Assumptions of Physics
Summer School 2025

Open Problems in Reverse Physics

Gabriele Carcassi and Christine A. Aidala

Physics Department
University of Michigan

Overview

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Classical mechanics

$$d_t \xi^a \omega_{ab} = \partial_b H$$

Essentially done, though improvements are always possible

Classical field theory

$$\mathcal{A} = \int \left[\frac{1}{2\kappa} R + \mathcal{L}_M \right] \sqrt{-g} d^4x$$

Vague general ideas, more technical work needed

Quantum mechanics

$$\imath\hbar d_t |\psi\rangle = H |\psi\rangle$$

Core ideas identified, need to be organized and written down

Quantum field theory

$$\mathcal{A} = \int \bar{\psi} (\imath \gamma^\mu D_\mu - m) \psi \, d^4x$$

No point in starting now, first finish QM and CFT

Thermodynamics

$$\Delta S \geq \int \frac{\delta Q}{T}$$

Some ideas identified, need to be finalized

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Classical field theory

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Assumptions of Physics

Status

- Actively looking for someone to help generalize Reverse Physics to classical field theory (from finitely many DOFs to continuously many DOFs)
  - Main conjecture: volumes in space provide a count of DOFs, in the same way that volumes in phase space provide a count of states/configurations
- Need to
  - Find a suitable Hamiltonian/symplectic formulation of field theory (starting with electromagnetism)
  - Understand how to generalize count of states/entropy/… to field theory
  - Adapt the arguments from classical mechanics to field theory

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Geometry of principle of least action (SDOF)

DR

$$\nabla \cdot \vec{S} = 0$$

No state is "lost" or
"created" as time evolves

$[p, 0, -H]$

$$\vec{S} = -\nabla \times \vec{\theta}$$

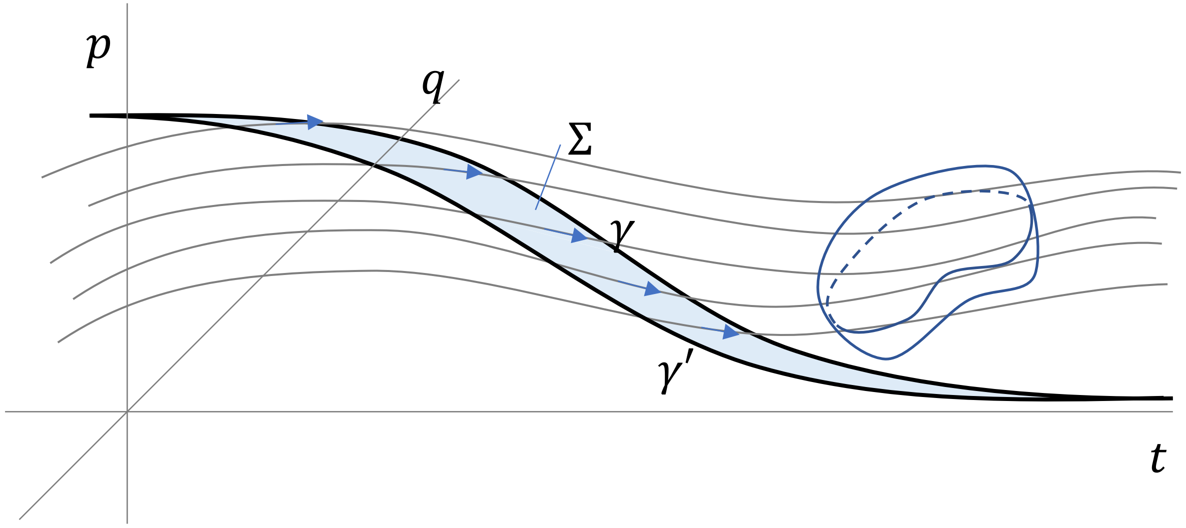
(Minus sign to match convention)

KE $\quad pd_tq + 0d_tp - Hd_tt$

$$\mathcal{S}[\gamma] = \int_\gamma L dt = \int_\gamma \vec{\theta} \cdot d_t \xi^a dt$$

Sci Rep **13**, 12138 (2023)

The action is the line integral of the vector potential (unphysical)



Variation of the action

$$\delta\mathcal{S}[\gamma] = \oint_{\partial\Sigma} \vec{\theta} \cdot d\vec{\gamma}$$
$$= -\iint_\Sigma \vec{S} \cdot d\vec{\Sigma}$$

Gauge independent,
physical!

Variation of the action measures the flow of states (physical).
Variation = 0 ⇒ flow of states tangent to the path.

