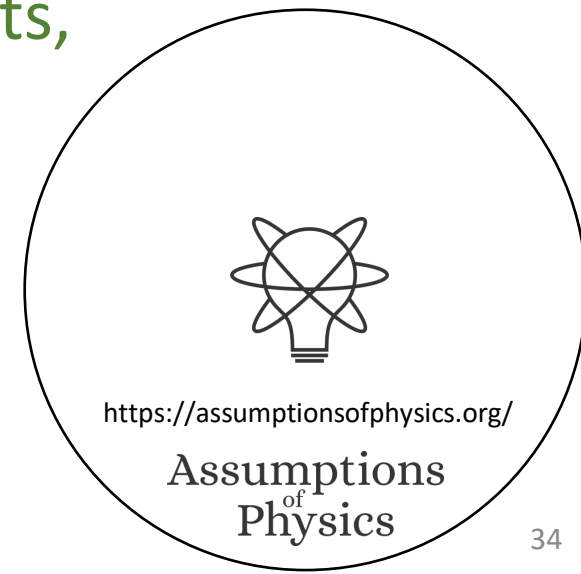


wikipedia

These problems are avoided if we restrict ourselves to Borel sets

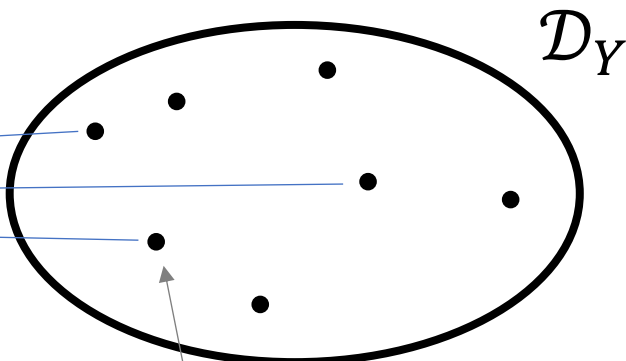
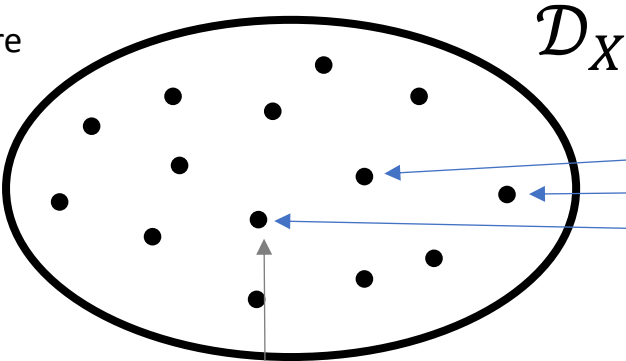
⇒ If we restrict ourselves to experimentally definable objects,
these paradoxes are avoided

Physical mathematics can give insight
to these foundational issues in mathematics



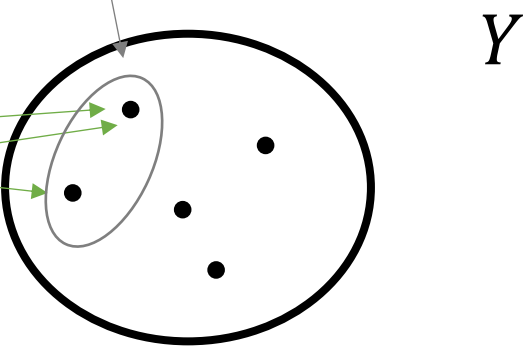
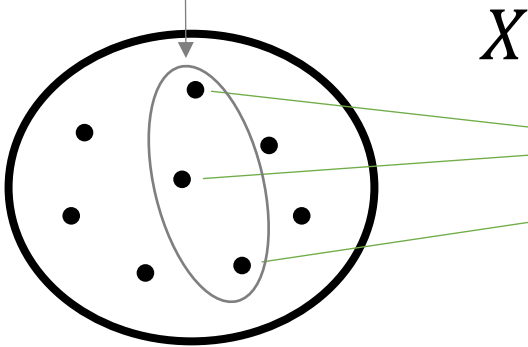
An **inference relationship** is a map $\mathcal{r}: \mathcal{D}_Y \rightarrow \mathcal{D}_X$ such that $\mathcal{r}(s) \equiv s$

e.g. the water temperature
is between 0 and 0.52
Celsius or between
7.6 and 9.12 Celsius

