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# Assumptions of Physics Summer School 2026

# Introduction to Reverse Physics

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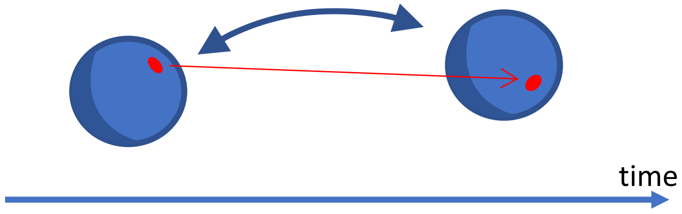
# Main goal of the Assumptions of Physics program

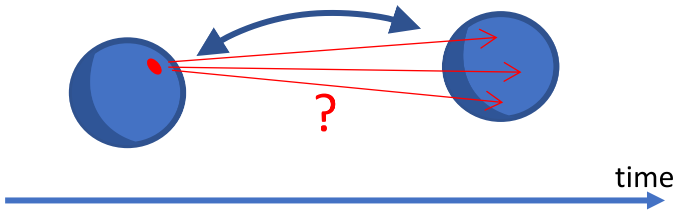
*Identify a handful of physical starting points from which the basic laws can be rigorously derived*

For example:

Infinitesimal reducibility ⇒ Classical state




Irreducibility ⇒ Quantum state

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This also requires rederiving all mathematical structures from physical requirements

For example:

Science is evidence based ⇒ scientific theory must be characterized by experimentally verifiable statements ⇒ topologies and $\sigma$-algebras

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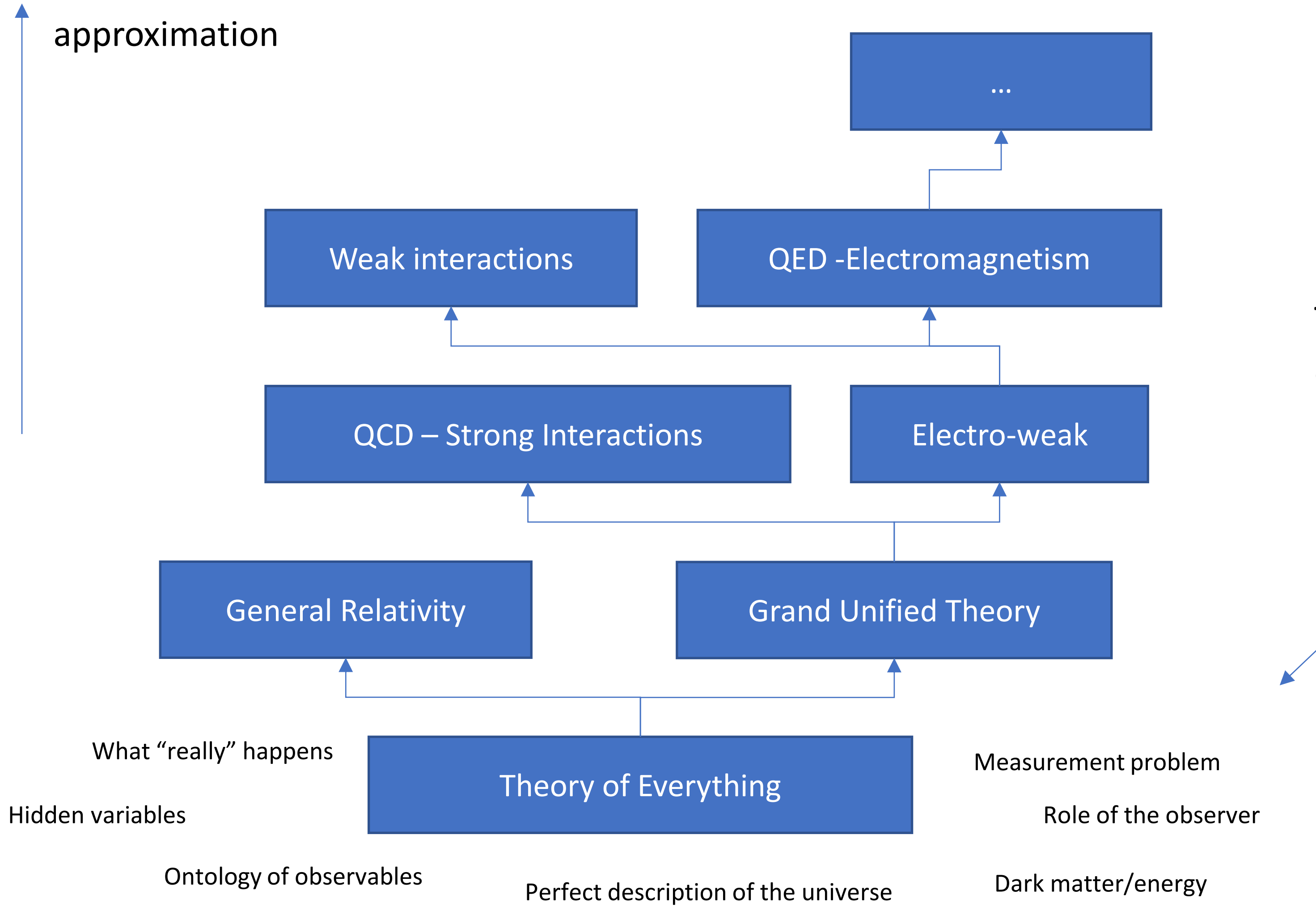
# Standard view of the foundations of physics




Goal of physics is to find the true laws of the universe!

The “real” physics!

The foundations of physics!

Everything else is an approximation

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# We found:

Experimental verifiability ⇒ topologies and $\sigma$-algebras
Geometric structures ⟺ Entropic structures
Hamiltonian evolution ⟺ det-rev/isolation + DOF independence
Massive particles and potential forces ⟺ + Kinematic eq

