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Assumptions of Physics
Summer School 2026

Results from Reverse Physics

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Reverse Physics

"When the math is derived from the right physical assumptions, the physical assumptions can be derived from the math."

Overview of Reverse Physics results for classical mechanics

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Assumptions of Physics

Assumption IR (Infinitesimal Reducibility). *The state of the system is reducible to the state of its infinitesimal parts. That is, specifying the state of the whole system is equivalent to specifying the state of its parts, which in turn is equivalent to specifying the state of its subparts and so on.*

Classical state is a distribution over the states of infinitesimal parts

Recovers pairs of conjugate variables for each DOF, frame-invariant entropy and count of states

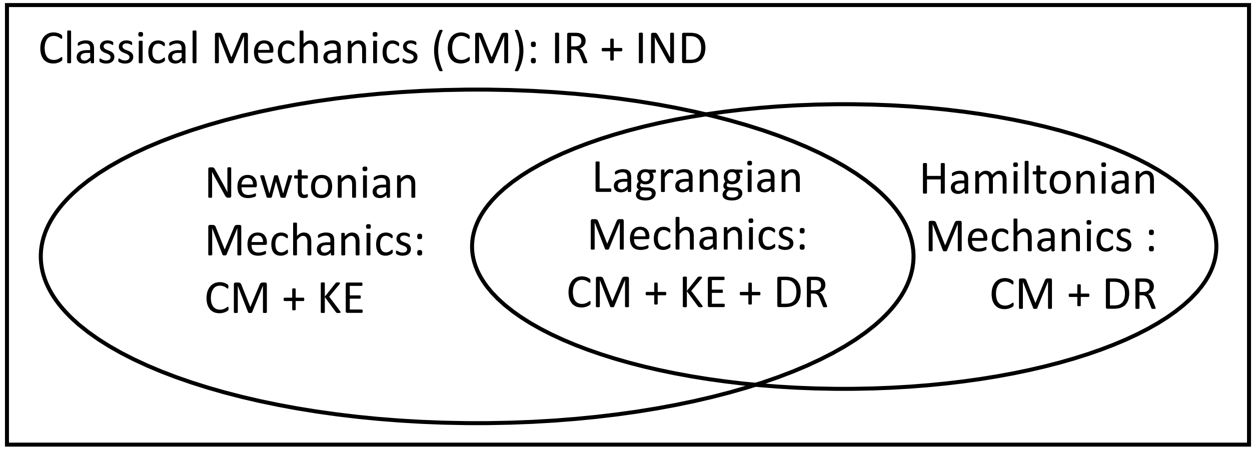
Assumption IND (Independent DOFs). *The system is decomposable into independent degrees of freedom. That is, the variables that describe the state can be divided into groups that have independent definition, units and count of states.*

Assumption DR (Determinism and Reversibility). *The system undergoes deterministic and reversible evolution. That is, specifying the state of the system at a particular time is equivalent to specifying the state at a future (determinism) or past (reversibility) time.*

Conservation of entropy/count of states and DOF independence recovers Hamiltonian evolution

Links trajectories in space with dynamics, recovering inertia, masses and scalar/vector potential forces

Assumption KE (Kinematic Equivalence). *The kinematics of the system is sufficient to reconstruct its dynamics and vice-versa. That is, specifying the motion of the system is equivalent to specifying its state and evolution.*



Assumptions for classical mechanics

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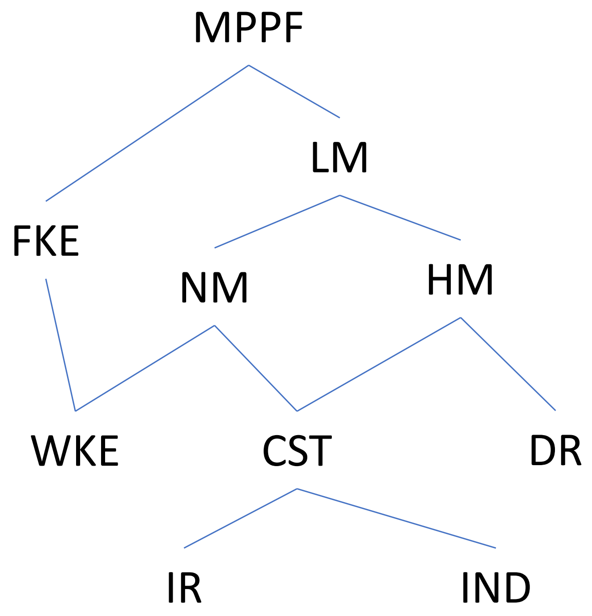
Assumptions of Physics

Classical conditions and their relationships



Core assumptions

IR: Infinitesimal Reducibility
IND: INDependent DOF
W/FKE: Weak/Full Kinematic Equivalence
DR: Determinism+Reversibility

Physical theories

CST: Classical STates
NM: Newtonian Mechanics
HM: Hamiltonian Mechanics
LM: Lagrangian Mechanics
MPPF: Massive Particles under Potential Forces

Each assumption is an equivalence class of multiple conditions

There is a linear relationship between conjugate momentum and velocity (FKE-LIN)

At each position, there exists a local inertial frame (FKE-INER)

The position fully defines the units of all state variables, and therefore an invertible transformation between momentum and velocity (FKE-UNIT)

Density expressed in velocity at the same position is proportional to the density over states (FKE-PROP)

Uniform distributions along momentum correspond to uniform distributions along velocity (FKE-UNIF)

The system under study is a massive particle under scalar and vector potential forces (FKE-POT)

The Jacobian of the transformation between state variables and kinematic variables is a non-singular function of position only. (FKE-NSIN)

Densities over phase space can be expressed in terms of position and velocity by rescaling the value at each point: $\rho(x^i, v^j)|J(x^i)| = \rho(q^i, p_j)$. (FKE-DEN)

Areas and volumes in phase space can be expressed in kinematic variables, and the transformation depends on position only: $dx^1 \cdots dx^n \, dv^1 \cdots dv^n = |J(x^i)| \, dq^1 \cdots dq^n \, dp_1 \cdots dp_n$. (FKE-VOL)

The symplectic form ω_{ab} can be expressed in kinematic variables, and its components are a linear function of velocity. (FKE-SYMP)

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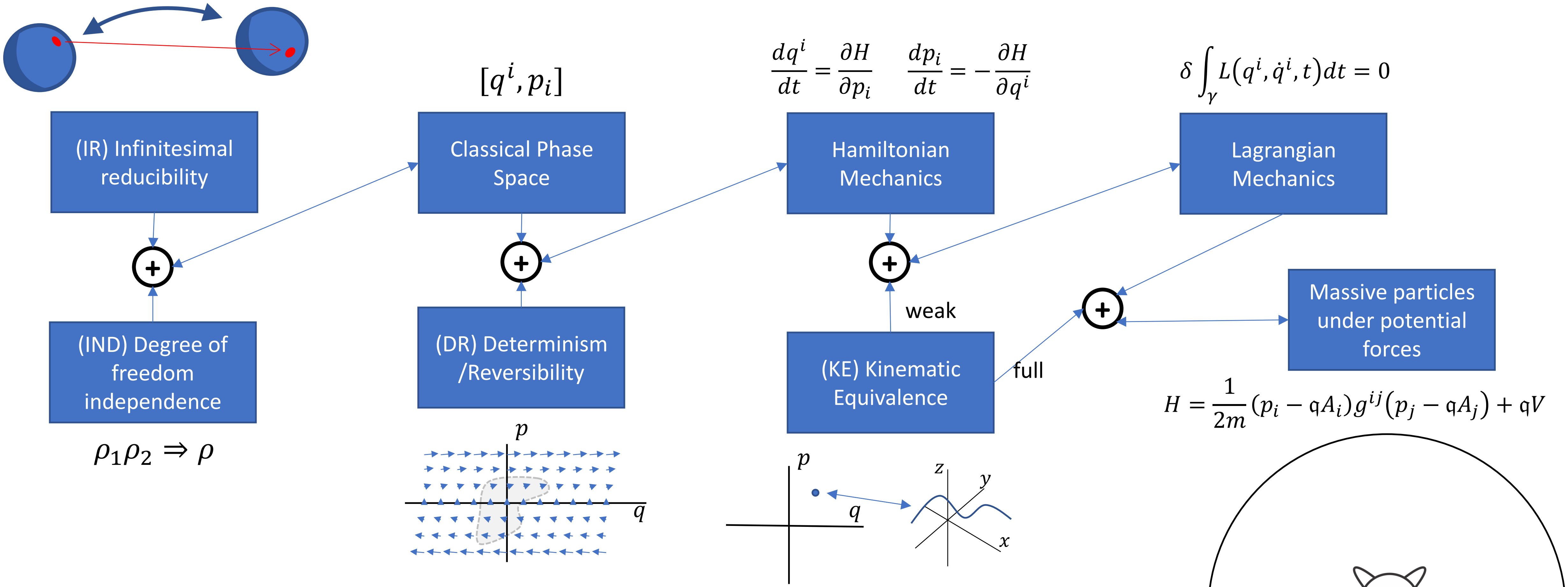
Assumptions of Physics

https://assumptionsofphysics.org/book/AssumptionsOfPhysicsV3.0.pdf#section.1.1.4

Assumptions of classical mechanics



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Assumptions of Physics

Principle of least action - one DOF

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Required background

- Hamiltonian mechanics
  - Hamilton’s equations: $\frac{dq}{dt} = \frac{\partial H}{\partial p} \qquad \frac{dp}{dt} = -\frac{\partial H}{\partial q}$
- Lagrangian mechanics
  - Action is the integral of the Lagrangian over a given path: $\mathcal{A}[\gamma] = \int_\gamma L(q(t), \dot{q}(t), t)dt$
  - Actual evolutions are the paths that make the action stationary: $\delta\mathcal{A} = 0$
  - Lagrangian is the Legendre transform of the Hamiltonian: $L = p\dot{q} - H$
- Vector calculus
  - Given a vector field $\vec{v}$, a field line is a line always tangent to the field
  - The divergence $\vec{\nabla} \cdot \vec{v}$ is the flow of the field through an infinitesimal closed region
  - A divergence-free field $\vec{\nabla} \cdot \vec{B} = 0$ admits a vector potential $\vec{A}$ such that $\vec{B} = \vec{\nabla} \times \vec{A}$

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Assumptions of Physics

$$\text{Action: } \mathcal{A}[\gamma] = \int_\gamma L(q(t), \dot{q}(t), t)dt$$

$$\text{Lagrangian: } L = T - U$$ Does not always work!