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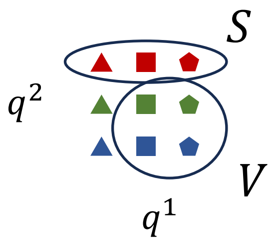
Counting states and configurations

Discrete case



$\#\text{conf}(S) = \#S \qquad \#\text{states}(V) = \#V \qquad \#\text{DOF}(I) = \#I$

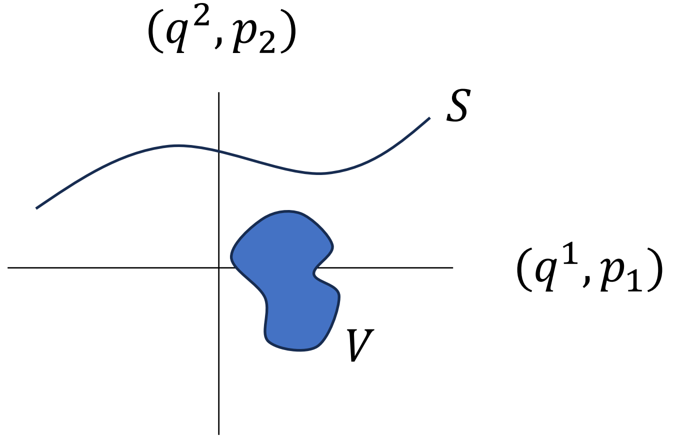
Continuous case

Configuration density

State density

$\neq \#V = \infty$

$\#\text{conf}(S) = \int_S \omega_{ab} d\xi^a d\xi^b \qquad \#\text{states}(V) = \int_V \wedge \omega \, d\xi^{a_1} \cdots d\xi^{a_n}$



$\#\text{DOF}(I) = \#I$

Ham Mech ⇒ Correct count of configurations/states on finitely many dense (i.e. continuous) DOFs

Field theory ⇒ DOFs themselves are dense (i.e. continuous)

$\#\text{DOF}(I) \neq \#I$

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Conjecture: GR ⇔ det/rev + DOF independence for infinitely many (dense) DOFs

$$\delta \int_\gamma L dt = \oint_{\partial\Sigma} \theta_a d\gamma^a = \iint_\Sigma \omega_{ab} d\xi^a d\gamma^b$$

$$\delta \int_\gamma \mathcal{L} d^4x = ???$$

Flow of states

$$\frac{\#\text{conf}}{\#\text{DOFs}} = ?$$

Line integral of the vector potential of the flow of state **density**?

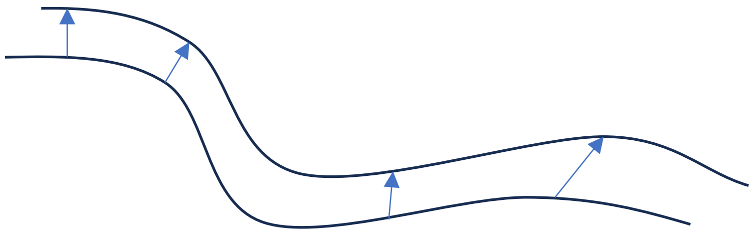
$$\int_U \sqrt{-g}\, d^3x$$

#DOFs?

$$\mathcal{L} = \mathcal{L}_g + \mathcal{L}_{matter}$$

Flow of DOFs?

Flow of configurations?



We are mapping values between Cauchy surfaces,
#DOFs are the points on the Cauchy surface,
#conf are the possible field values at each point

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Thermodynamics

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Assumptions of Physics

Status

- Key ideas in Reverse Physics for Thermodynamics have (probably) been identified, though the complete investigation needs to be carried out
  - Entropy in physics is connected to the count of evolutions (fluctuations)
  - Statistical equilibrium maximizes the count of evolutions, thermodynamic equilibrium is when all parts are in statistical equilibrium with each other
- Equilibrium is fundamental as it is equivalent to requiring that the dynamics of the system does not depend on the internal dynamics or the environment
- ⇒ In this sense, thermodynamics is a more fundamental theory than classical or quantum mechanics
- We give an overview of how it should work

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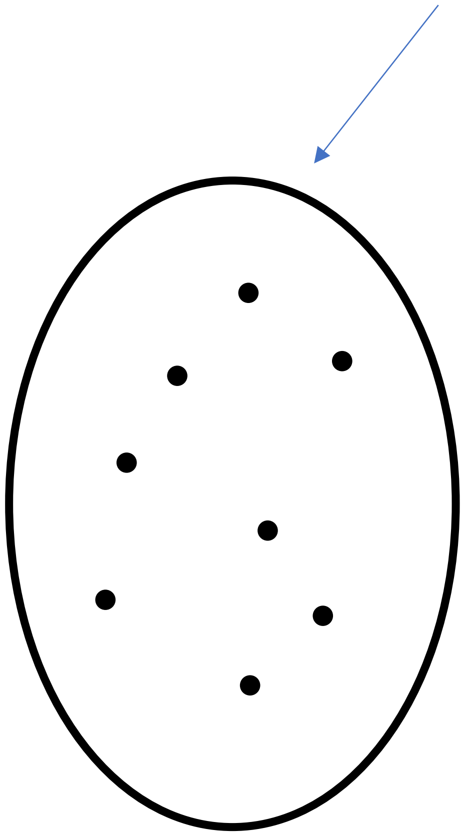
Entropy as logarithm of the count of evolutions