Physical requirements and assumptions drive most of the theoretical apparatus

~~Goal of physics is to find the true laws of the universe!~~

Less productive point of view

Goal of physics is to find models that can be empirically tested

More productive point of view

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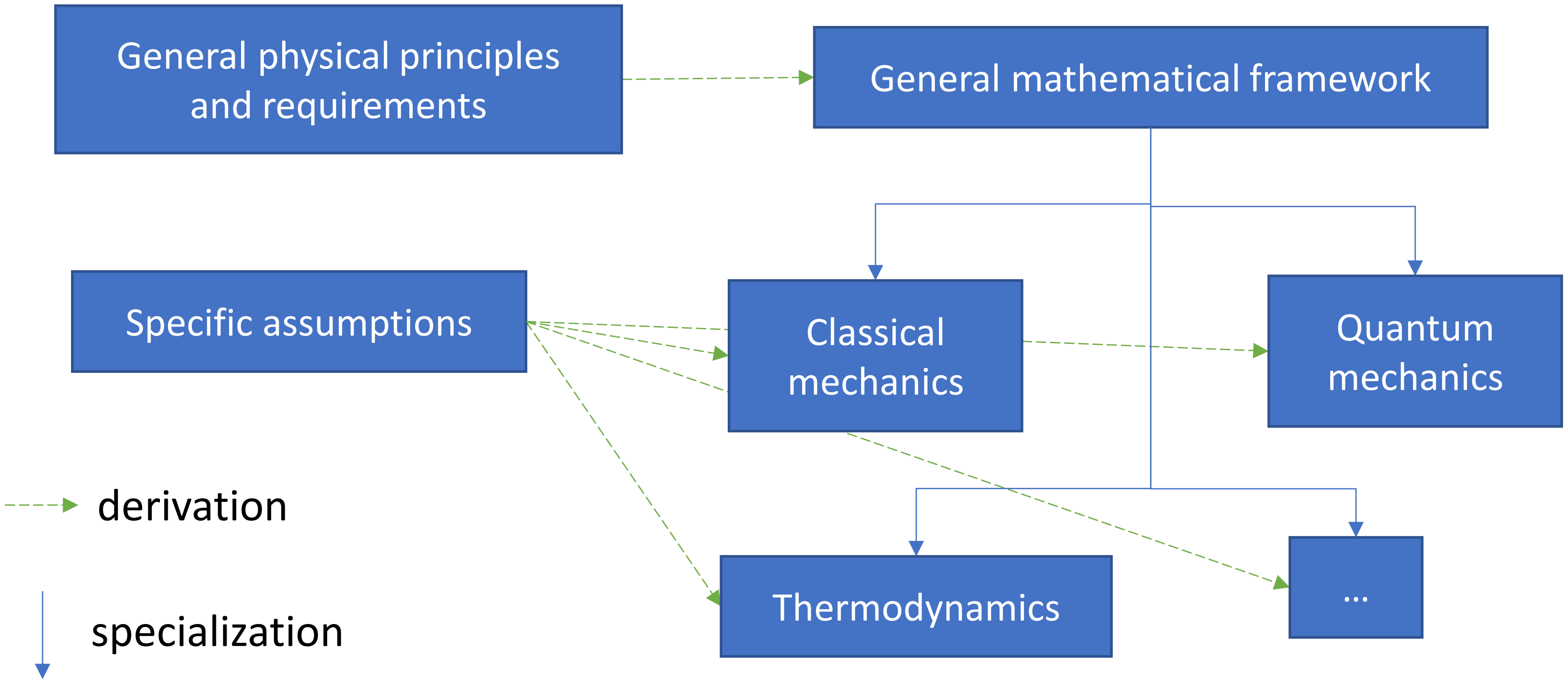
# Our view of the foundations of physics




Foundations of physics
⇕
The theory of physical models

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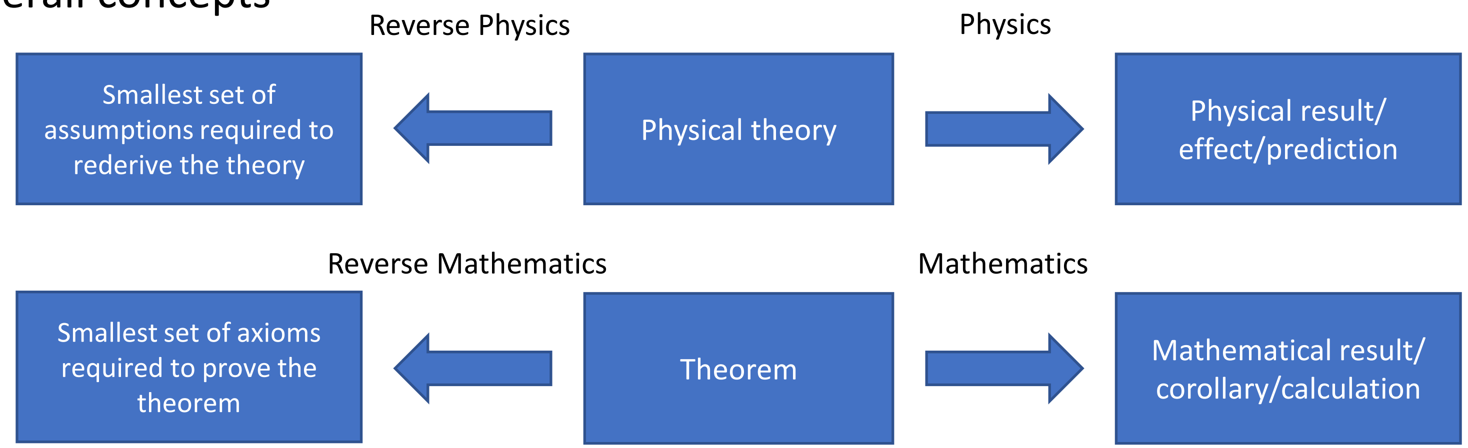
Find the right overall concepts

# *Reverse Physics*:

Start with the equations, reverse engineer physical assumptions/principles

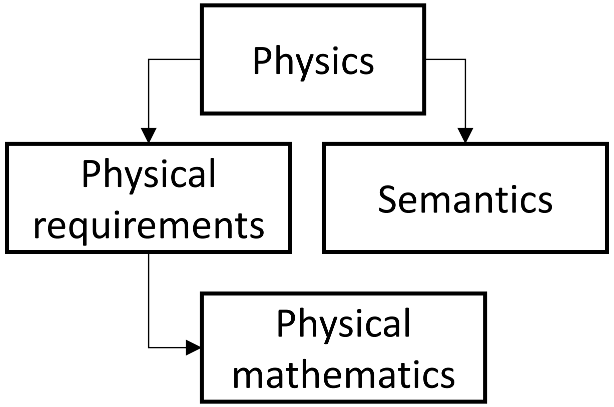
*Found. Phys.* **52**, 40 (2022)




Goal: find the right overall physical concepts, “elevate” the discussion from mathematical constructs to physical principles

# *Physical Mathematics*:

Start from scratch and rederive all mathematical structures from physical requirements




Goal: get the details right, perfect one-to-one map between mathematical and physical objects

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# Reverse Physics

*"When the math is derived from the right physical assumptions, the physical assumptions can be derived from the math."*

Reverse Physics aims to adopt approaches that are more in line with the foundations of mathematics, to break physics into a set of physical and mathematical conditions with well-understood logical relationships.