Lagrangian for particle with charge q:

$$L = \underbrace{\frac{1}{2}mv^2}_{\text{Kinetic}} + \underbrace{\mathrm{q}\vec{v} \cdot \vec{A}}_{???} - \underbrace{\mathrm{q}V}_{\text{Potential}}$$

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Displacement field

Tells us how states "move"

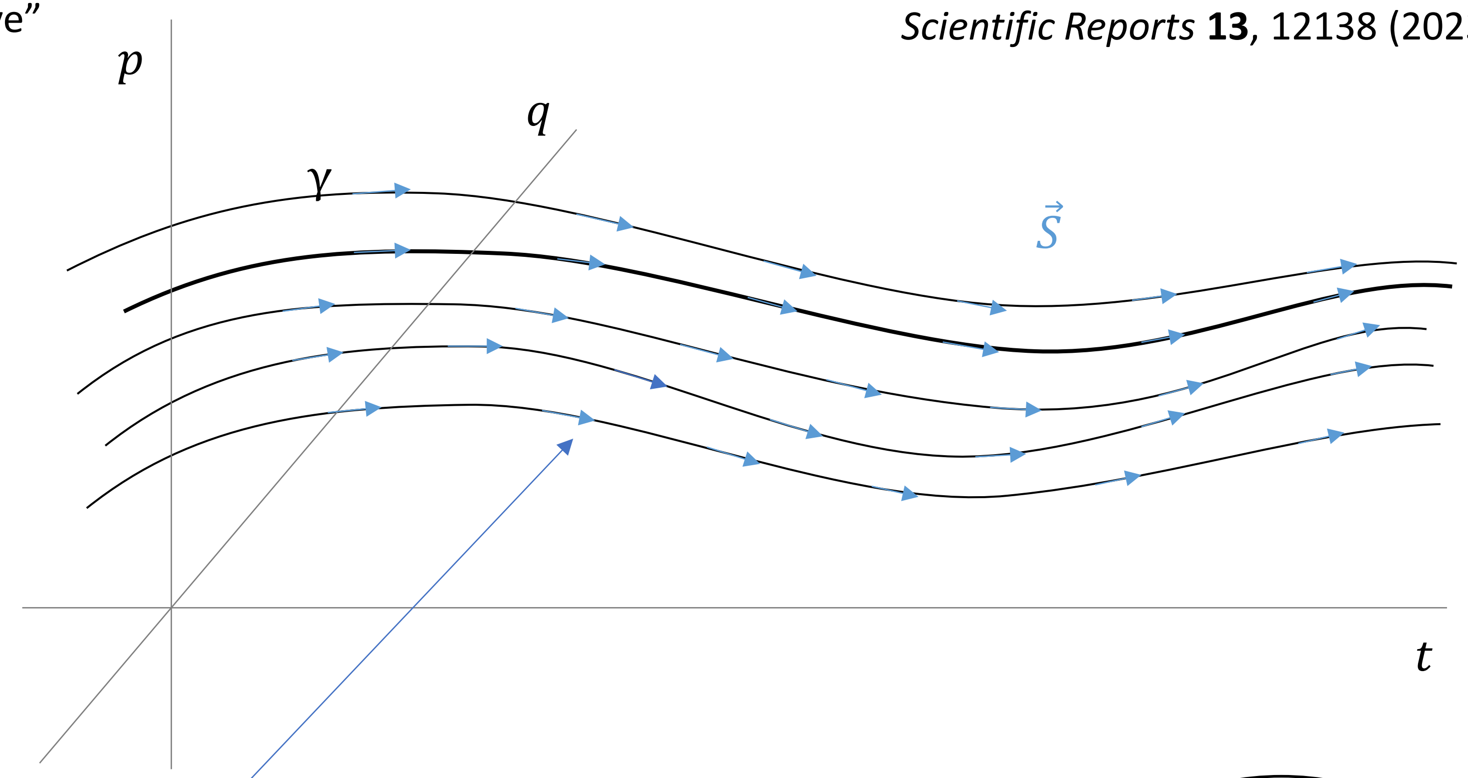
$$\vec{S} = \left[\frac{dq}{dt}, \frac{dp}{dt}, \frac{dt}{dt}\right]$$

Evolution

$$\gamma = [q(t), p(t), t]$$

Time is both a parameter and a variable

Scientific Reports **13**, 12138 (2023)



Evolutions are field lines of the displacement field

Always tangent to the field

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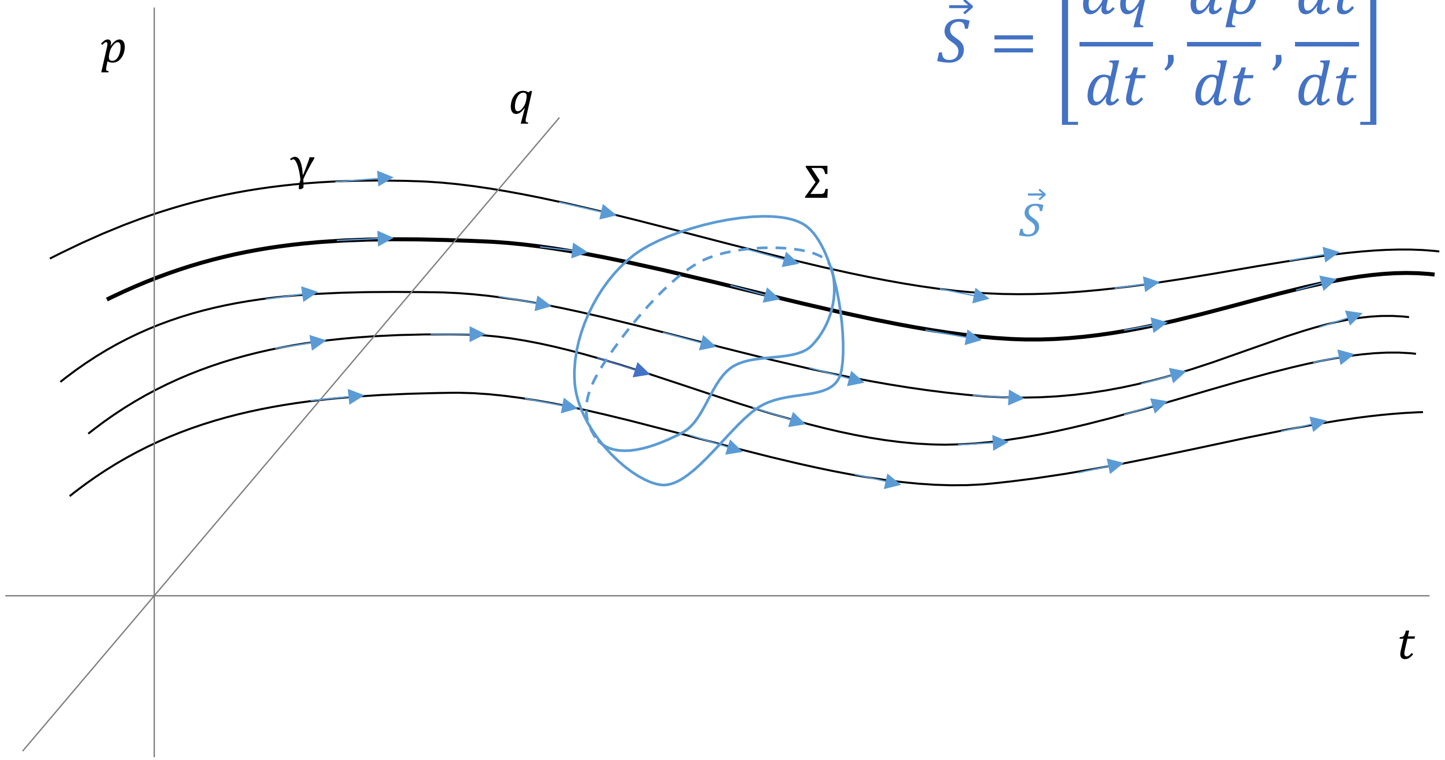
Assumptions of Physics

https://arxiv.org/pdf/2208.06428
https://www.nature.com/articles/s41598-023-39145-y

(DR) Determinism and Reversibility

$$\vec{S} = \left[\frac{dq}{dt}, \frac{dp}{dt}, \frac{dt}{dt}\right]$$



No states are lost or created
(count of states is preserved in time)

$$\oiint_{\Sigma} \vec{S} \cdot d\vec{\Sigma} = 0$$

⇒ Divergence-free displacement

$$\vec{\nabla} \cdot \vec{S} = 0$$

Vector potential

$$\vec{S} = -\vec{\nabla} \times \vec{\theta}$$

Minus sign added to match conventions

Displacement admits a vector potential

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Assumptions of Physics

$$\vec{\theta} = [\theta_q, \theta_p, \theta_t]$$

Fix gauge: $\theta_a \to \theta_a - \partial_a \int_0^p \theta_p dp$

$$\vec{\theta} = [\theta_q, 0, \theta_t]$$

$S_t = \partial_p \theta_q - \partial_q \theta_p = \frac{dt}{dt} = 1$

$$\vec{\theta} = [p, 0, \theta_t]$$

Rename θ_t

$$\vec{\theta} = [p, 0, -H]$$

$$\vec{S} = \left[\frac{dq}{dt}, \frac{dp}{dt}, \frac{dt}{dt}\right]$$

Without loss of generality

$$\vec{S} = -\vec{\nabla} \times \vec{\theta}$$

$$\vec{\theta} = [p, 0, -H]$$

similar to four-momentum

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Assumptions of Physics

$$\vec{S} = \left[\frac{dq}{dt}, \frac{dp}{dt}, \frac{dt}{dt}\right]$$

$$\vec{S} = -\vec{\nabla} \times \vec{\theta} \qquad\qquad \vec{\theta} = [p, 0, -H(q,p)]$$

$$S^q = \frac{dq}{dt} = -\left(\frac{\partial}{\partial p}\theta_t - \frac{\partial}{\partial t}\theta_p\right) = \frac{\partial H}{\partial p}$$

$$S^p = \frac{dp}{dt} = -\left(\frac{\partial}{\partial t}\theta_q - \frac{\partial}{\partial q}\theta_t\right) = -\frac{\partial H}{\partial q}$$

$$S^t = \frac{dt}{dt} = -\left(\frac{\partial}{\partial q}\theta_p - \frac{\partial}{\partial p}\theta_q\right) = 1$$

Hamilton's equations

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Line integral of $\vec{\theta}$