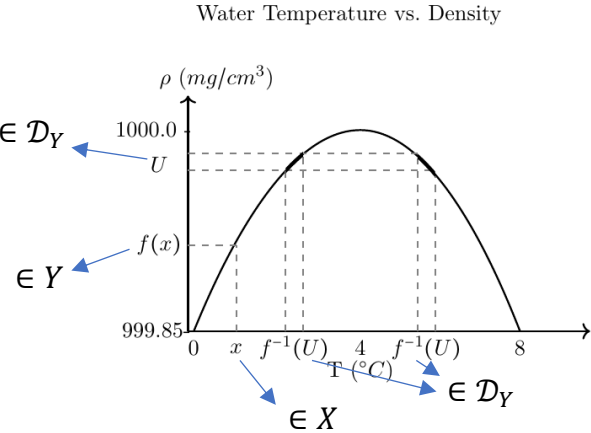


e.g. the water density is
between 999.8 and
999.9 kg/m³

e.g. the water temperature
is exactly 4 Celsius

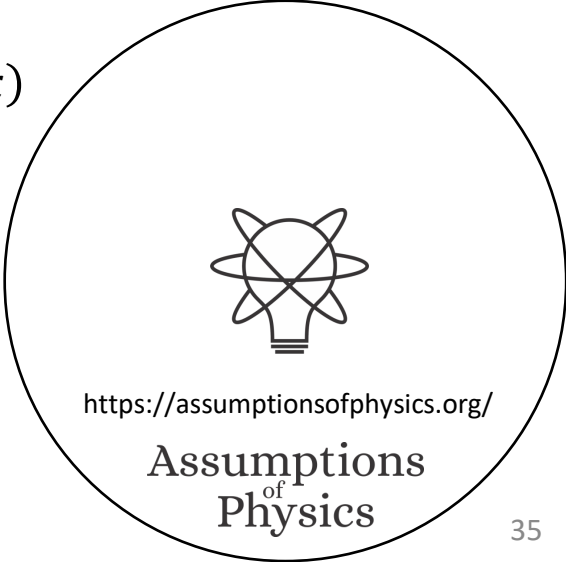


e.g. the water density is
exactly 1 kg/m³

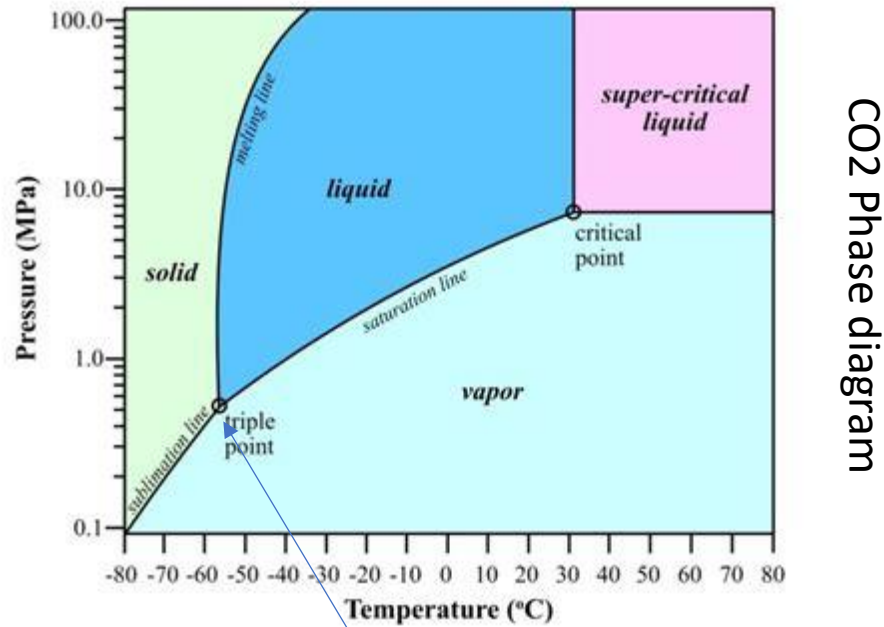


A **causal relationship** is a map $f: X \rightarrow Y$ such that $x \preceq f(x)$