*A new **methodology** for the foundations of physics*

**Assumptions of Physics**, *Michigan Publishing* (v3 2025)
https://doi.org/10.3998/mpub.12204707

*Found. Phys.* **52** 40 (2022)
https://link.springer.com/article/10.1007/s10701-022-00555-z

*J. Phys. Commun.* **2** 045026 (2018)
http://iopscience.iop.org/article/10.1088/2399-6528/aaba25

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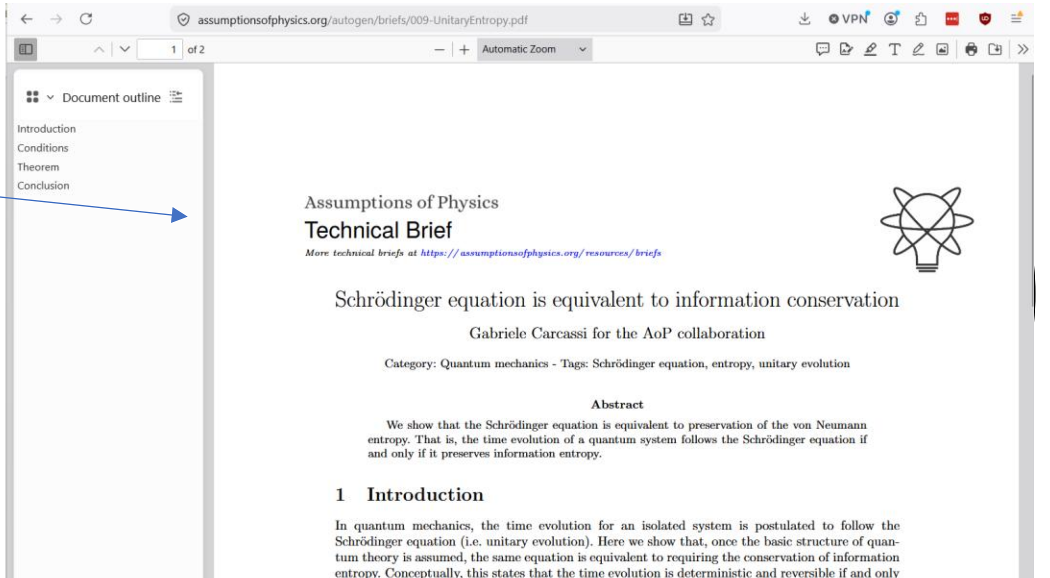
# Goals of Reverse Physics

- Framework/interpretation/formulation/theory neutral
  - Allow for each result to be independently formulated from the others
- Uncover “paradoxes” and develop standards of rigor appropriate for physics
- Clarify the logical dependencies of physical/mathematical conditions
- Identify core constitutive requirements of physical theories
  - The “big four” of Reverse Physics

Where relevant throughout lecture, link to papers or technical briefs



Reverse Physics

# Examining the actual content of mathematical structures

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# Hamiltonian mechanics 1 DOF $\Longleftrightarrow$ Det/Rev

$d_t\mu(U) = 0$ $\Longleftrightarrow$ $d_t \log \mu(U) = 0$ $\Longleftrightarrow$ $d_t \int -\rho \log \rho \, d\mu = 0$

Conservation of state count

Conservation of thermodynamic entropy

Conservation of information entropy

$d_t\omega_{ab} = 0$
symplectomorphism

**Determinism and reversibility** $\Longleftrightarrow$ Conservation of uncertainty

$d_t\left|cov\left(\xi^a, \xi^b\right)\right| \approx 0$
(for peaked distributions)

$d_t q = \partial_p H$
$d_t p = -\partial_q H$
Hamilton's equations

All equivalent conditions!!!

Base theory:

State: $\xi^a = [\, q \; p \,] \in \mathbb{R}^2$

Count of states: $\mu(U) = \int_U dqdp = \int_U \omega$

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# Schrödinger equation $\Longleftrightarrow$ Det/Rev

$\psi(0) = \phi$ iff $\psi(t) = e^{\frac{H\Delta t}{\imath\hbar}}\phi$ $\Longleftrightarrow$ $d_t \mathrm{tr}(-\rho \log \rho) = 0$ $\Longleftrightarrow$ ...

Bijective map of state and probability

Conservation of information entropy

...

$$U(\Delta t)|\psi\rangle = e^{\frac{H\Delta t}{\imath\hbar}}|\psi\rangle$$

unitary evolution

$$\imath\hbar d_t|\psi\rangle = H|\psi\rangle$$

Schrödinger equation

## Determinism and reversibility

All equivalent conditions!!!

Base theory: State: $\psi \in P(\mathcal{H})$

Probability of $\psi$ given $\phi$: $p(\psi|\phi) = |\langle\psi|\phi\rangle|^2$

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https://assumptionsofphysics.org/autogen/briefs/009-UnitaryEntropy.pdf

# Reversing the principle of stationary action

DR

$$\nabla \cdot \vec{S} = 0$$

No state is "lost" or "created" as time evolves

$[p, 0, -H(q,p)]$

$$\vec{S} = -\nabla \times \vec{\theta}$$

(Minus sign to recover Ham eq)

KE

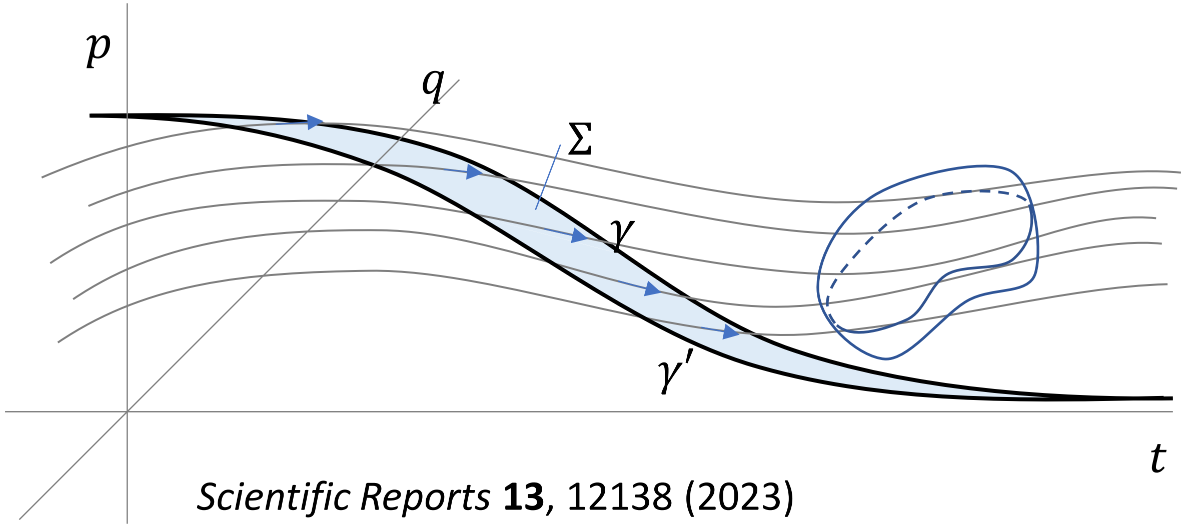
$p\frac{dq}{dt} + 0\frac{dp}{dt} - H\frac{dt}{dt}$

$$\mathcal{A}[\gamma] = \int_\gamma L dt = \int_\gamma \vec{\theta} \cdot d\vec{\gamma}$$

*Sci Rep* **13**, 12138 (2023)

unphysical

## The action is the line integral of the vector potential of the flow of states




*Scientific Reports* **13**, 12138 (2023)

### Variation of the action

$$\delta\mathcal{A}[\gamma] = \oint_{\partial\Sigma} \vec{\theta} \cdot d\vec{\gamma}$$

$$= -\iint_\Sigma \vec{S} \cdot d\vec{\Sigma}$$

Gauge independent, physical!