$$\int_\gamma \vec{\theta} \cdot d\vec{\gamma}$$

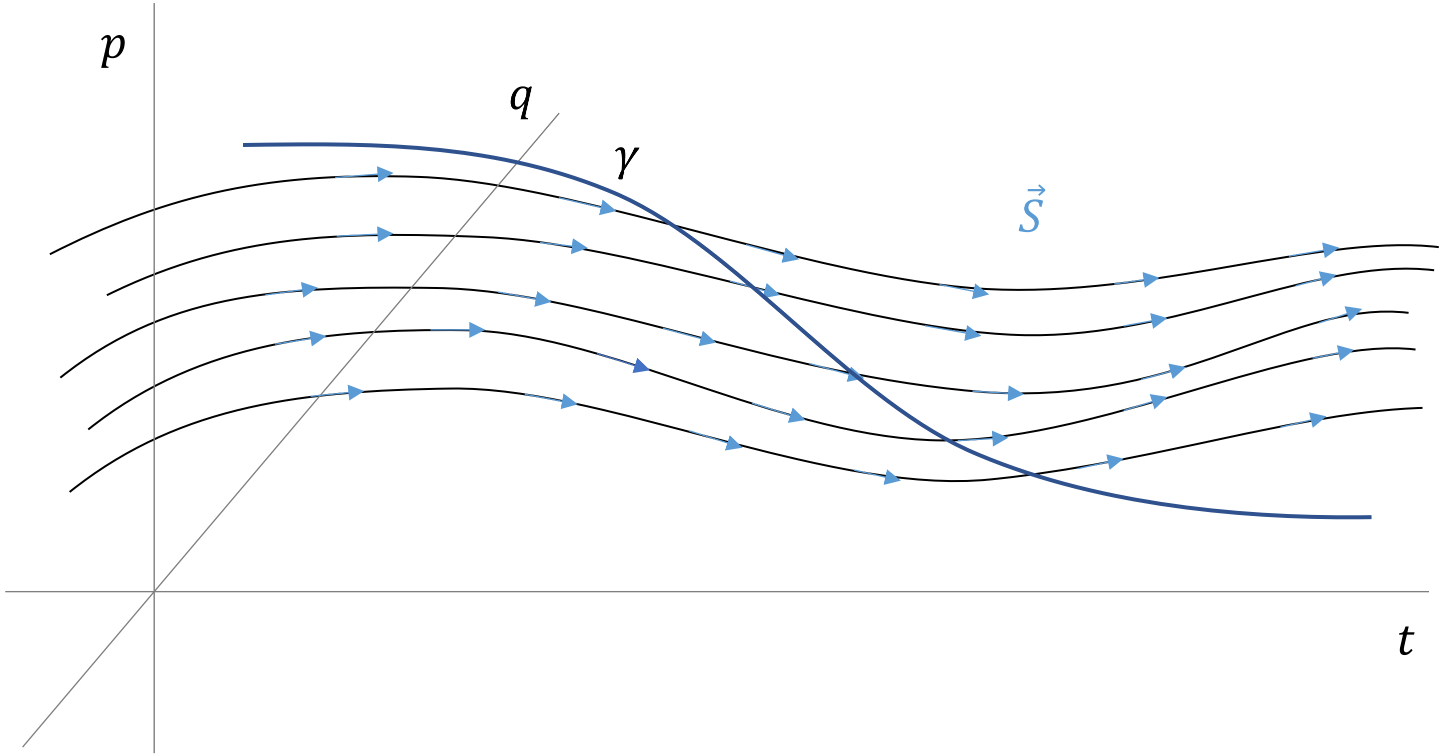
Express in components

$$= \int_\gamma [p, 0, -H] \cdot \left[\frac{dq}{dt}, \frac{dp}{dt}, \frac{dt}{dt}\right] dt$$

$$= \int_\gamma \left(p \frac{dq}{dt} - H\right) dt$$

$$= \int_\gamma L(q, \dot{q}, t)\, dt$$

$$= \mathcal{A}[\gamma]$$



Assuming (KE) Kinematic Equivalence:
$p = p(q, \dot{q}, t)$

The action is the line integral
of the vector potential of the flow of states

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Assumptions of Physics

Flow between a path γ and its variation γ'

$$\Phi[\Sigma] = -\iint_{\Sigma} \vec{S} \cdot d\vec{\Sigma}$$

Hamilton equations

$$= \iint_{\Sigma} \vec{\nabla} \times \vec{\theta} \cdot d\vec{\Sigma}$$

Stokes' theorem

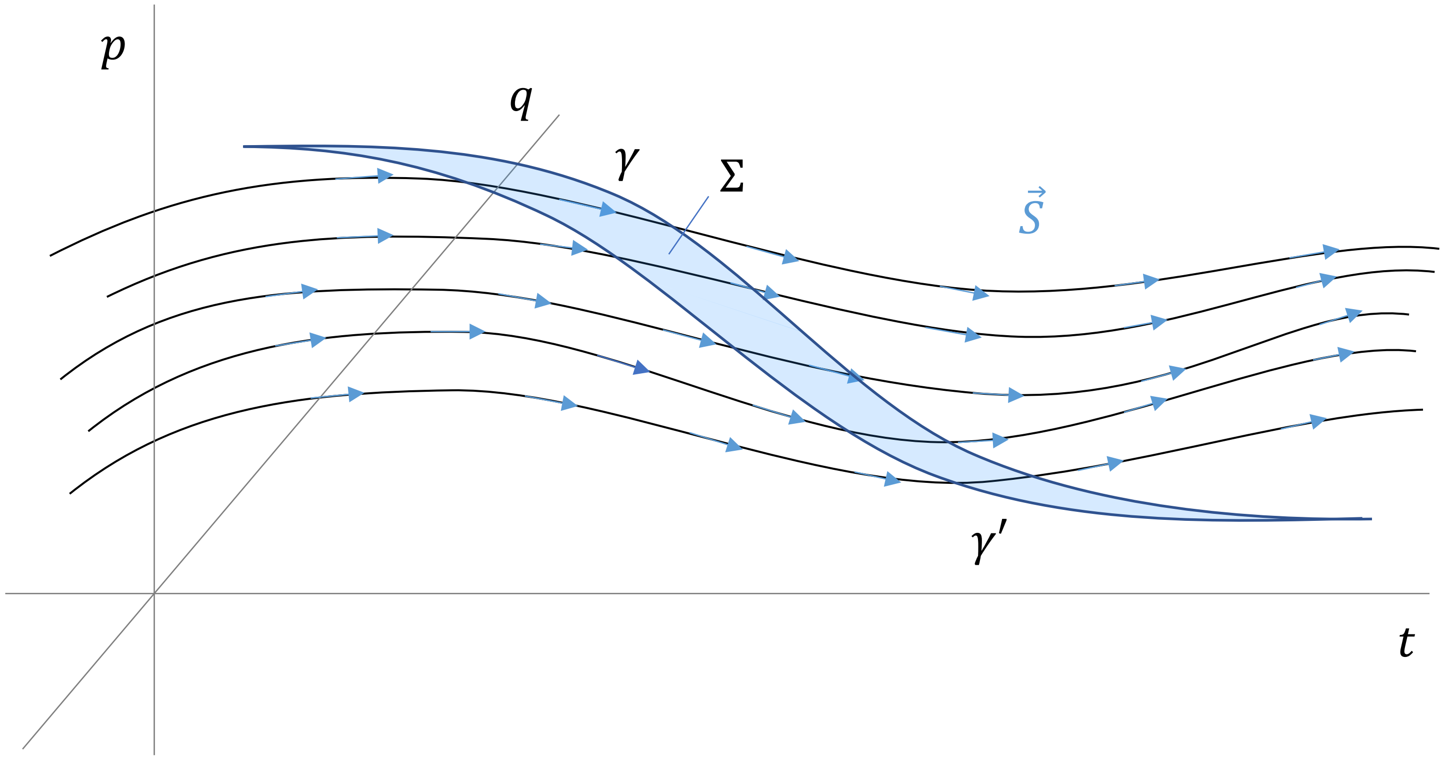
$$= \oint_{\partial\Sigma} \vec{\theta} \cdot d\vec{\gamma}$$

$$= \int_{\gamma} \vec{\theta} \cdot d\vec{\gamma} - \int_{\gamma'} \vec{\theta} \cdot d\vec{\gamma}$$

$$= \delta \int_{\gamma} \vec{\theta} \cdot d\vec{\gamma}$$

$$= \delta \int_{\gamma} L \, dt$$

$$= \delta \mathcal{A}[\gamma]$$



The variation of the action is the flow between the path and its variation

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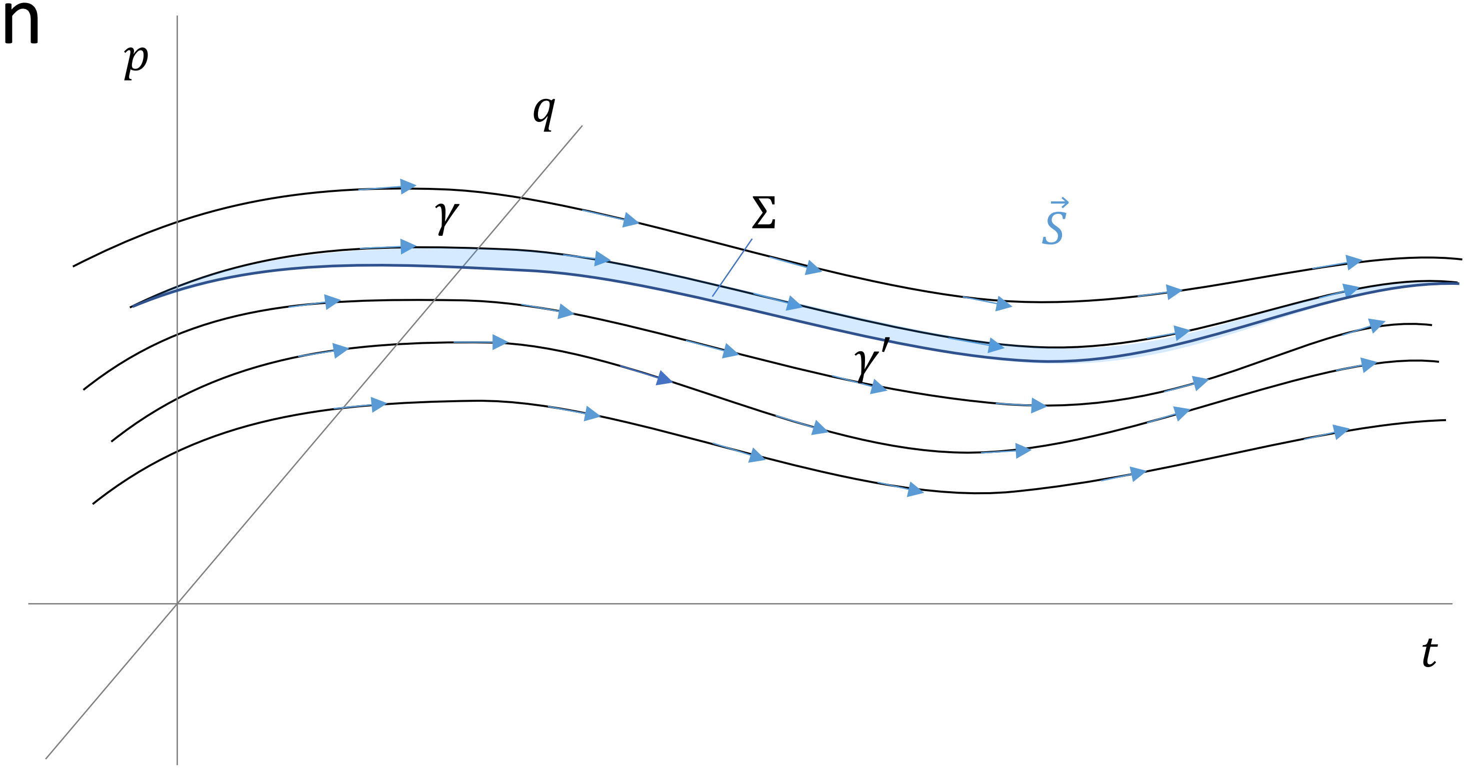
Assumptions of Physics

If the path γ is an evolution of the system

$$\Phi[\Sigma] = -\iint_\Sigma \vec{S} \cdot d\vec{\Sigma}$$
$$= \delta \int_\gamma \vec{\theta} \cdot d\vec{\gamma} = 0$$

flow is tangent



If the path γ is not an evolution of the system, we can always find a variation γ' such that

$$-\iint_\Sigma \vec{S} \cdot d\vec{\Sigma} = \delta \int_\gamma \vec{\theta} \cdot d\vec{\gamma} \neq 0$$

Action is stationary over and only over evolutions

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Assumptions of Physics

1 DOF of classical phase space

+

Determinism/Reversibility

+

Kinematic equivalence

$\Updownarrow$

Principle of stationary action

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Relativistic mechanics

https://assumptionsofphysics.org/book/AssumptionsOfPhysicsV3.0.pdf#section.1.1.9

J. Phys. Commun. **2** 045026 (2018)
https://arxiv.org/abs/1702.07052
http://iopscience.iop.org/article/10.1088/2399-6528/aaba25