(e.g. classical microstates, quantum state, state of a dynamical system, …)

State space of a system



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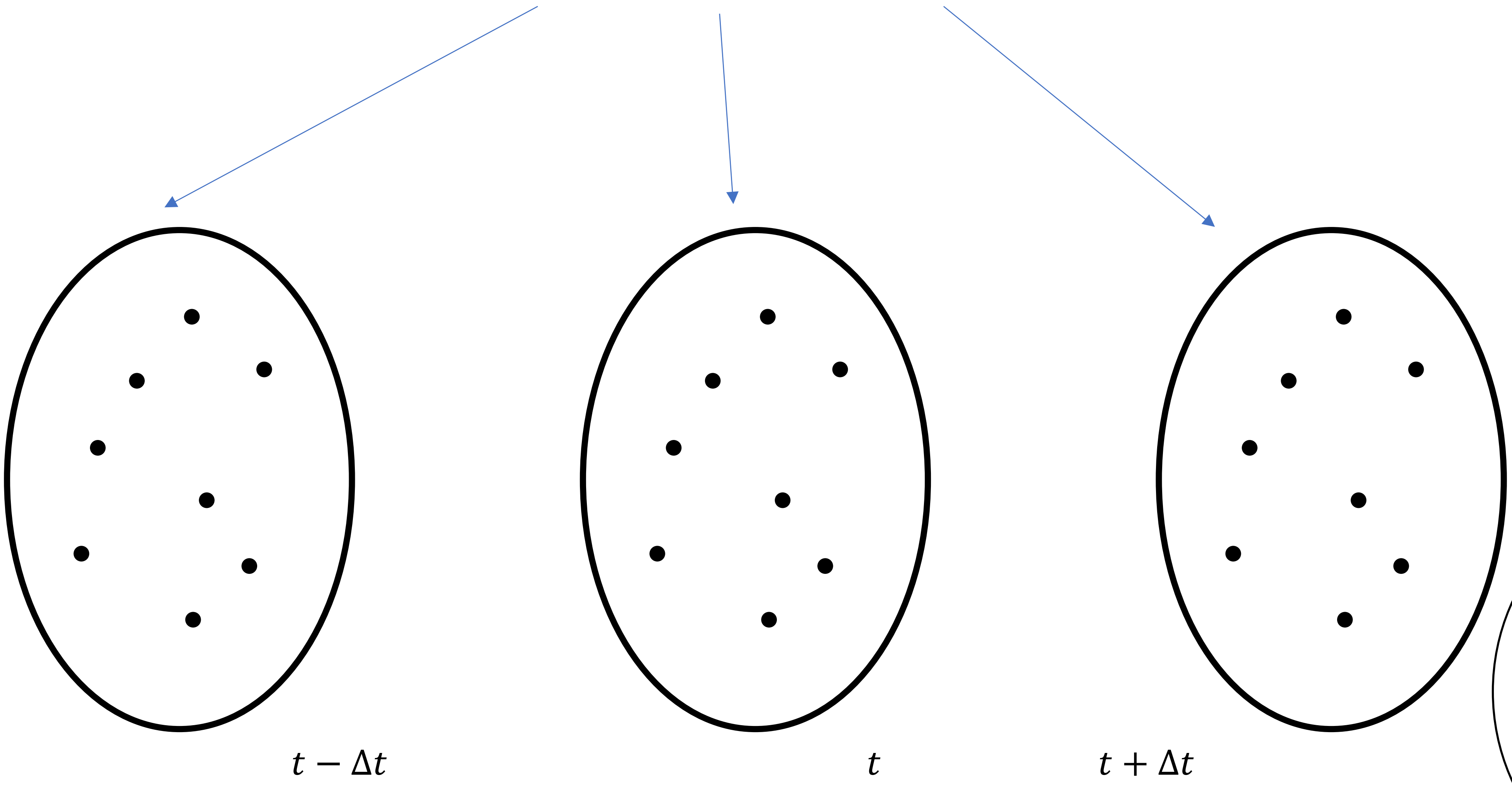
State space of a system at different times