Variation of the action measures the flow of states (physical).
Variation = 0 ⇒ flow of states tangent to the path.

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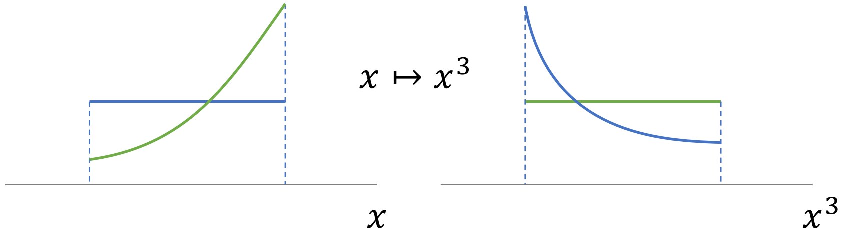
https://arxiv.org/pdf/2208.06428
https://www.nature.com/articles/s41598-023-39145-y

# Reversing phase space

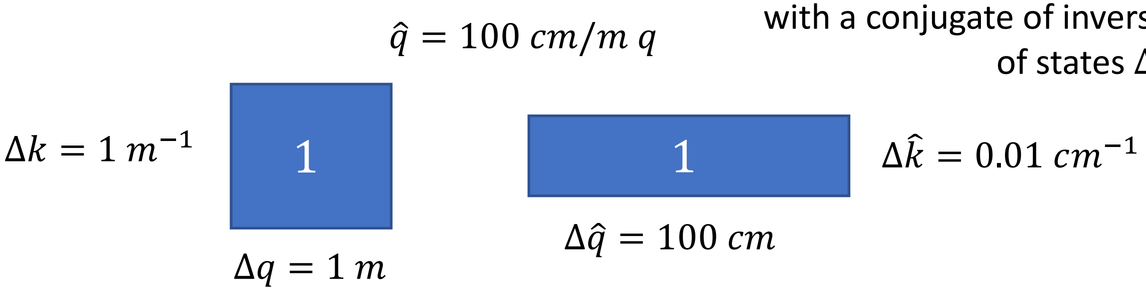
*Erkenntnis* **90**, 443 (2025)

*Stud Hist Phil Sci B* **71**, 082020, 60-71 (2020)




Density, entropy, uniform distributions NOT in general coordinate invariant

Each unit variable (i.e. coordinate) paired with a conjugate of inverse units: number of states $\Delta q \Delta k$ is invariant




Phase-space (symplectic) structure is the only one that supports coordinate-invariant density, entropy, state count

| | |
|---|---|
| Phase space volumes are invariant under equal-time changes of coordinates $q^i$ | (PSV-VOL) |
| The Jacobian for the transformation induced by equal-time changes of coordinates $q^i$ is unitary | (PSV-JAC) |
| Densities over phase space are invariant under equal-time changes of coordinates $q^i$ | (PSV-DEN) |
| Thermodynamic entropy is invariant under equal-time changes of coordinates $q^i$ | (PSV-THER) |
| Information entropy is invariant under equal-time changes of coordinates $q^i$ | (PSV-INFO) |
| Uncertainty of peaked distributions is invariant under equal-time changes of coordinates $q^i$ | (PSV-UNC) |

| | |
|---|---|
| The system allows statistically independent distributions over each DOF under any choice of coordinates $q^i$ | (PSI-DEN) |
| The system allows informationally independent distributions over each DOF under any choice of coordinates $q^i$ | (PSI-INFO) |
| The system allows peaked distributions where the uncertainty is the product of the uncertainty on each DOF under any choice of coordinates $q^i$ | (PSI-UNC) |

## Invariance of entropy/state count + DOF independence $\Longleftrightarrow$ structure of phase space (i.e. conjugate pairs)

https://philsci-archive.pitt.edu/22202/1/Hamiltonian_Privilege_preprint.pdf

https://link.springer.com/article/10.1007/s10670-023-00708-0

https://assumptionsofphysics.org/

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# Geometry is entropy

## Classical mechanics

$$S(\rho_U) = \log \mu(U) = \log \int_U \omega$$

Symplectic form determines and is determined by the entropy of all uniform joint and marginal (DOF) distributions

$$\partial_\alpha g_{\beta\gamma} - \partial_\beta g_{\alpha\gamma}$$

$$\omega_{ab} = \begin{bmatrix} -mG_{\alpha\beta\gamma}u^{\gamma} - qF^{\alpha\beta} & mg_{\alpha\beta} \\ -mg_{\alpha\beta} & 0 \end{bmatrix}$$

Metric tensor appears as the position-velocity part of the symplectic form

## Quantum mechanics

$$S\left(\frac{1}{2}\rho_\psi + \frac{1}{2}\rho_\phi\right) = I\left(\frac{1+|\langle\psi|\phi\rangle|}{2}, \frac{1-|\langle\psi|\phi\rangle|}{2}\right)$$

$$= -\frac{1+|\langle\psi|\phi\rangle|}{2}\log\frac{1+|\langle\psi|\phi\rangle|}{2} - \frac{1-|\langle\psi|\phi\rangle|}{2}\log\frac{1-|\langle\psi|\phi\rangle|}{2}$$

Born rule determines and is determined by the entropy of all equal mixtures of pairs of pure states

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*Found. Phys.* **52**, 40 (2022)