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Assumptions of Physics

Required background

- Special relativity
  - Space and time coordinates are grouped together: $q^\alpha = \left[ct, q^i\right]$
  - Metric tensor $g_{\alpha\beta}$ defines the geometry of space-time (i.e. lengths and angles)
  - Using $(-,+,+,+)$ signature: $ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$
  - Energy and momentum are grouped together (four-momentum): $p_\alpha = \left[-\frac{E}{c}, p_i\right]$
  - Four-velocity is the velocity with respect to proper time (time in the rest frame) and its norm is $-c^2$: $\quad u^\alpha = d_\tau q^\alpha \quad u^\alpha g_{\alpha\beta} u^\beta = -c^2$
  - For a free particle, relationship between four-momentum and four-velocity: $p_\beta g^{\beta\alpha} = p^\alpha = m u^\alpha$

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Assumptions of Physics

Want to extend Hamiltonian mechanics to handle change of coordinates that mix space and time variables

$q^\alpha = [t, q^i]$

$[q^i(t)] \to [t(s), q^i(s)]$

Separate time-the-variable from time-the-parameter

Hamiltonian mechanics on the extended phase space

Recall

$\theta_{q^i} \qquad \theta_t$

$$\theta_a = [p_i, 0, -H]$$

Form a covector

$$p_\alpha = [-E, p_i]$$

$\Rightarrow$ only possible choice for conjugates

$$\frac{dq^\alpha}{ds} = \frac{\partial \mathcal{H}}{\partial p_\alpha} \qquad \frac{dp_\alpha}{ds} = -\frac{\partial \mathcal{H}}{\partial q^\alpha}$$

Hamiltonian constraint

$$\mathcal{H}(t, q^i, E, p_i) = 0$$

Not just a conserved quantity

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Assumptions of Physics

$$q^{\alpha} = [t, q^i]$$
$$p_{\alpha} = [-E, p_i]$$

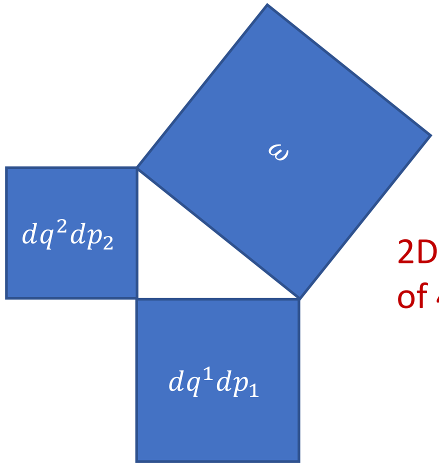
$$\omega = dq^i dp_i - dt dE$$

Energy-momentum covector and negative sign for temporal DOF appear in the extended phase space without knowledge of space-time geometry (or c)

Lorentzian relativity is the only "correct" one

It extends the count of configurations per DOF to the temporal DOF in the correct way

$$\omega = dq^1 dp_1 + dq^2 dp_2$$

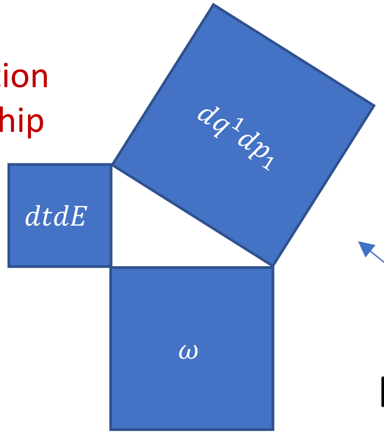


Independent DOFs are orthogonal

2D representation of 4D relationship

$$\omega = dq^1 dp_1 - dt dE$$

$$dq^1 dp_1 = \omega + dt dE$$



States are counted at equal time: temporal DOF orthogonal to ω

Pythagoras's formula still present

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Example: free particle in an inertial frame

$$\mathcal{H} = \frac{1}{2m}\left(p_i\delta^{ij}p_j - \left(\frac{E}{c}\right)^2 + m^2c^2\right)$$

This Hamiltonian constraint is the generator for proper time (i.e. affine parameter s corresponds to proper time)

$$\frac{dt}{ds} = \frac{\partial\mathcal{H}}{\partial(-E)} = -\frac{1}{2mc^2}\frac{\partial}{\partial(-E)}(-E)^2 = -\frac{1}{2mc^2}(-2E) = \frac{E}{mc^2}$$

$$\frac{dq^i}{ds} = \frac{\partial\mathcal{H}}{\partial p_i} = \frac{1}{2m}\frac{\partial}{\partial p_i}\left(p_i\delta^{ij}p_j\right) = \frac{2\delta^{ij}p_j}{2m} = \frac{p_i}{m}$$

$$\frac{dE}{ds} = -\frac{d(-E)}{ds} = \frac{\partial\mathcal{H}}{\partial t} = 0$$

$$\frac{dp_i}{ds} = -\frac{\partial\mathcal{H}}{\partial q^i} = 0$$

four momentum

$$\frac{E}{c} = m\frac{d(ct)}{ds}$$

$$p_i = m\frac{dq^i}{ds}$$

four velocity

proper time!

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Assumptions of Physics

Compare to "standard" way

$$H = c\sqrt{m^2c^2 + p_i\delta^{ij}p_j}$$

Hamiltonian is time generator

$$\frac{dq^i}{dt} = \frac{\partial H}{\partial p_i} = c\frac{\partial}{\partial p_i}\sqrt{m^2c^2 + p_i\delta^{ij}p_j}$$

$$= \frac{c}{2}\frac{1}{\sqrt{m^2c^2 + p_i\delta^{ij}p_j}}\frac{\partial}{\partial p_i}\left(m^2c^2 + p_i\delta^{ij}p_j\right)$$

$$= \frac{1}{2}\frac{2\delta^{ij}p_j}{\sqrt{m^2 + \frac{p_i\delta^{ij}p_j}{c^2}}} = \frac{p_i}{\sqrt{m^2 + \frac{|p_i|^2}{c^2}}}$$

$$\frac{dp_i}{dt} = -\frac{\partial H}{\partial q^i} = 0$$

Generator for proper time

Nice and quadratic

$$\mathcal{H} = \frac{1}{2m}\left(p_i\delta^{ij}p_j - \left(\frac{E}{c}\right)^2 + m^2c^2\right)$$

$$\frac{dq^i}{dt} = \frac{ds}{dt}\frac{dq^i}{ds} = \frac{mc^2}{E}\frac{p_i}{m}$$

Hamiltonian mechanics on the extended phase-space comes out more naturally and is a nicer formulation

Wait… quadratic?
Can E be negative?

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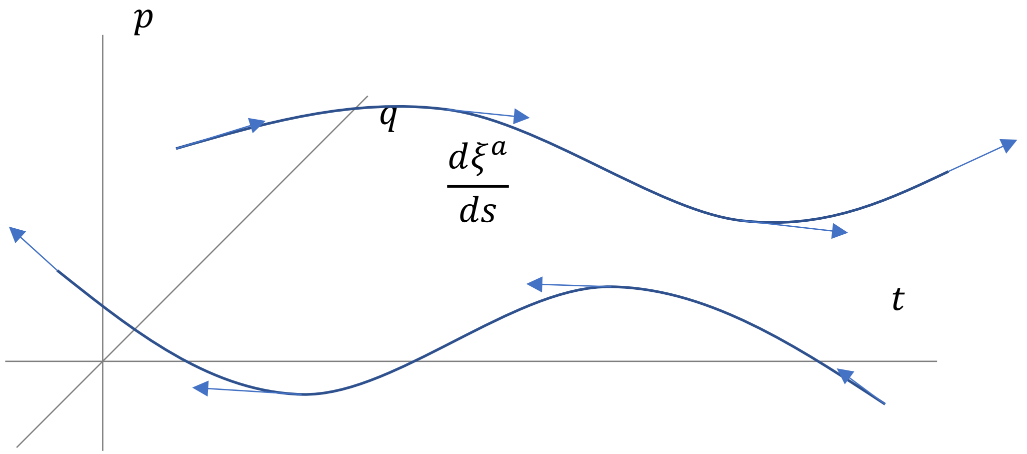
Classical anti-particles

This can be negative

$$d_s t = \partial_{-E}\mathcal{H} \qquad d_s(-E) = \partial_t \mathcal{H}$$
$$d_s q^i = \partial_{p_i}\mathcal{H} \qquad d_s p_i = \partial_{q^i}\mathcal{H}$$



Particle: affine parameter aligned with time

Anti-particle: affine parameter anti-aligned with time

Parametrization flows backwards with respect to time

An evolution cannot change time alignment

Free particle

$$d_s t = \frac{E}{mc^2}$$

Anti-particle = negative energy

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Assumptions of Physics

Recover standard Hamiltonian

Over valid states $\mathcal{H} = 0$ and $E = H(q^i, p_i, t)$ $\qquad \mathcal{H} = \lambda(H - E)$

Assume no E dependence

$$E = H$$

$$d_t t = 1 = d_t s \, d_s t = d_t s \, \partial_{-E} \mathcal{H} = d_t s \, \lambda$$

$$d_t s = \frac{1}{\lambda}$$

$= 0$

$$d_t q^i = d_t s \, d_s q^i = d_t s \, \partial_{p_i} \mathcal{H} = \frac{1}{\lambda} \left(\partial_{p_i} \lambda (H - E) + \lambda \partial_{p_i} H \right) = \partial_{p_i} H$$

$$d_t p_i = d_t s \, d_s p_i = -d_t s \, \partial_{q^i} \mathcal{H} = -\frac{1}{\lambda} \left(\partial_{q^i} \lambda (H - E) + \lambda \partial_{q^i} H \right) = -\partial_{q^i} H$$

$$d_t E = d_t s \, d_s E = d_t s \, \partial_t \mathcal{H} = \frac{1}{\lambda} \left(\partial_t \lambda (H - E) + \lambda \partial_t H \right) = \partial_t H$$

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Assumptions of Physics

Hamiltonian constraint can "hide" multiple Hs

Free particle Hamiltonian constraint

$$\mathcal{H} = \frac{1}{2mc^2}\Big(\underbrace{\sqrt{c^2|p_i|^2 + (mc^2)^2} + E}_{\text{Negative energy}}\Big)\Big(\underbrace{\sqrt{c^2|p_i|^2 + (mc^2)^2} - E}_{\text{Positive energy}}\Big)$$

$$\mathcal{H} = (H_1 - E)(H_2 - E)\,\frac{-1}{2mc^2}$$

$$H_1 = -H_2$$

$$d_s t = \frac{1}{2mc^2}(E + E) = \frac{E}{mc^2}$$

Classical-quantum parallel

Free particle Hamiltonian constraint

$$\mathcal{H} = \frac{1}{2m}\left(p_\alpha \eta^{\alpha\beta} p_\beta + m^2c^2\right) = \frac{1}{2m}\left(p_i \delta^{ij} p_j - (E/c)^2 + m^2c^2\right)$$

Hamiltonian constraint becomes Klein-Gordon equation in QM

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 + \frac{m^2c^2}{\hbar^2}\right)\psi(t, \mathbf{x}) = 0$$

minus sign from $\imath^2$

Also note: $$\mathcal{H}\rho = 0$$ Density can be non-zero only where $\mathcal{H} = 0$