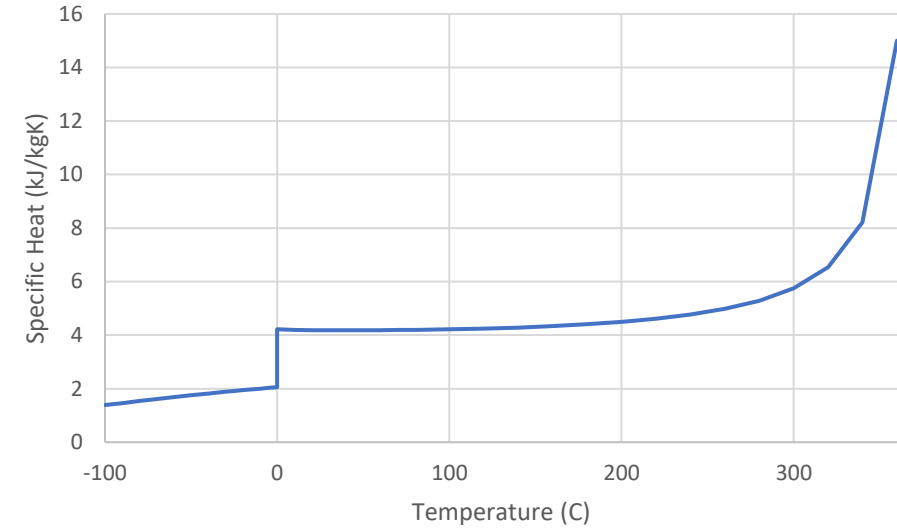
- 1) Two domains admit an inference relationship if and only if they admit a causal relationship
- 2) The causal relationship must be a continuous map in the natural topology



Why functions are well-behaved



Water heat capacity



data from <https://www.engineeringtoolbox.com>

Topological continuity $\neq$ Analytical continuity

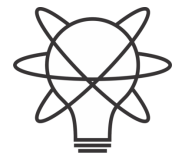
We can verify we are at the triple point

We measure the equilibrium of three phases,
not the pressure/temperature

Before 2019, the triple point of water was used to define the kelvin, the base unit of thermodynamic temperature in the International System of Units (SI).^[3] The kelvin was defined so that the triple point of water is exactly 273.16 K, but that changed with the 2019 revision of the SI, where the kelvin was redefined so that the Boltzmann constant is exactly $1.380\,649 \times 10^{-23} \text{ J} \cdot \text{K}^{-1}$, and the triple point of water became an experimentally measured constant.

from https://en.wikipedia.org/wiki/Triple_point

Analytical discontinuity
can only happen in
regions that are
experimentally decidable

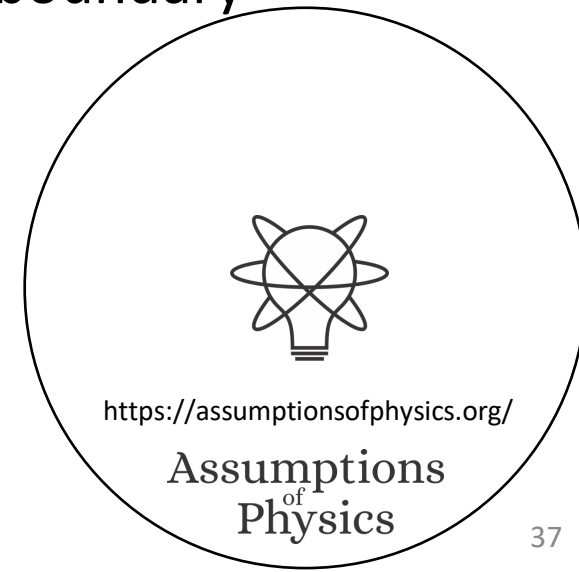


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Assumptions
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Physics