# Classical uncertainty principle

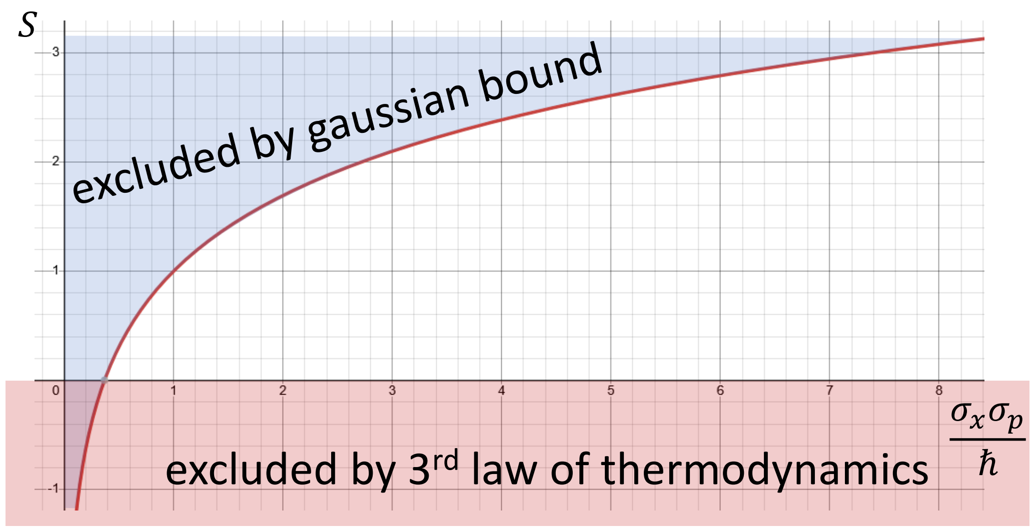
$$S(\rho) = -\int \rho \log h\rho \, dq dp$$

Fixes units

Uniform distribution over phase-space area $h$ has zero entropy

$$S(\rho) \leq \log 2\pi e \frac{\sigma_q \sigma_p}{h}$$

Gaussian maximizes entropy
for a given uncertainty




$$\sigma_q \sigma_p \geq \frac{h}{2\pi e} e^{S(\rho)} = \frac{\hbar}{e} e^{S(\rho)}$$

Entropy puts a lower bound
on the uncertainty

$$S \geq 0 \Rightarrow \sigma_q \sigma_p \geq \frac{\hbar}{e}$$

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https://assumptionsofphysics.org/autogen/briefs/004-ClassicalUncertaintyPrinciple.pdf
https://link.springer.com/article/10.1007/s10701-022-00555-z

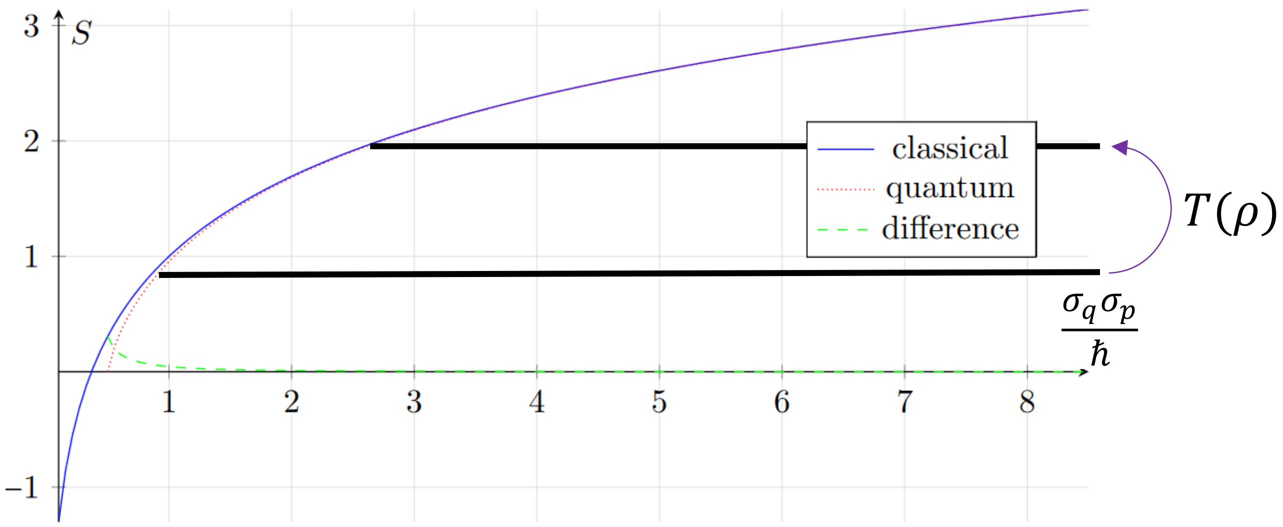
# Classical mechanics as high-entropy limit of quantum mechanics

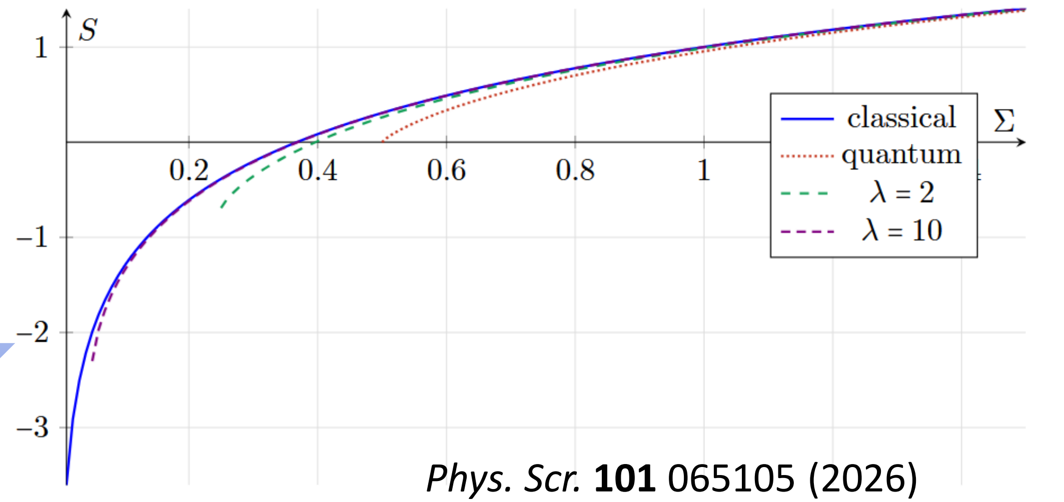
## Physical limit: $S \gg 0$

like $\frac{v}{c} \ll 1$

like $c \to \infty$

## Mathematical limit: $\hbar \to 0$




*Phys. Scr.* **101** 065105 (2026)

$$[X, P] = \imath\hbar \qquad [T(X), T(P)] = \lambda\imath\hbar$$

$$[T(\hat{X}), T(\hat{P})] = \imath\hbar$$

$$[\hat{X}, \hat{P}] = \frac{\imath\hbar}{\lambda}$$

Physical limit: increase the entropy of mixed states

Mathematical limit: decrease the entropy of pure states

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https://arxiv.org/pdf/2411.00972

https://iopscience.iop.org/article/10.1088/1402-4896/ae3a20

# Non-linear representation in quantum mechanics

## This map introduces a total phase based on all coefficients

const. $\theta \in (0,2\pi)$

$$m(c_+|z^+\rangle + c_-|z^-\rangle) = e^{\iota\theta \frac{|c_+|^2}{|c_+|^2+|c_-|^2}}(c_+|z^+\rangle + c_-|z^-\rangle)$$