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Assumptions of Physics

Hamiltonian constraint for charged particles

pure four-velocity kinetic term

$$\mathcal{H} = \frac{1}{2m}(p_\alpha - \mathfrak{q}A_\alpha)g^{\alpha\beta}(p_\beta - \mathfrak{q}A_\beta) + \frac{1}{2}mc^2 = \frac{1}{2}mu^\alpha g_{\alpha\beta}u^\beta + \frac{1}{2}mc^2$$

$$mu_\alpha = mu^\beta g_{\beta\alpha} = p_\alpha - \mathfrak{q}A_\alpha$$

Kinetic momentum

Momentum gauge and EM gauge must cancel out

Compare to EM Klein-Gordon equation

$$[c^2(\partial_\alpha - \imath\mathfrak{q}A_\alpha)\eta^{\alpha\beta}(\partial_\alpha - \imath\mathfrak{q}A_\alpha) + m^2c^4]\psi = [c^2 D_\alpha \eta^{\alpha\beta} D_\beta + m^2c^4]\psi = 0$$

$$D_\alpha = \partial_\alpha - \imath\mathfrak{q}A_\alpha$$

Gauge-covariant derivative is kinetic momentum

Takeaways

- Relativistic mechanics is recovered without additional assumptions
  - Relativity is needed to make densities/entropy invariant over time transformations
- Hamiltonian mechanics on the extended phase space is a more relativistic formulation
  - Hamiltonian constraint is the generator of the affine parameter, and can be taken to generate proper time for particles and minus proper time for anti-particles
- Multiple connections to quantum mechanics
  - Anti-particles already present in classical Hamiltonian particle mechanics
  - Hamiltonian constraint related to Klein-Gordon (and Dirac) operators
  - Gauge considerations are already present in classical particle mechanics

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Assumptions of Physics

Classical mechanics – Summary

- All the core elements of classical mechanics can be rederived and understood using four physically meaningful assumptions
  - Infinitesimal Reducibility (IR), DOF independence (IND), Determinism and Reversibility (DR) and Kinematic Equivalence (KE)
- Many physical ideas are a consequence of those assumptions
  - Principle of stationary action, massive particles under scalar/vector potential forces, relativistic mechanics, phase space (conjugate quantities), …
- A great number of better insights from a theory that is generally considered as “understood”
  - Probably even better insights to be found

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Assumptions of Physics

Why does classical mechanics fail?
What is quantum mechanics fixing?

Reverse Physics

"When the math is derived from the right physical assumptions, the physical assumptions can be derived from the math."

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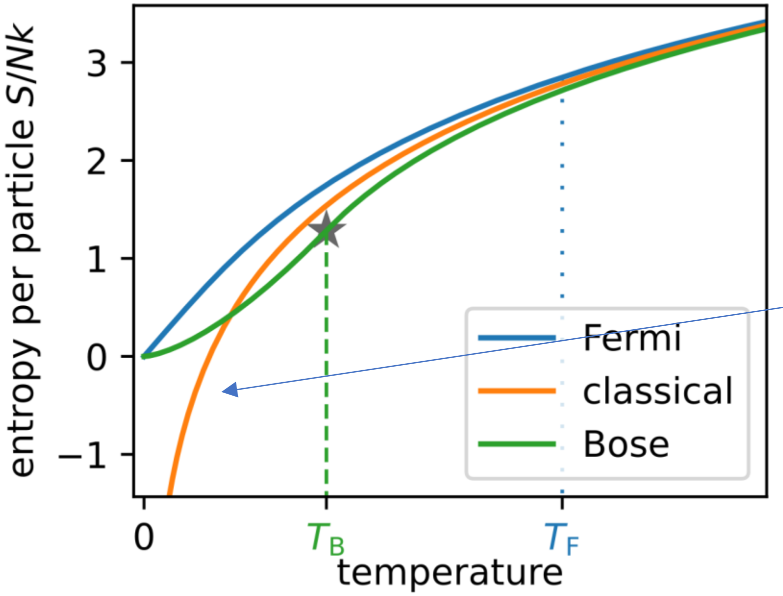
Assumptions of Physics

Wikipedia



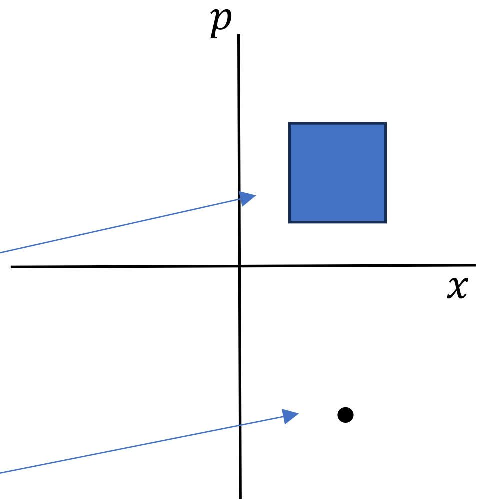
Temperature/entropy curve for ideal gases

to minus infinity!

Third law of thermodynamics

Every substance has a finite positive entropy, but at the absolute zero of temperature the entropy may become zero, and does so become in the case of perfect crystalline substances.

G. N. Lewis and M. Randall, Thermodynamics and the free energy of chemical substances (McGraw-Hill, 1923)



$$S = \log \Delta x \Delta p$$

$$S \geq 0$$

For a single microstate

$$S = \log 0 = -\infty$$

Classical physics allows arbitrarily small entropy…

… but thermodynamics doesn’t!!!

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Assumptions of Physics

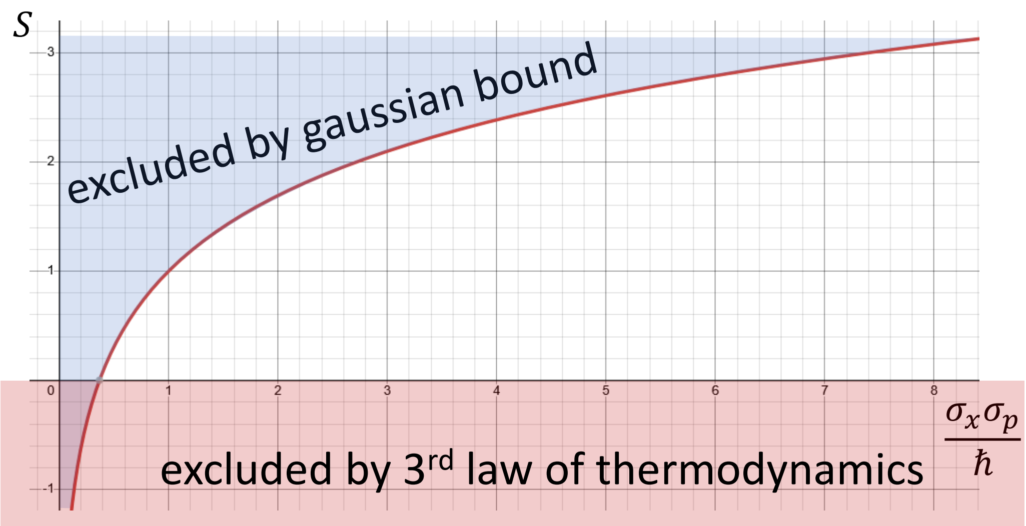
Found. Phys. **52**, 40 (2022)

Classical uncertainty principle

$$S(\rho) = -\int \rho \log h\rho \, dq dp$$

Fixes units

Uniform distribution over phase-space area h has zero entropy



$$S(\rho) \leq \log 2\pi e \frac{\sigma_q \sigma_p}{h}$$

Gaussian maximizes entropy for a given uncertainty

$$\sigma_q \sigma_p \geq \frac{h}{2\pi e} e^{S(\rho)} = \frac{\hbar}{e} e^{S(\rho)}$$

Entropy puts a lower bound on the uncertainty

$$S \geq 0 \;\Rightarrow\; \sigma_q \sigma_p \geq \frac{\hbar}{e}$$

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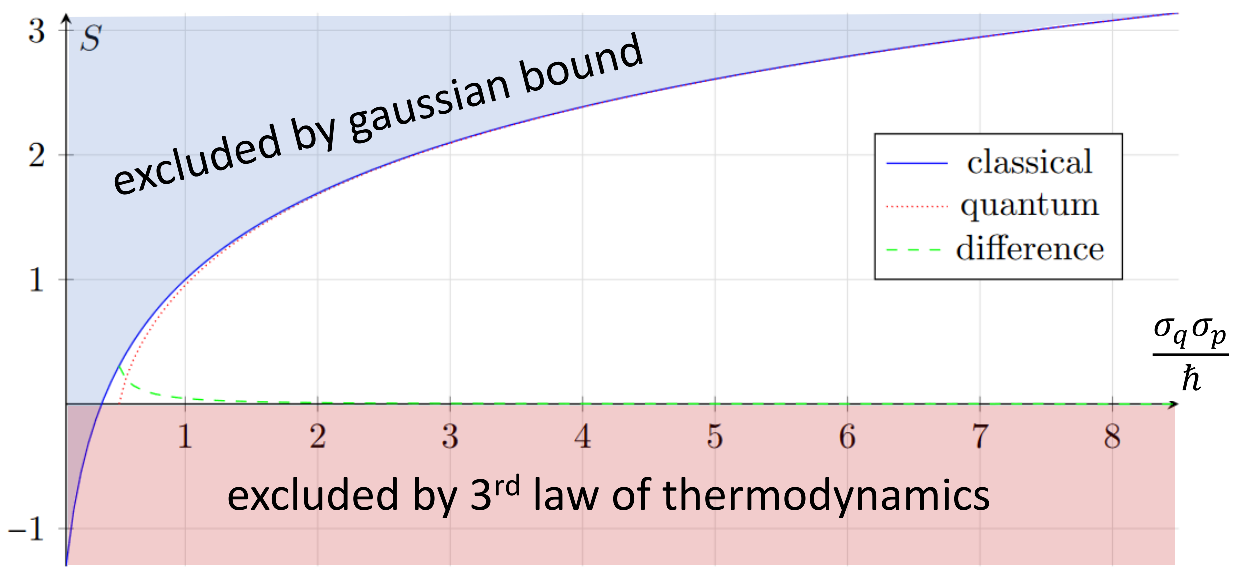
https://assumptionsofphysics.org/autogen/briefs/004-ClassicalUncertaintyPrinciple.pdf
https://link.springer.com/article/10.1007/s10701-022-00555-z

Comparing theories

classical $\sigma_q \sigma_p \geq \frac{\hbar}{e}$

2.71828…

quantum $\sigma_q \sigma_p \geq \frac{\hbar}{2}$



$$S_C = \ln e\sigma$$

$$S_Q = \left(\sigma + \frac{1}{2}\right) \ln\left(\sigma + \frac{1}{2}\right) - \left(\sigma - \frac{1}{2}\right) \ln\left(\sigma - \frac{1}{2}\right)$$

Entropy of quantum states is already non-negative

The gaussian bound quickly becomes very similar across theories

Quantum mechanics incorporates the third law.
Classical mechanics does not.

Is this the only difference?

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Assumptions of Physics

Suppose the lower bound on the entropy is the only difference, then in the limit of high entropy of quantum mechanics we should recover classical mechanics

Can we?

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Assumptions of Physics

Classical mechanics as high entropy limit?

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greetings

Manuele Landini <manulando@gmail.com>
To carcassi@umich.edu

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Wed 7/10/2024 6:15 AM

You replied to this message on 7/10/2024 10:00 AM.

Caro Gabriele,

Mi chiamo Manuele Landini e lavoro a Innsbruck (Austria) come senior scientist in un gruppo di fisica atomica sperimentale. Puoi vedere di cosa ci occupiamo sul nostro sito: https://quantummatter.at.

Ho visto un po' dei tuoi video su youtube. Mi sembra un progetto molto ambizioso, ma promettente. Mi farebbe piacere riuscire a spiegare agli studenti in futuro in termini piu' fisici concetti come le sovrapposizioni o il teorema spin-statistica.