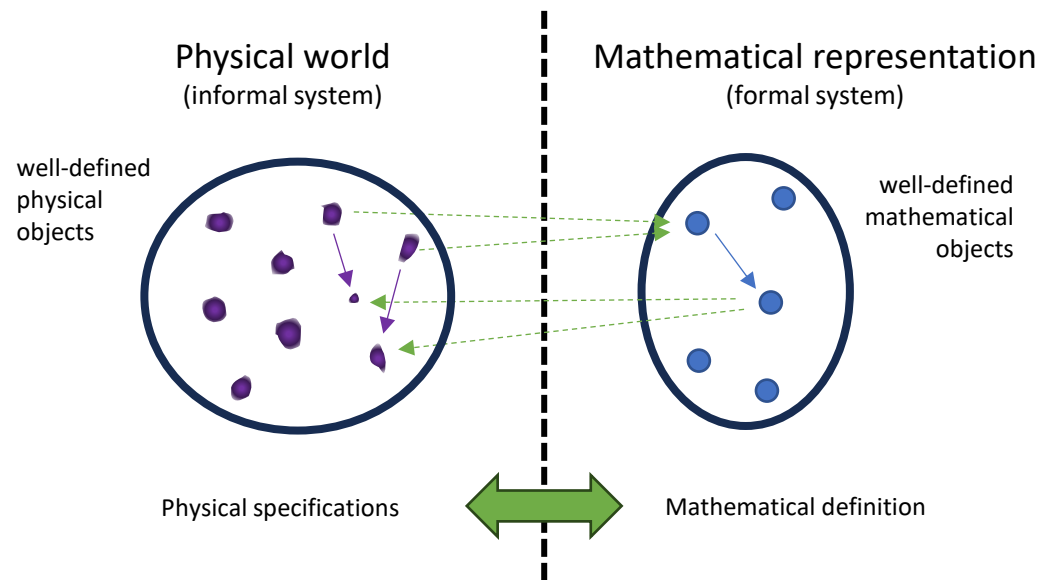
Takeaway

- Requiring experimental verifiability in a physical theory leads to topologies and σ -algebras
 - Open sets correspond to verifiable statements, continuous functions preserve experimental verifiability, Borel sets correspond to statements associated with tests, ...
- All proofs can be understood as describing arguments on experimental verifiability
 - Limits (truth sequences of verifiable statements become constants), topological distinguishability (experimental distinguishability), interior/exterior/boundary (verifiable/falsifiable/undecidable), ...
- Further constructions become more meaningful
 - Probability measure defined on σ -algebra: we assign probability to statements with a test; topological groups: transformations we can experimentally identify/define; ...



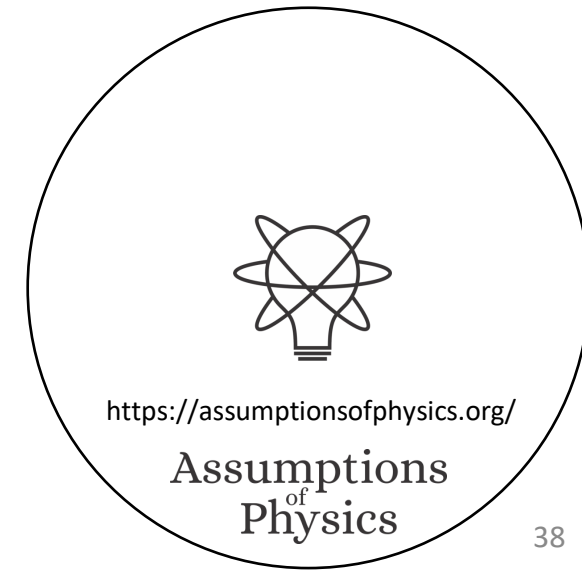
It is possible to develop a foundation of physics that is both mathematically rigorous and physically meaningful

The mathematical definitions ARE the physical requirements and assumptions



No issues of “interpretations”

Clear realm of applicability of
mathematical tools



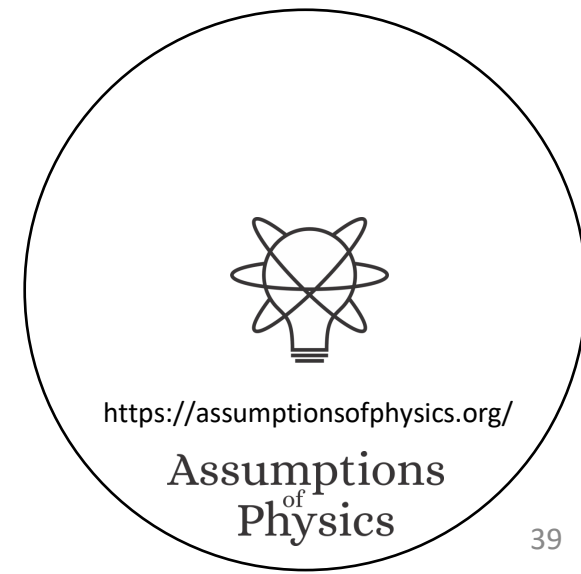
There is no “just math”