Non-linear map…

… that preserves the physics
(i.e. Born rule)

$$\frac{\langle m(\psi)|m(\phi)\rangle\langle m(\phi)|m(\psi)\rangle}{\langle m(\psi)|m(\psi)\rangle\langle m(\phi)|m(\phi)\rangle} = \frac{\langle\psi|\phi\rangle\langle\phi|\psi\rangle}{\langle\psi|\psi\rangle\langle\phi|\phi\rangle}$$

Linearity is a mathematical convenience,
not a physical necessity

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https://assumptionsofphysics.org/autogen/briefs/008-NonLinearQuantum.pdf

# Superposition is multi-decomposition

Property of the **vector space representation** of states in quantum mechanics

Superposition: $|\phi\rangle = c_1|\psi_1\rangle + c_2|\psi_2\rangle$

Impossible in classical probability
(i.e. Choquet simplex)

$\Updownarrow$

Multi-decomposition: $\rho = p_{\phi}\rho_{\phi} + p_{\hat{\phi}}\rho_{\hat{\phi}} = p_1\rho_{\psi_1} + p_2\rho_{\psi_2}$

Property of **distinguishability of preparations**

Linear transformations

$\Updownarrow$

Output of mixtures = mixture of outputs

Indistinguishability of different mixtures

$\Updownarrow$

No physical process can depend on them

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https://assumptionsofphysics.org/autogen/briefs/002-SuperpositionAndMultipleMixtures.pdf

Reverse Physics

# Illuminating paradoxes and developing standards of rigor

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# Poor dimensional checking in statistical mechanics

Definition of Gibbs entropy

$$-\int_X \rho \log \rho \, dq^i \, dp_j \quad \Rightarrow -\int_X \rho \log h^n \rho \, dq^i \, dp_j$$

Density is not a pure number

When deriving Sackur–Tetrode equation for the entropy of a monatomic ideal gas

$$\Omega_N = \frac{1}{N!}\frac{V^N}{h^{3N}}\frac{2\pi^{3N/2}}{(\frac{3N}{2}-1)!}(\sqrt{2mU})^{3N-1} \approx \frac{1}{N!}\frac{V^N}{h^{3N}}\frac{\pi^{3N/2}}{(3N/2)!}(\sqrt{2mU})^{3N}.$$

You can't "approximate physical dimensions"!!!

$$length^N \neq length^{N-1}$$

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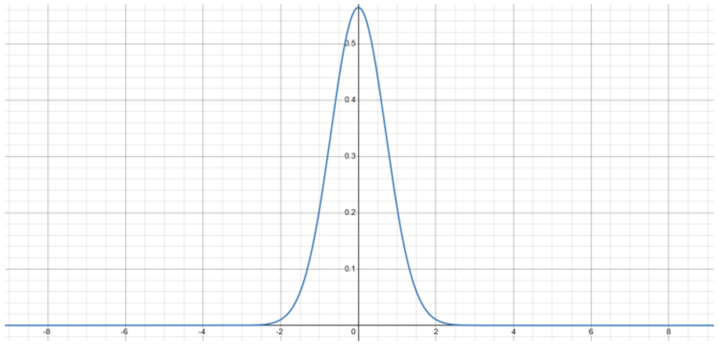
# Inconsistent expectations in Hilbert spaces

*Quantum Stud.: Math. Found.* **12**, 13 (2025)

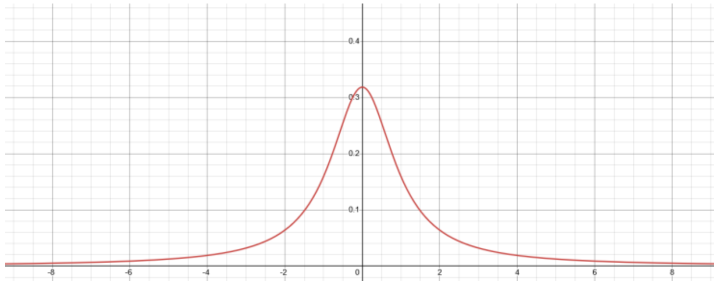
$$\psi(x) = \sqrt{\frac{e^{-x^2}}{\sqrt{\pi}}} \qquad \int |\psi|^2 dx = 1$$

$$\rho_\psi(x) = \frac{e^{-x^2}}{\sqrt{\pi}} \qquad \langle X^2 \rangle_\psi = \frac{1}{2}$$



$$\phi(y) = \sqrt{\frac{1}{\pi(y^2+1)}} \qquad \int |\phi|^2 dy = 1$$

$$\rho_\phi(y) = \frac{1}{\pi(y^2+1)} \qquad \langle Y^2 \rangle_\phi \to \infty$$



Different observers see finite/infinite expectation

$$y = \tan\left(\frac{\pi}{2}\operatorname{erf}(x)\right)$$

$$\psi(y) = \psi(x)\sqrt{\frac{dx}{dy}}$$

Expectation can have finite/infinite oscillations

$$x(x_0, t) = x_0 \cos^2\frac{\pi t}{2} + \tan\left(\frac{\pi}{2}\operatorname{erf}(x_0)\right)\sin^2\frac{\pi t}{2}$$

Every continuous linear operator defined on the whole Hilbert space is bounded ⇒ position/momentum/energy/number of particles are not defined on the whole Hilbert space!!!

https://arxiv.org/pdf/2308.06669
https://link.springer.com/article/10.1007/s40509-024-00357-0

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# Inconsistent convergence in Hilbert spaces

Consider the sequence of states given by

$$|\psi_i\rangle = \sqrt{\frac{i-1}{i}}|0\rangle + \sqrt{\frac{1}{i}}|i\rangle$$

$$\langle\psi_i|N|\psi_i\rangle = \frac{i-1}{i}\langle 0|N|0\rangle + \frac{1}{i}\langle i|N|i\rangle \to 1$$

$$|\psi_i\rangle \to |0\rangle$$

The state converges to the ground state
while the energy level converges to the first level

https://assumptionsofphysics.org/autogen/briefs/006-InconsistentConvergence.pdf

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# Inconsistent units for quantum observables