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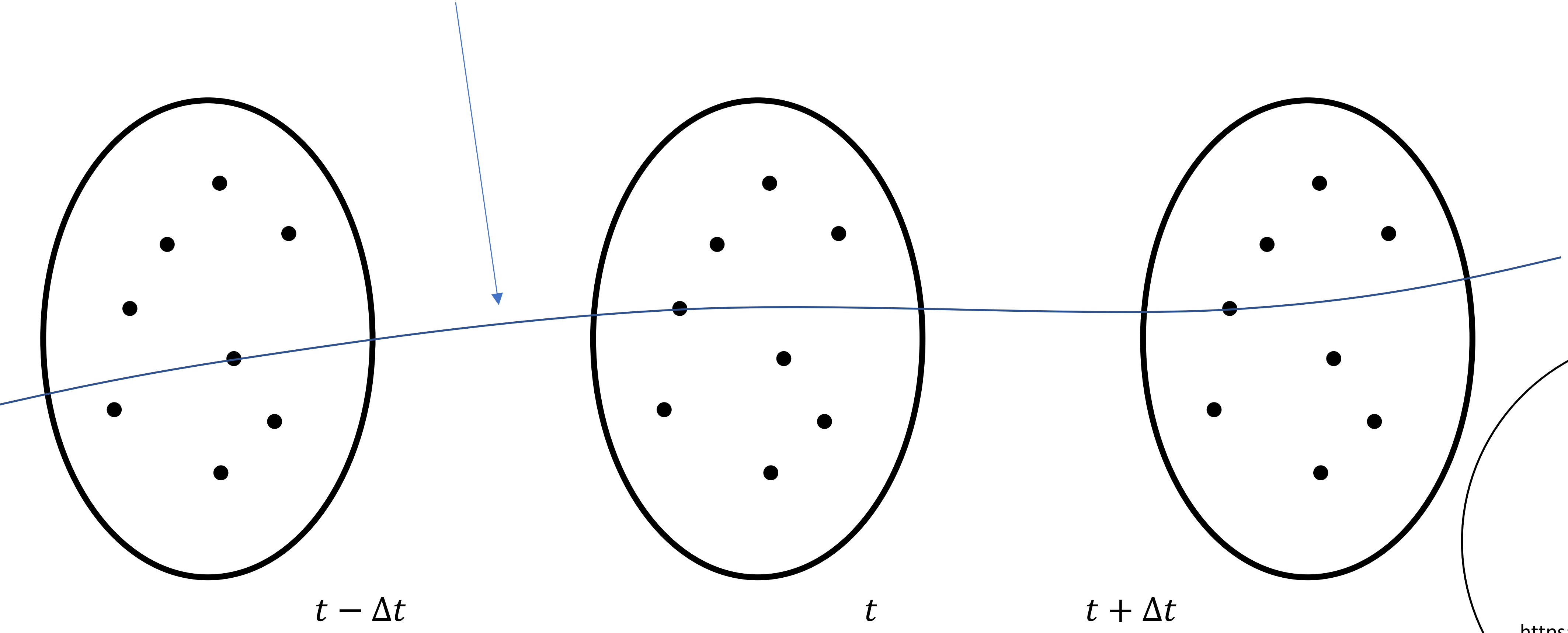
Evolution: complete description of the system at all times



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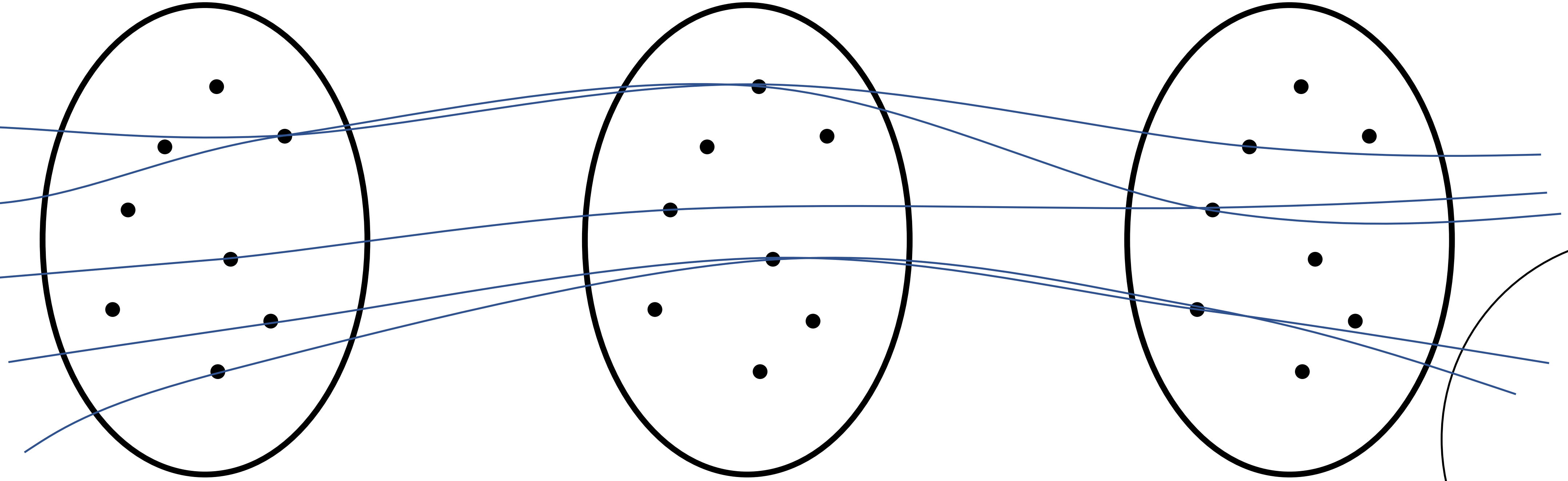
A process defines the set of all possible evolutions