Per la storia della metrica, da quel che ho capito hai bisogno di una metrica che non sia basata sull'entropia, visto che vuoi definire una distanza a entropia costante. Ci sono varie opzioni, ma la trace distance Trace distance - Wikipedia funziona perche' ha una propireta' fondamentale che puoi usare. Chiamala: T(rho,sigma)

Se parti da stati puri, si riduce a (1-<psi|phi>)^(1/2). Quindi per massimizzarla, scegli due stati ortogonali (non importa quali). Il massimo e' T_0=1. Una volta che hai questi stati, che hanno entropia 0, li puoi trasformare in stati con entropia finita (in particolare quelli con massima distanza) tramite una trace preserving map M.

Siccome T si contrae, hai che T(M(rho),M(sigma))<=T(rho,sigma). L'uguale vale se la mappa e' unitaria. Cosi' definisci un serie di step in cui la distanza massima decresce T_n+1<T_n, fino ad arrivare a 0 per stati fully mixed.

arXiv > quant-ph > arXiv:2411.00972

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Quantum Physics

[Submitted on 1 Nov 2024 (v1), last revised 3 Dec 2024 (this version, v2)]

Classical mechanics as the high-entropy limit of quantum mechanics

Gabriele Carcassi, Manuele Landini, Christine A. Aidala

We show that classical mechanics can be recovered as the high-entropy limit of quantum mechanics. That is, the high entropy masks quantum effects, and mixed states of high enough entropy can be approximated with classical distributions. The mathematical limit $\hbar \to 0$ can be reinterpreted as setting the zero entropy of pure states to $-\infty$, in the same way that non-relativistic mechanics can be recovered mathematically with $c \to \infty$. Physically, these limits are more appropriately defined as $S \gg 0$ and $v \ll c$. Both limits can then be understood as approximations independently of what circumstances allow those approximations to be valid. Consequently, the limit presented is independent of possible underlying mechanisms and of what interpretation is chosen for both quantum states and entropy.

Comments: 14 pages, 3 figures
Subjects: **Quantum Physics (quant-ph)**
Cite as: arXiv:2411.00972 **[quant-ph]**
(or arXiv:2411.00972v2 **[quant-ph]** for this version)
https://doi.org/10.48550/arXiv.2411.00972

Submission history
From: Christine Aidala [view email]
[v1] Fri, 1 Nov 2024 18:48:04 UTC (19 KB)
[v2] Tue, 3 Dec 2024 13:52:45 UTC (20 KB)

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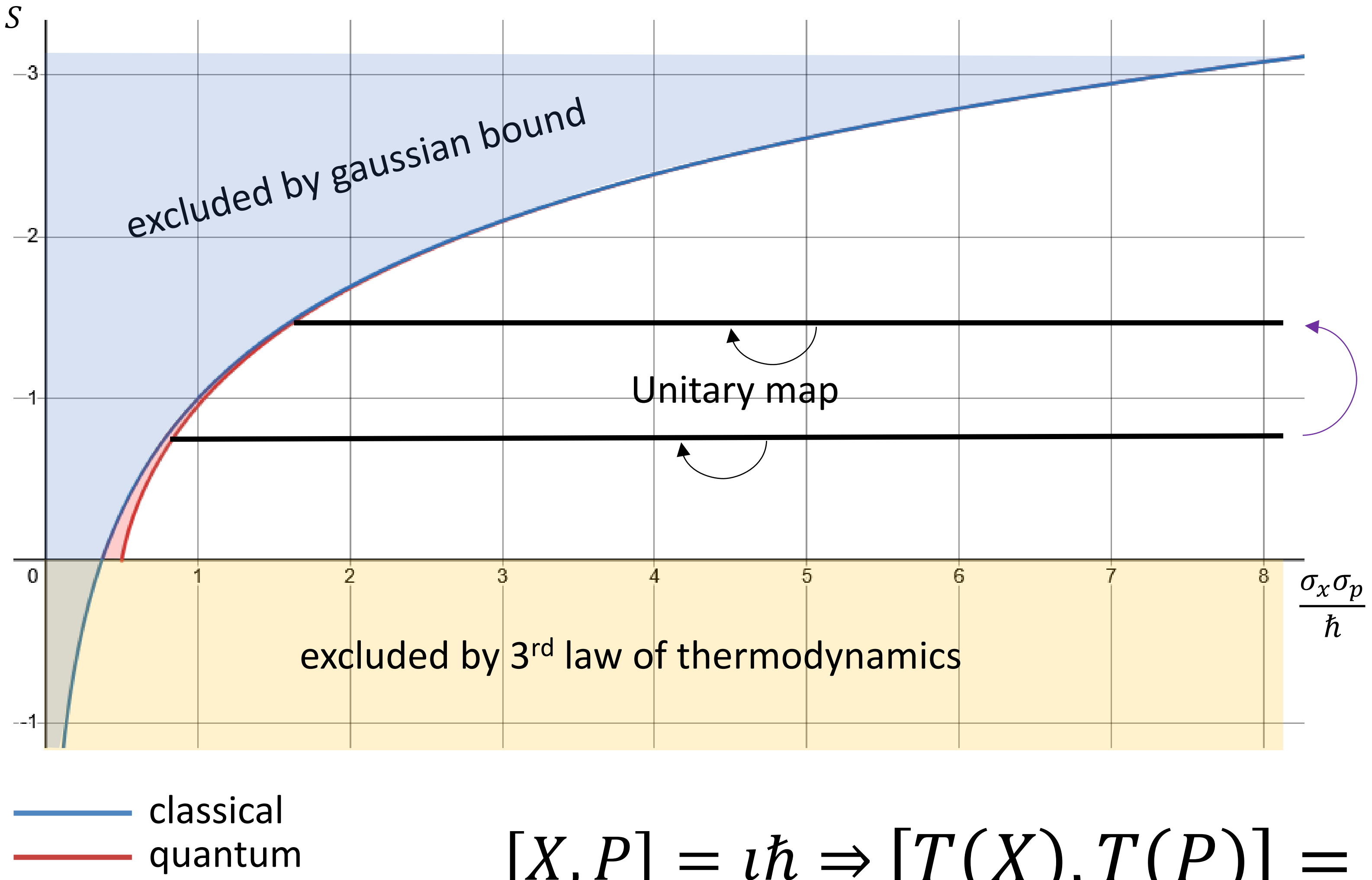
Assumptions of Physics

Looking for a map $R(\rho)$ that increases entropy of all mixed states, such that every level set of entropy maps to another level set

In the limit!



⇒ Unitary trans. must be mapped to unitary trans.

$R(\rho)$ Entropy increasing map

$$[X, P] = \imath\hbar \Rightarrow [T(X), T(P)] = \lambda\imath\hbar$$

Physica Scripta 101, 065105 (2026)

https://arxiv.org/pdf/2411.00972

https://iopscience.iop.org/article/10.1088/1402-4896/ae3a20

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Assumptions of Physics

Another perspective: move the pure states to minus infinite entropy

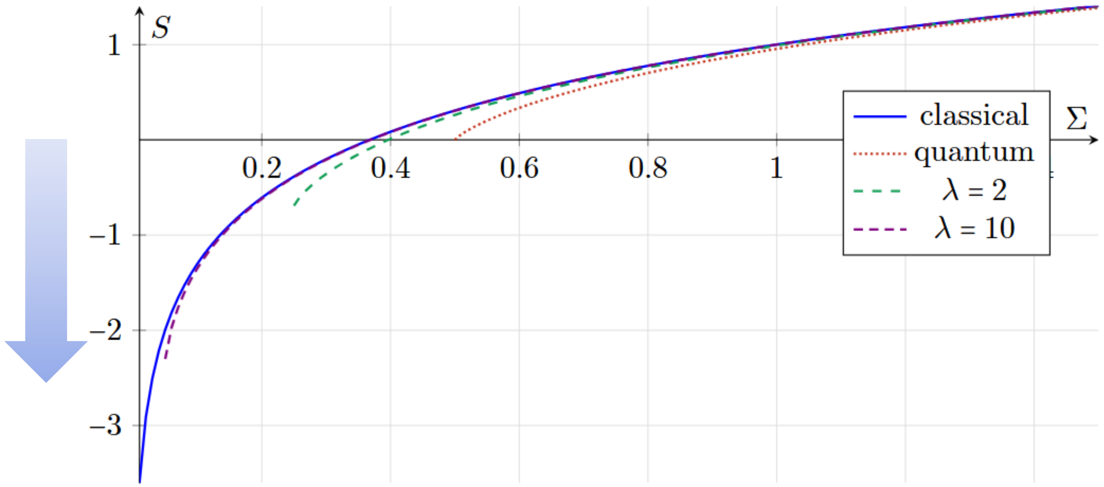
Instead of

$$[X, P] = \imath\hbar \quad [T(X), T(P)] = \lambda \imath\hbar$$

Redefine original space such that

$$[\hat{X}, \hat{P}] = \frac{\imath\hbar}{\lambda} \quad [T(\hat{X}), T(\hat{P})] = \imath\hbar$$

$$\lambda \to \infty \Rightarrow \frac{\hbar}{\lambda} \to 0$$



Mathematically equivalent to lowering the entropy of a pure state to $-\infty$, or $\hbar \to 0$ (group contraction)

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Assumptions of Physics

Physica Scripta 101, 065105 (2026)

“$c \to \infty$” Speed

“$\hbar \to 0$” Entropy

Classical Mechanics	Relativistic Mechanics
Quantum Mechanics	Quantum Field Theory

No-mechanism limit
(same as non-relativistic limit)

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Assumptions of Physics

$$\{A, B\} \longrightarrow \frac{[A, B]}{\imath\hbar}$$

Dirac's correspondence principle: give me a theory with an entropic lower bound that recovers the classical one at high entropy

Only one way to do it

Moyal bracket is the unique one-parameter Lie-algebraic deformation of the Poisson bracket

Quantizing a classical theory means putting a lower bound on the entropy

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Assumptions of Physics

Geometry is entropy!