Suppose $X$ is an observable

Expectation $\langle\psi|X|\psi\rangle$ should have units of $X$

But then $X|\psi\rangle$ cannot have the same units of $|\psi\rangle$

But then $X|\psi\rangle + |\psi\rangle$ makes no physical sense

Therefore, $X$ cannot be a map within the same space

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Reverse Physics

# Identifying logical dependencies and developing/clarifying base theories

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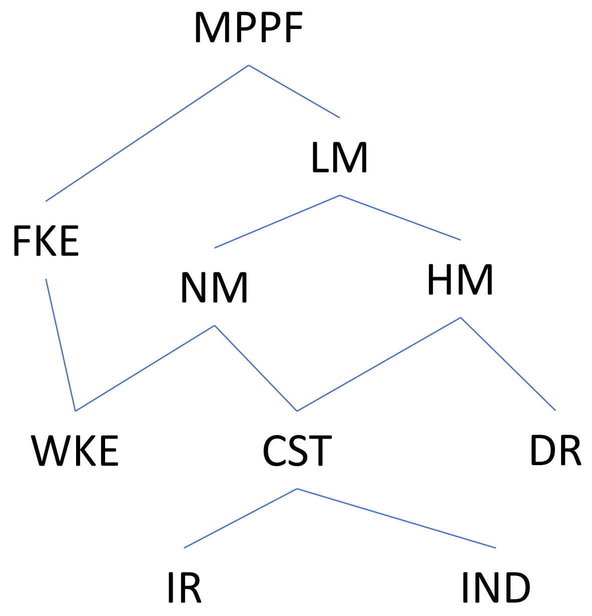
# Classical conditions and their relationships

## Core assumptions

IR: Infinitesimal Reducibility
IND: INDependent DOF
W/FKE: Weak/Full Kinematic Equivalence
DR: Determinism+Reversibility

## Physical theories

CST: Classical STates
NM: Newtonian Mechanics
HM: Hamiltonian Mechanics
LM: Lagrangian Mechanics
MPPF: Massive Particles under Potential Forces




## Each assumption is an equivalence class of multiple conditions

There is a linear relationship between conjugate momentum and velocity (FKE-LIN)

At each position, there exists a local inertial frame (FKE-INER)

The position fully defines the units of all state variables, and therefore an invertible transformation between momentum and velocity (FKE-UNIT)

Density expressed in velocity at the same position is proportional to the density over states (FKE-PROP)

Uniform distributions along momentum correspond to uniform distributions along velocity (FKE-UNIF)

The system under study is a massive particle under scalar and vector potential forces (FKE-POT)

The Jacobian of the transformation between state variables and kinematic variables is a non-singular function of position only. (FKE-NSIN)

Densities over phase space can be expressed in terms of position and velocity by rescaling the value at each point: $\rho(x^i, v^j)|J(x^i)| = \rho(q^i, p_j)$. (FKE-DEN)

Areas and volumes in phase space can be expressed in kinematic variables, and the transformation depends on position only: $dx^1 \cdots dx^n \, dv^1 \cdots dv^n = |J(x^i)| \, dq^1 \cdots dq^n \, dp_1 \cdots dp_n$. (FKE-VOL)

The symplectic form $\omega_{ab}$ can be expressed in kinematic variables, and its components are a linear function of velocity. (FKE-SYMP)

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https://assumptionsofphysics.org/book/AssumptionsOfPhysicsV3.0.pdf#chapter.1.1

# Goal: Create a base theory for all of physics

Collection of conditions that MUST be true in all physical theories.

Additional, different conditions or assumptions will then lead to different physical theories, e.g. classical Hamiltonian particle mechanics, nonrelativistic quantum mechanics, …

Any new, yet-undiscovered physical theory MUST still be built on the base theory

Base theory $\neq$ Theory of everything

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# Foundational pillars

The “big four” of Reverse Physics—core constitutive requirements of physical theories:

- Experimental verifiability
- Reproducibility and ensembles
- Ensemble variability
- Equilibria

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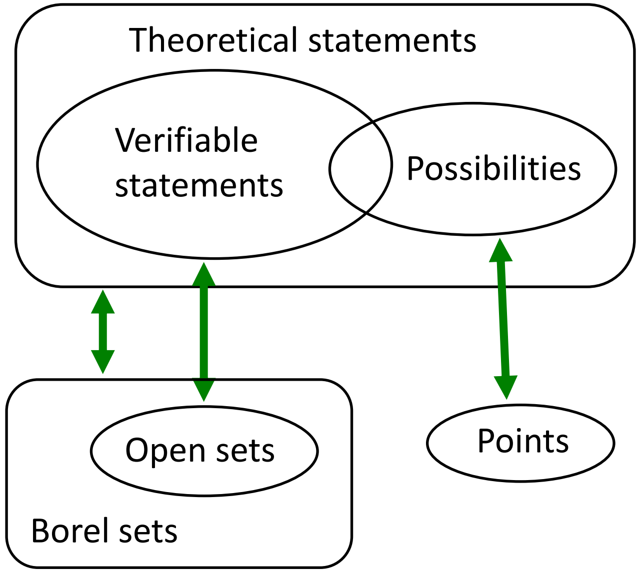
# Experimental verifiability

## Science is evidence based ⇒ Statements are associated to experimental tests

| **Verifiable statement** | Test Result |
|---|---|
| T | SUCCESS (in finite time) |
| F | UNDEFINED |
| | FAILURE (in finite time) |



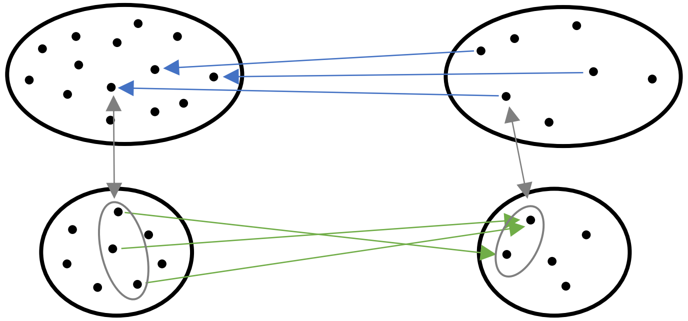

| $s$ | Test |
|---|---|
| T | S |
| | U |
| F | U |
| | F |

$int(A)$ corresponds to the verifiable part of a statement

$\partial A$ corresponds to the undecidable part of a statement

$ext(A)$ corresponds to the falsifiable part of a statement

Inference relationship $r: \mathcal{D}_Y \to \mathcal{D}_X$ such that $r(s) \equiv s$