$t - \Delta t$ $\quad$ t $\quad$ $t + \Delta t$

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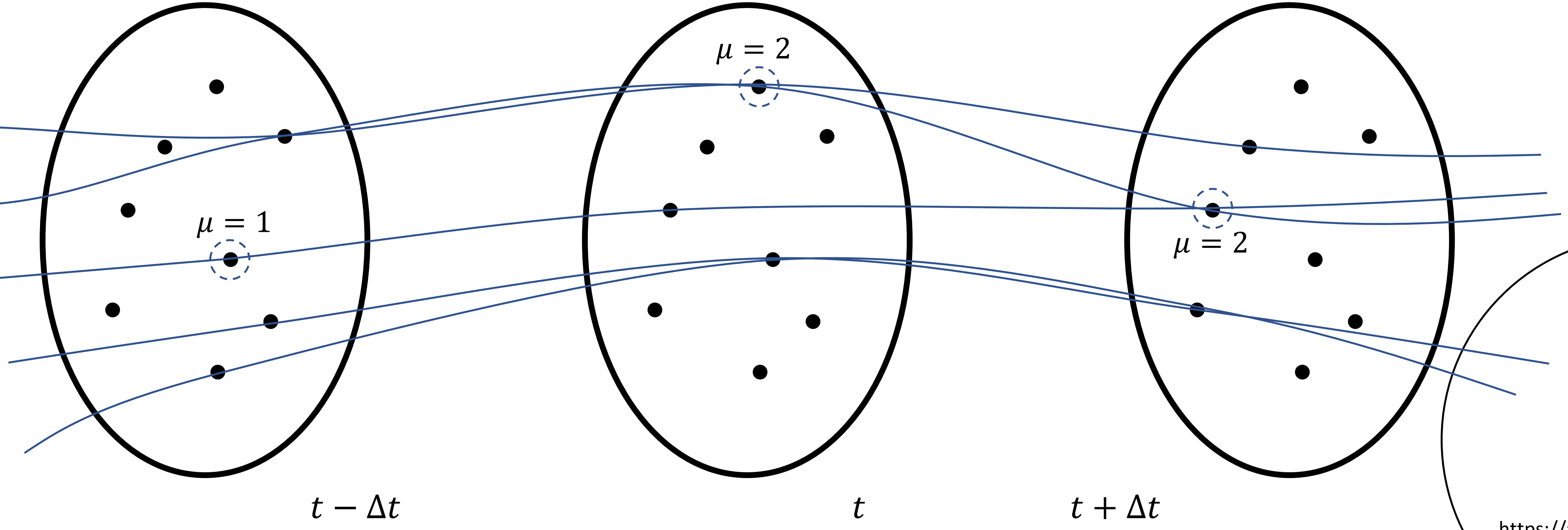
Each description at each time corresponds to a set of evolutions

We can count the number of evolutions for a particular state s at a particular time with a measure* $\mu(s)$ since s identifies a set of evolutions.