Either the math represents
physical objects, then it's
describing physics

Or it doesn't, and therefore it
should be stripped away from
the physical theory

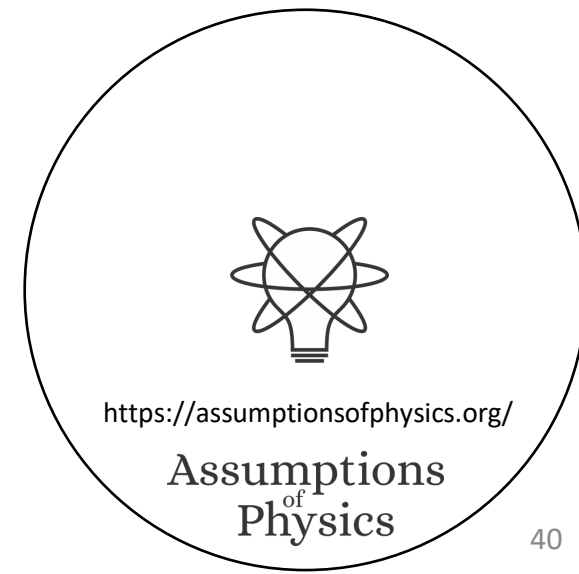
Only by understanding the full details of
the math and physics (and philosophy)
can you make that determination

If you do not know what the well-ordering of the reals is, you are precisely a person that cannot determine whether it is physically significant or not



Wrapping it up

- Assumptions of Physics: different approach to the foundations of physics
 - No interpretations, no theories of everything: physically meaningful starting points from which we can rederive the laws and the mathematical frameworks they need
 - Physical theories are models
 - Need to clarify exactly what the realm of applicability of each model is
- Physical mathematics: derive the math required from physical requirements
 - In physics, mathematics is used to model physical systems, therefore we need mathematics that is designed specifically for that purpose
- You need to start at the lowest level of mathematics
 - Rigor, precision, meaning, correctness cannot be “sprinkled on top”
 - “Big systems that work evolve from small systems that work, never from big systems that do not work” (Gall’s law)



Assumptions of Physics is an open research program

Our main output is an open access book: <https://assumptionsofphysics.org/book/>

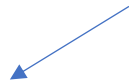
All our material is developed on GitHub: <https://github.com/assumptionsofphysics>

One YouTube channel dedicated to publicize results: <https://www.youtube.com/user/gcarcassi>

Another YouTube channel dedicated to research: <https://www.youtube.com/@AssumptionsofPhysicsResearch>

Activities coordinated through a Discord server (contact info@assumptionsofphysics.org for an invite)

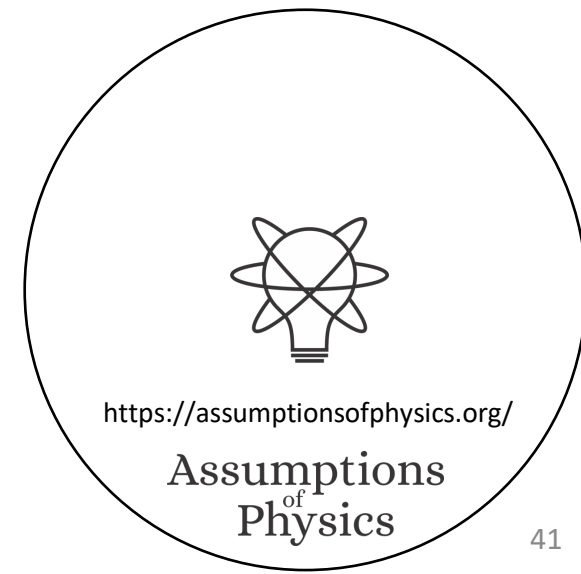
Livestream
discussions

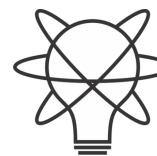


Always looking for experts to gain insights and/or help

Always looking for collaborations

Always looking for editors/journals/conferences that are sympathetic to the mission





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