The geometric structures of both classical and quantum mechanics are equivalent to the entropic structure

$\mu(U) = \int_U \omega^n$

Symplectic manifold

ρ uniform over U

Standard probability

$\rho(x) = \frac{1}{\mu(U)}$

Information theory

$S(\rho) = \log \mu(U)$

Thermodynamics/Statistical mechanics are not built on top of mechanics

Mechanics is the ideal case of thermodynamics/statistical mechanics

$\langle \psi | \phi \rangle$

Projective Hilbert space

$\rho = \frac{1}{2}\rho_\psi + \frac{1}{2}\rho_\phi$

Quantum probability

$p(\psi|\phi) = |\langle \psi | \phi \rangle|^2$

Quantum information theory

$S(\rho) = S\left(\frac{1+\sqrt{p}}{2}, \frac{1-\sqrt{p}}{2}\right)$

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Assumptions of Physics

Reverse Physics tells us there are not many "independent concepts" in a physical theory

Extracting principles/assumptions behind the laws gives us solid intuition that cuts across fields and leads to new insights/results

But: the only way to be sure we got all the concepts is to derive all the math from scratch
→ Physical Mathematics

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Assumptions of Physics

Assumptions of Physics is an open research program

Our main output is an open access book: https://assumptionsofphysics.org/book/

All our material is developed on GitHub: https://github.com/assumptionsofphysics

One YouTube channel dedicated to publicize results: https://www.youtube.com/user/gcarcassi

Livestream discussions

Another YouTube channel dedicated to research: https://www.youtube.com/@AssumptionsofPhysicsResearch

Activities coordinated through a Discord server (contact info@assumptionsofphysics.org for an invite)

Always looking for experts to gain insights and/or help

Always looking for collaborations

Always looking for editors/journals/conferences that are sympathetic to the mission

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Assumptions of Physics

Extra

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Assumptions of Physics

Action principle multiple DOF

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Assumptions of Physics

Required background

- Elements of differential geometry
  - A differential n-form takes n vectors and returns a number that depends on the volume of the parallelepiped (must be anti-symmetric):
    $W[\gamma] = \int_\gamma dW = \int_\gamma F_a dx^a$
    $\Phi[\Sigma] = \int_\Sigma d\Phi = \int_\Sigma B_{ab} v^a w^b$
    $m[V] = \int_V dm = \int_{\mathrm{V}} \rho_{abc} v^a w^b u^c$
  - Pseudo-vectors are really two-forms in 3D: $\vec{B} \to B_{ij}$
  - The exterior derivative generalizes gradient, curl, divergence:
    $\nabla f \to \partial_a \wedge f \qquad \nabla \times \vec{\theta} \to \partial_a \wedge \theta_b \qquad \nabla \cdot \vec{B} \to \partial_a \wedge B_{bc}$
- Generalized properties
  - Generalized Stokes theorem: $\int_\Sigma \partial_a \wedge \theta_b \, d\Sigma^{ab} = \int_{\partial\Sigma} \theta_b d\gamma^a$
  - Define closed form as having zero exterior derivative: $\partial_a \wedge \omega_{ab} = 0$
  - A closed form (locally) admits a vector potential:

$$\omega_{ab} = \partial_a \wedge \theta_b = \partial_a \theta_b - \partial_b \theta_a$$

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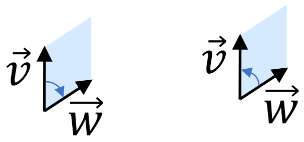
Assumptions of Physics

Characterize configuration flow

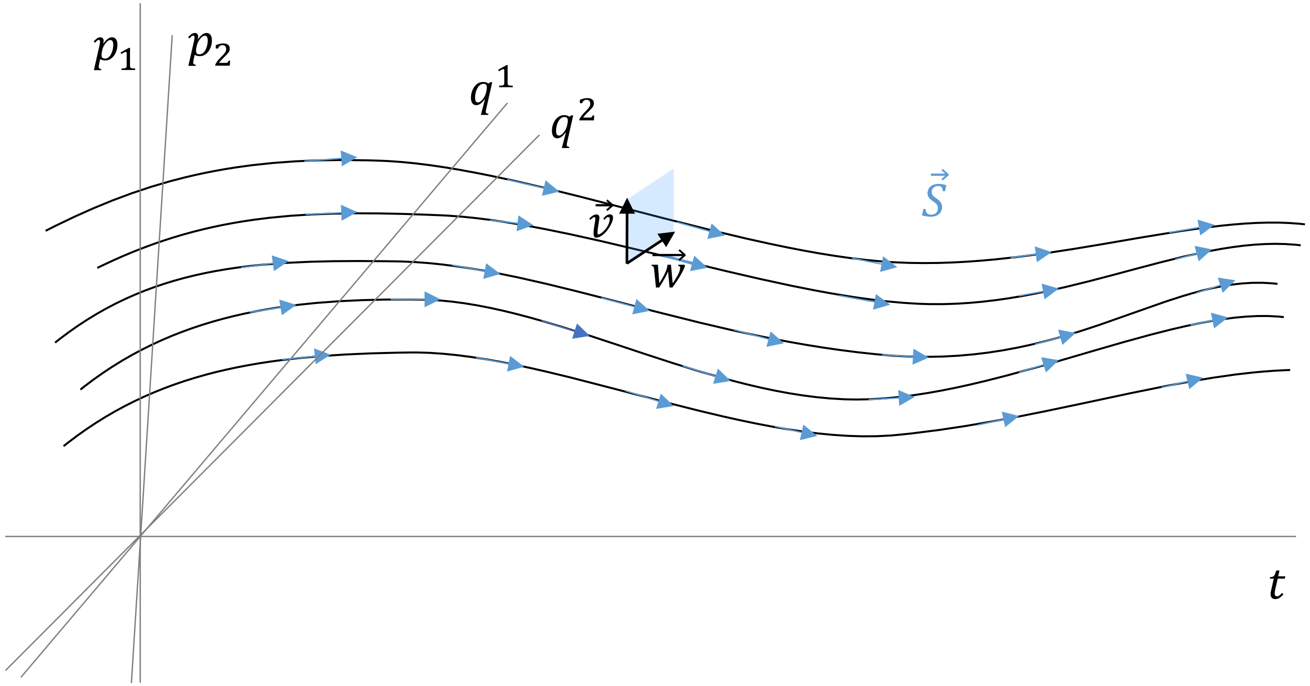
Flow of configurations through an infinitesimal surface

$$d\Phi = v^a \omega_{ab} w^b$$

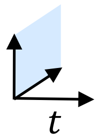
Function of surface: $\omega_{ab} = -\omega_{ba}$



change of order = change of orientation

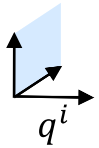


(DR) Determinism and reversibility: $\partial_t \wedge \omega_{ab} = 0$



Configurations are conserved through time

(IND) DOF independence: $\partial_{q^i} \wedge \omega_{ab} = 0$
$\partial_{p_i} \wedge \omega_{ab} = 0$



Configurations are conserved across DOF

IND + DR: ω_{ab} closed two-form $(\partial_a \wedge \omega_{bc} = 0)$

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Assumptions of Physics

$$\partial_a \wedge \omega_{bc} = 0$$

Closed form (DR) + (IND)

$$\omega_{ab} = -\partial_a \wedge \theta_b = -(\partial_a\theta_b - \partial_b\theta_a)$$

Vector potential

θ is the potential of ω

$$\theta_a = \begin{bmatrix} p_i & 0 & -H \end{bmatrix}$$

without loss of generality

$$\begin{aligned}
\partial_a\theta_b - \partial_b\theta_a &= \begin{bmatrix} \partial_{q^i}\theta_{q^j} - \partial_{q^j}\theta_{q^i} & \partial_{q^i}\theta_{p_j} - \partial_{p_j}\theta_{q^i} & \partial_{q^i}\theta_t - \partial_t\theta_{q^i} \\ \partial_{p_i}\theta_{q^j} - \partial_{q^j}\theta_{p_i} & \partial_{p_i}\theta_{p_j} - \partial_{p_j}\theta_{p_i} & \partial_{p_i}\theta_t - \partial_t\theta_{p_i} \\ \partial_t\theta_{q^j} - \partial_{q^j}\theta_t & \partial_t\theta_{p_j} - \partial_{p_j}\theta_t & \partial_t\theta_t - \partial_t\theta_t \end{bmatrix} \\
&= \begin{bmatrix} \partial_{q^i}p_j - \partial_{q^j}p_i & \partial_{q^i}0 - \partial_{p_j}p_i & \partial_{q^i}(-H) - \partial_t p_i \\ \partial_{p_i}p_j - \partial_{q^j}0 & \partial_{p_i}0 - \partial_{p_j}0 & \partial_{p_i}(-H) - \partial_t 0 \\ \partial_t p_j - \partial_{q^j}(-H) & \partial_t 0 - \partial_{p_j}(-H) & \partial_t(-H) - \partial_t(-H) \end{bmatrix} \\
&= \begin{bmatrix} 0 - 0 & 0 - \delta_i^j & -\partial_{q^i}H - 0 \\ \delta_j^i - 0 & 0 - 0 & -\partial_{p_i}H - 0 \\ 0 + \partial_{q^j}H & 0 + \partial_{p_j}H & -\partial_t H + \partial_t H \end{bmatrix} \\
&= \begin{bmatrix} 0 & -\delta_i^j & -\partial_{q^i}H \\ \delta_j^i & 0 & -\partial_{p_i}H \\ \partial_{q^j}H & \partial_{p_j}H & 0 \end{bmatrix}.
\end{aligned}$$

$$\omega_{ab} = \begin{bmatrix} \omega_{q^iq^j} & \omega_{q^ip_j} & \omega_{q^it} \\ \omega_{p_iq^j} & \omega_{p_ip_j} & \omega_{p_it} \\ \omega_{tq^j} & \omega_{tp_j} & \omega_{tt} \end{bmatrix} = \begin{bmatrix} 0 & \delta_j^i & \partial_{q^i}H \\ -\delta_i^j & 0 & \partial_{p_i}H \\ -\partial_{q^j}H & -\partial_{p_j}H & 0 \end{bmatrix}$$

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Assumptions of Physics

Displacement kills the flow

Displacement only direction that "kills" the flow

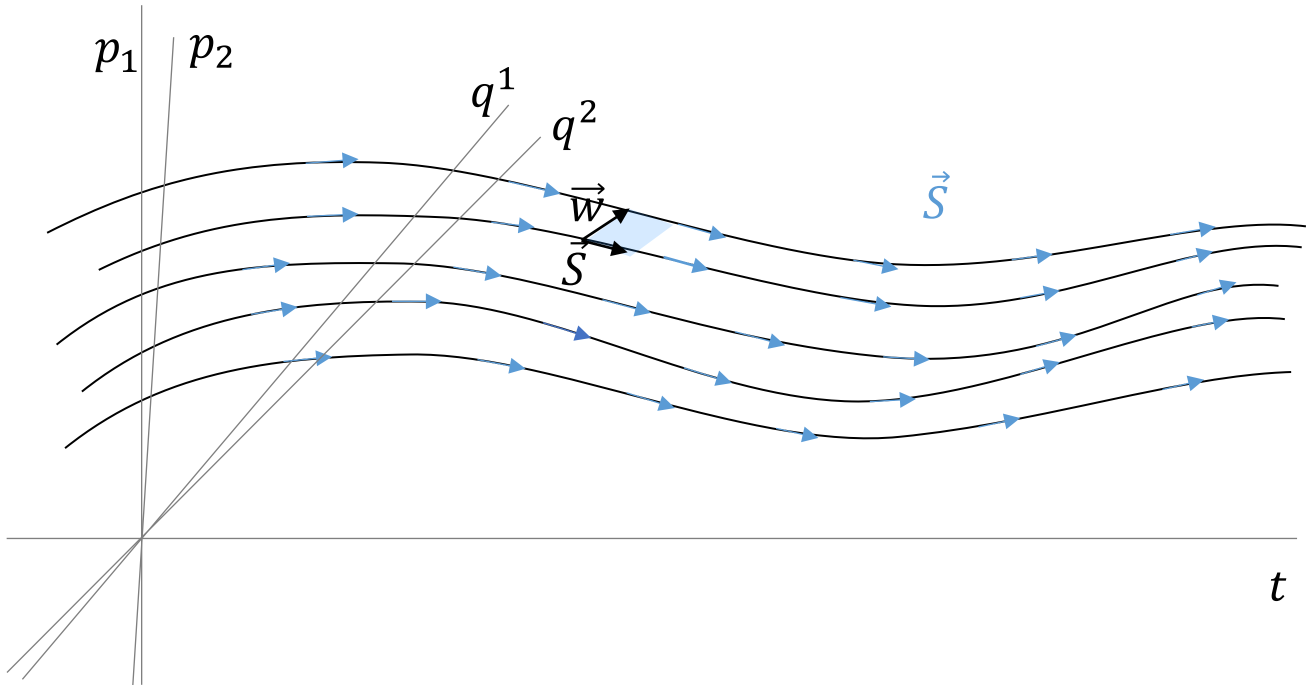
$$S^a \omega_{ab} w^b = 0$$

For all $\vec{w}$

Flow is tangent (and only tangent) to the displacement

$$S^a \omega_{ab} = 0$$

Let's calculate!