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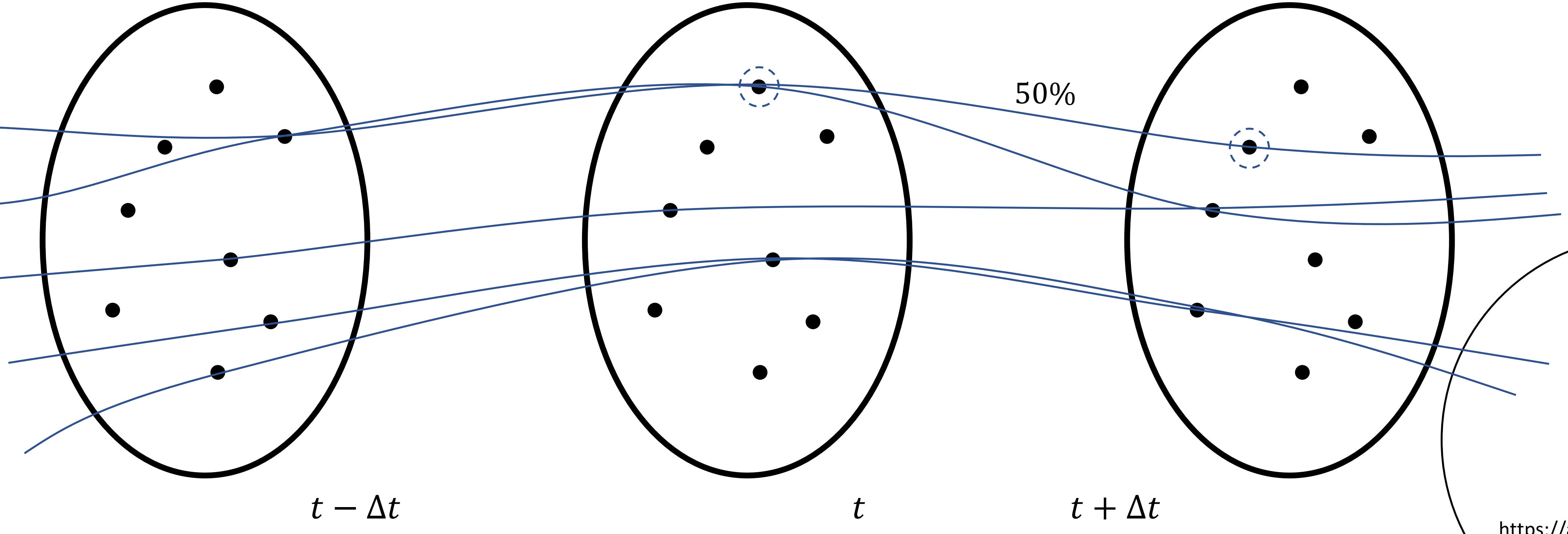
Assumptions of Physics

** The full mathematical characterization may be a bit more complicated (e.g. preorder that leads to a family of measures)…*

The probability $P(s_1|s_0)$ of having s_1 given s_0 corresponds to the fraction of evolutions that go from state s_0 to state s_1

That is, $P(s_1|s_0) = \frac{\mu(s_0 \cap s_1)}{\mu(s_0)}$



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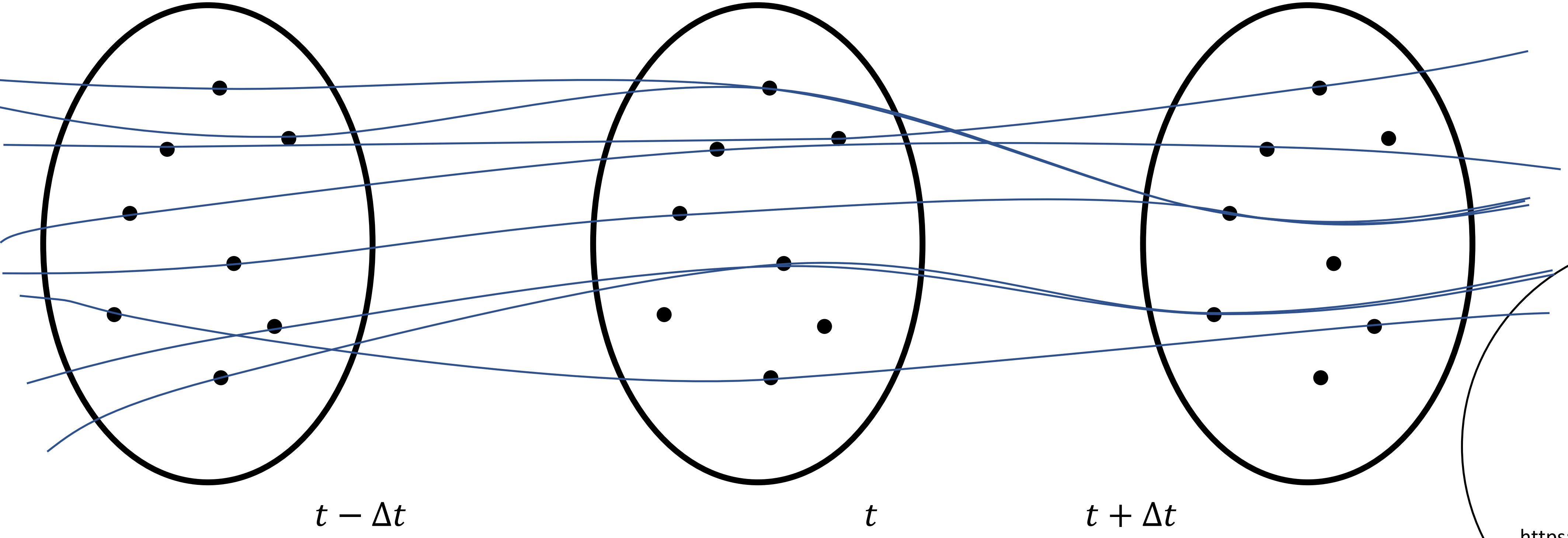
Assumptions of Physics