Causal relationship $f: X \to Y$ such that $x \preccurlyeq f(x)$

## Experimental verifiability ⇒ topology and $\sigma$-algebra

Experimental relationships must be topologically continuous

https://assumptionsofphysics.org/book/AssumptionsOfPhysicsV3.0.pdf#section.2.1.1

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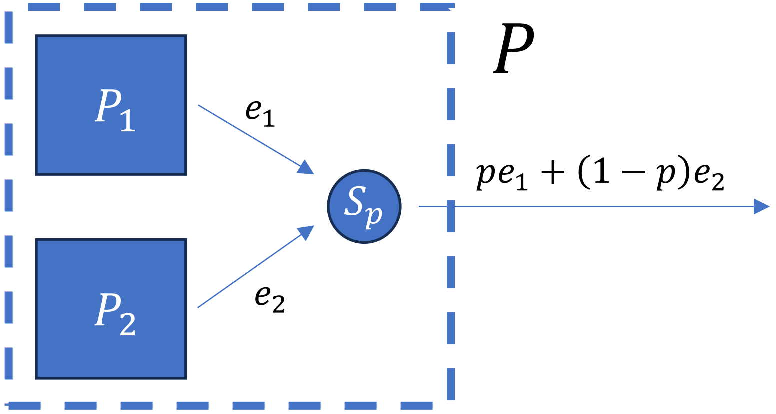
# Reproducibility and ensembles

## Scientific laws describe reproducible relationships $\Rightarrow$ Ensembles




Ensembles are outputs of repeatable processes

**Statistical mixing $\Rightarrow$ Linear structures in physics**

Note: mixing coefficients are parameters of the preparation,
NOT probabilities in the ordinary sense

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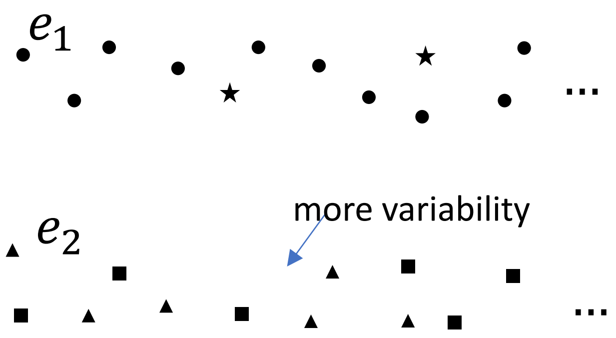
# Ensemble variability

## Variability of the elements within an ensemble ⇒ Entropy




Mutually exclusive ⟺ orthogonality

⇕

Maximal variability increase during mixing

## **Variability ⇒ Probabilities, inner product and geometry**

Probability ⟺ Mixtures of mutually exclusive ensembles

Classical mechanics/probability ⟺ All pure states are mutually exclusive

Pure states ⟺ Best preparations ⇎ No variability

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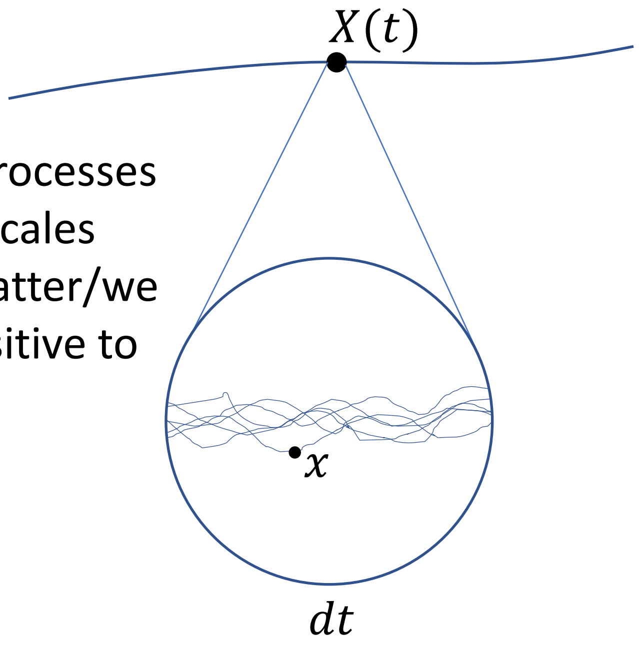
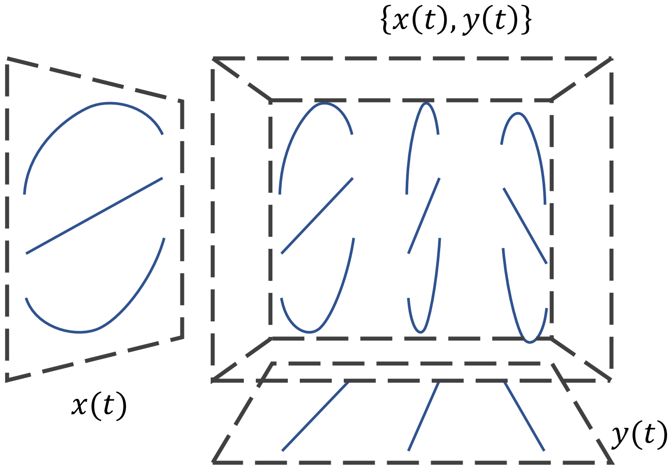
# Equilibria

## States must exist for at least a short time ⇒ Equilibria

Of internal processes
at faster scales
that don't matter/we
are not sensitive to




Of system/environment
interactions
that don't matter/we
are not sensitive to

Least developed part

This requirement allows us to properly talk about different physical objects

**Equilibria ⇒ Symmetries, differentiability, Lie algebras**

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Assumptions of Physics

# Wrapping it up

- Assumptions of Physics: different approach to the foundations of physics
  - No interpretations, no theories of everything: physically meaningful starting points from which we can rederive the laws and the mathematical frameworks they need
  - Physical theories are models
    - Need to clarify exactly what the realm of applicability of each model is
- Reverse Physics: reverse engineer principles from the known laws
  - Current laws are given through mathematical structures, which make it hard to understand the physics
  - We have to find physical principles and assumptions from which to rederive the laws
  - These starting points must have a clear intuitive meaning for the majority of physicists and engineers
  - Investigate physics as a whole (no theory is "more fundamental")

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Assumptions of Physics

Reverse Physics gives us a powerful new methodology for the foundations of physics, more in line with foundational approaches of other disciplines, offering a lot more—and more precise—insights and opening new avenues of research

Reverse Physics

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Assumptions of Physics

# Assumptions of Physics is an open research program

Our main output is an open access book: https://assumptionsofphysics.org/book/

All our material is developed on GitHub: https://github.com/assumptionsofphysics

Livestream discussions

One YouTube channel dedicated to publicize results: https://www.youtube.com/user/gcarcassi

Another YouTube channel dedicated to research: https://www.youtube.com/@AssumptionsofPhysicsResearch

Activities coordinated through a Discord server (contact info@assumptionsofphysics.org for an invite)

Always looking for experts to gain insights and/or help

Always looking for collaborations

Always looking for editors/journals/conferences that are sympathetic to the mission

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