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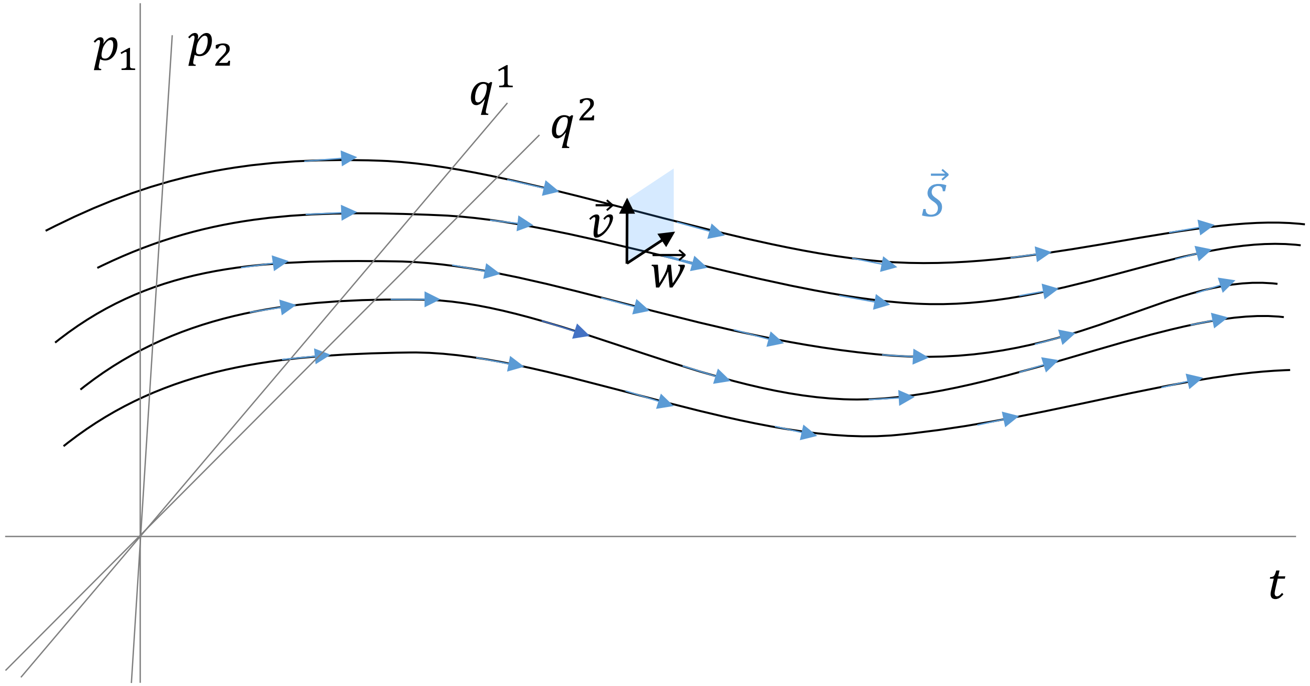
Assumptions of Physics

Displacement kills the flow

$$S^a \omega_{ab} = 0$$

$$\omega_{ab} = \begin{bmatrix} 0 & \delta^i_j & \partial_{q^i} H \\ -\delta^j_i & 0 & \partial_{p_i} H \\ -\partial_{q^j} H & -\partial_{p_j} H & 0 \end{bmatrix}$$

$$\begin{aligned} S^a \omega_{aq^j} &= S^{q^i} \omega_{q^i q^j} + S^{p_i} \omega_{p_i q^j} + S^t \omega_{t q^j} \\ &= -S^{p_j} - S^t \partial_{q^j} H = -S^{p_j} - \partial_{q^j} H = 0 \\ S^a \omega_{a p_j} &= S^{q^i} \omega_{q^i p_j} + S^{p_i} \omega_{p_i p_j} + S^t \omega_{t p_j} \\ &= S^{q^j} - S^t \partial_{p_j} H = S^{q^j} - \partial_{p_j} H = 0 \\ S^a \omega_{at} &= S^{q^i} \omega_{q^i t} + S^{p_i} \omega_{p_i t} + S^t \omega_{tt} \\ &= S^{q^i} \partial_{q^i} H + S^{p_i} \partial_{p_i} H \\ &= \partial_{p_i} H \partial_{q^i} H - \partial_{q^i} H \partial_{p_i} H = 0. \end{aligned}$$



$$S^{p_i} = \frac{dp_i}{dt} = -\frac{\partial H}{\partial q^i}$$

$$S^{q^i} = \frac{dq^i}{dt} = \frac{\partial H}{\partial p_i}$$

Recovers Hamilton's equations

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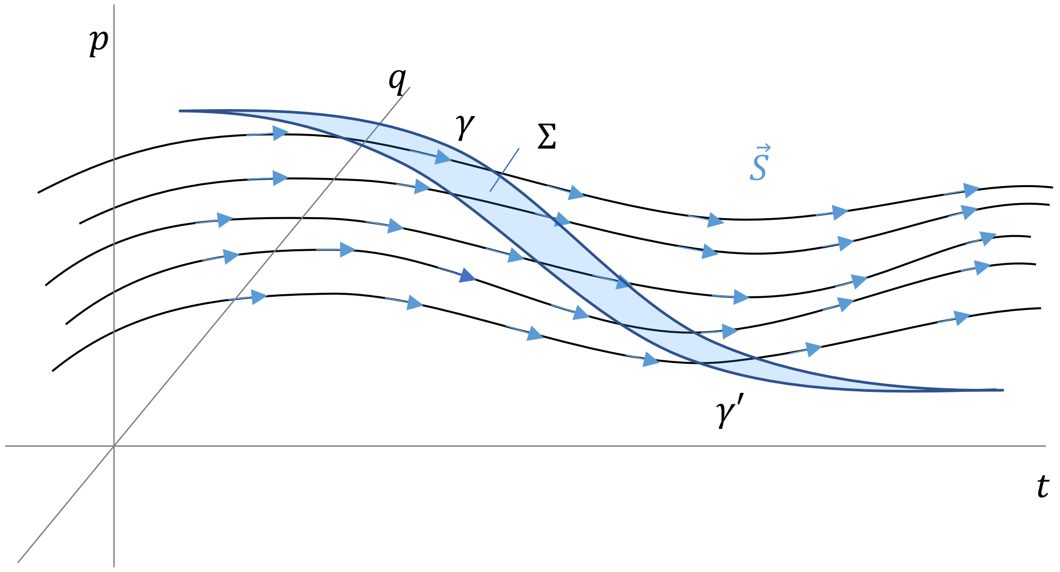
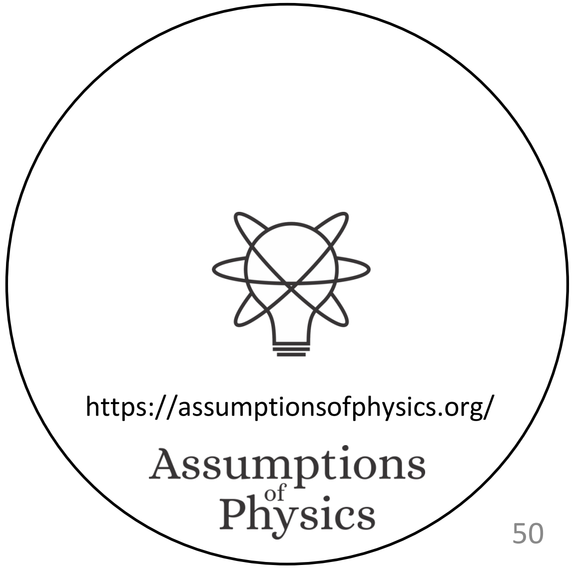
Assumptions of Physics

$$\xi^a = [q^i, p_i, t] \qquad \theta_a = [p_i, 0, -H]$$

$$\int_\gamma \theta_a d\gamma^a = \int_\gamma \theta_a d_t \xi^a dt = \int_\gamma \left(p_i d_t q^i - H\right) dt = \int_\gamma L dt = \mathcal{A}[\gamma]$$

Action is the line integral of the vector potential





$$\delta\mathcal{A}[\gamma] = \delta \int_\gamma L dt = \delta \int_\gamma \theta_a d_t \xi^a dt = \oint_{\partial\Sigma} \theta_a d\xi^a = \int_\Sigma \partial_a \wedge \theta_b d\xi^a d\eta^b = -\int_\Sigma d\xi^a \omega_{ab} d\eta^b = -\Phi[\Sigma]$$

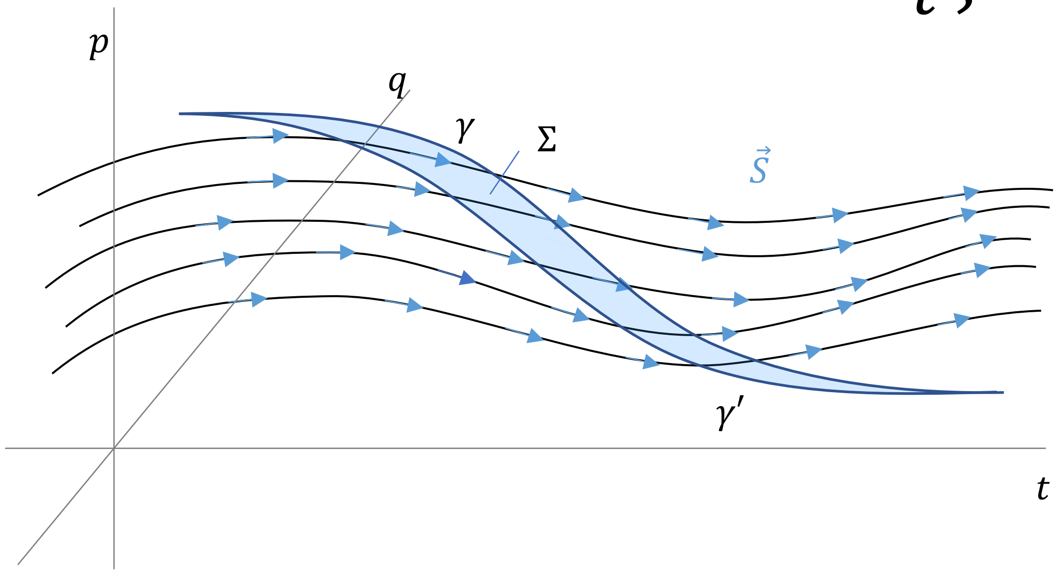
The variation of the action is the flow between path and its variation

$$\delta\mathcal{A}[\gamma] = 0 \iff d\xi^a \omega_{ab} d\eta^b = dt d_t \xi^a \omega_{ab} d\eta^b = 0 \ \forall d\eta\text{^}b$$

$$\iff d_t \xi^a \omega_{ab} = 0$$

$$d_t \xi^a = S^a$$

Action is stationary when the path kills the flow



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Assumptions of Physics

DOF independence

+

Determinism/Reversibility

+

Kinematic equivalence

$\Updownarrow$

Principle of stationary action

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Assumptions of Physics