A process is deterministic if knowing the state at a time allows us to predict the state at a future time

For these processes, we can properly write a law of evolution $s(t + \Delta t) = f\big(s(t)\big)$



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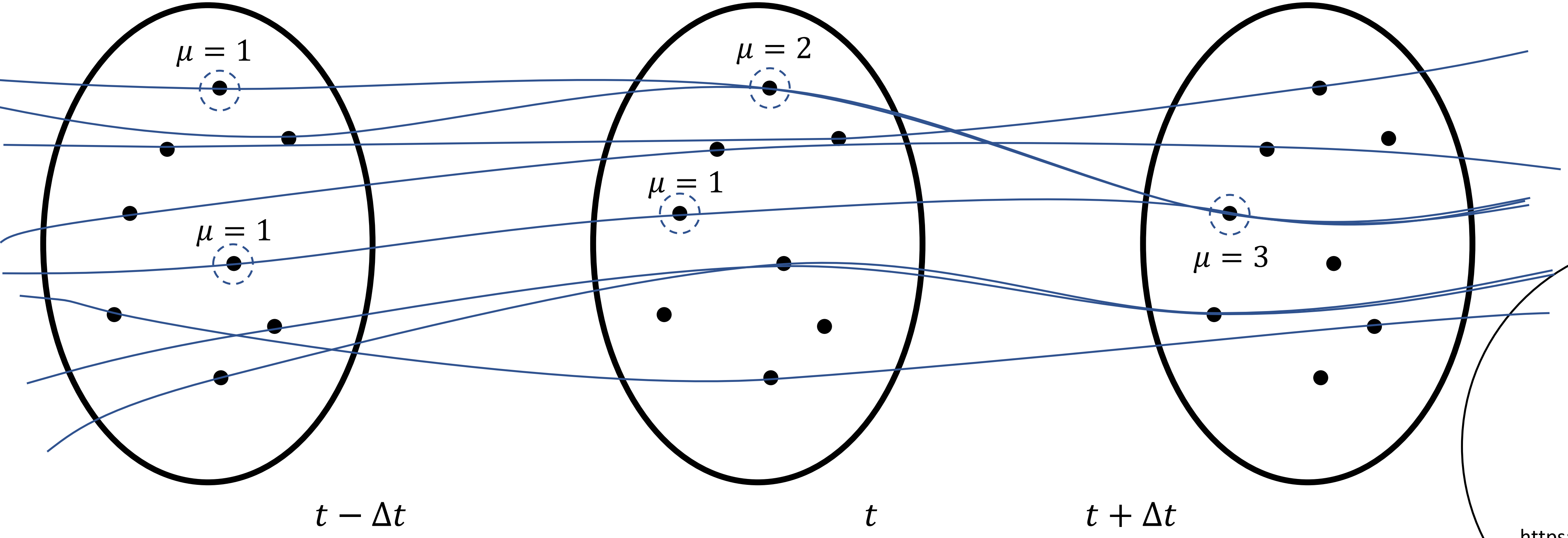
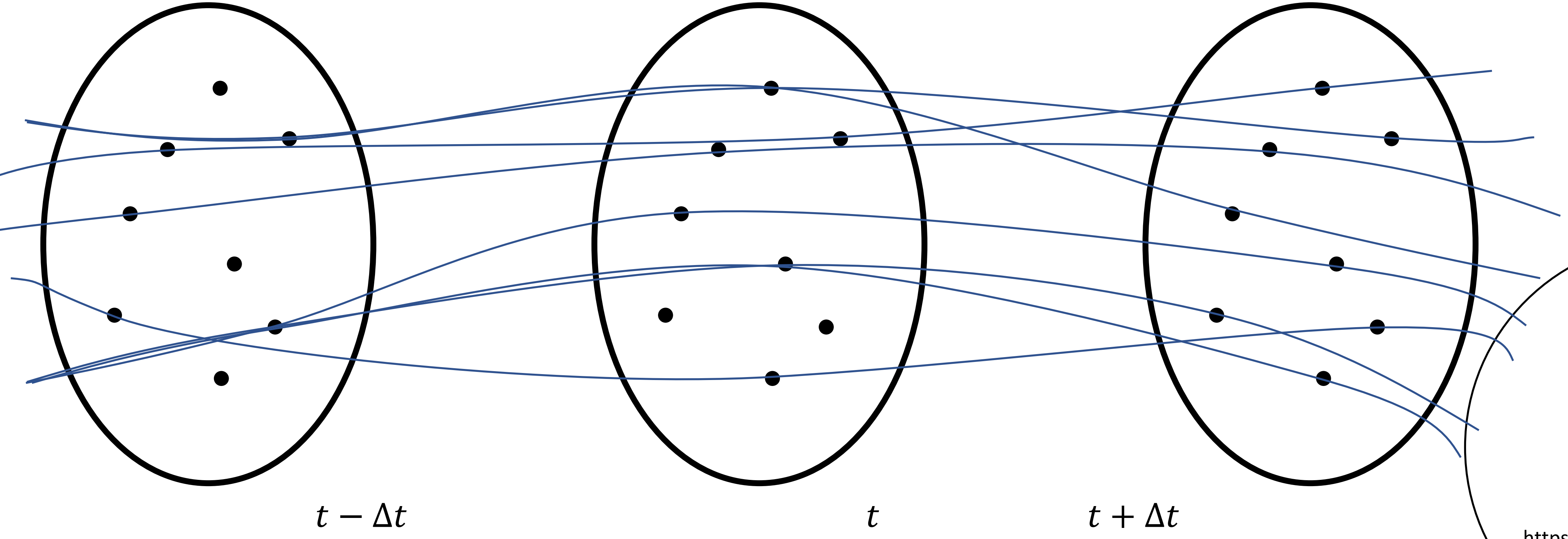
Assumptions of Physics

In a deterministic process, the evolutions can never split, only merge

That is, $\mu\big(s(t+\Delta t)\big) \geq \mu\big(s(t)\big)$



A process is reversible if knowing the state at a time allows us to reconstruct the state at a past time



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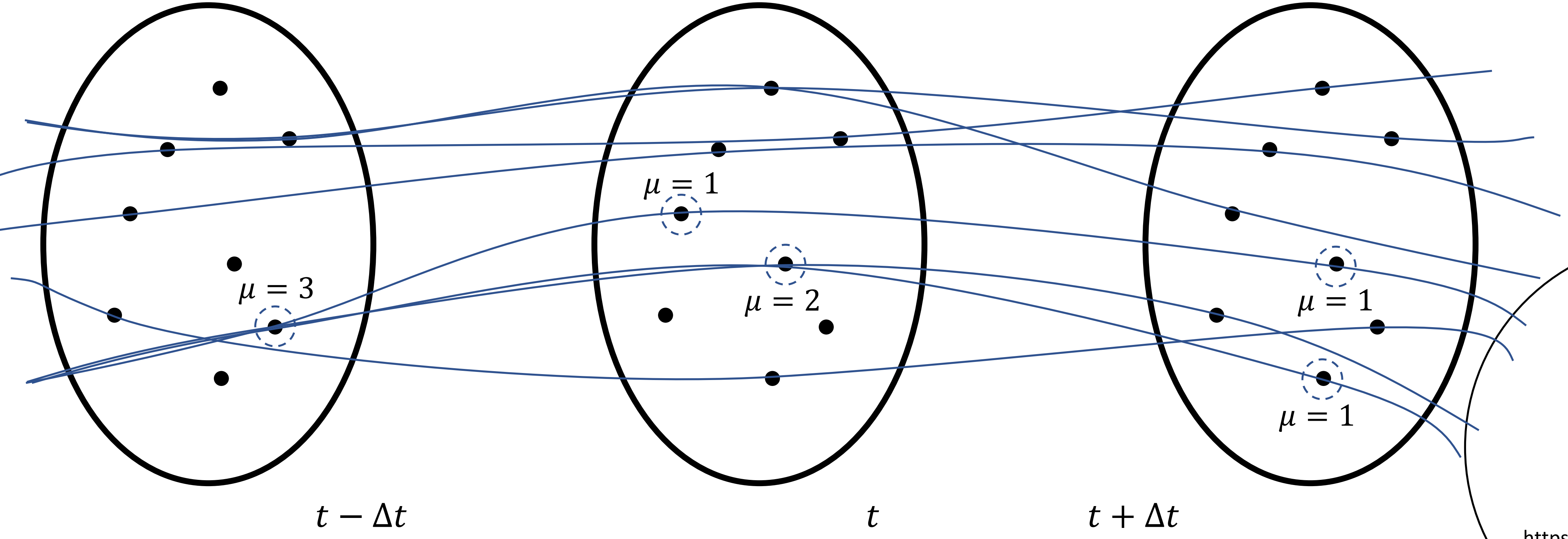
Assumptions of Physics

In a reversible process, the evolutions can never merge, only split

That is, $\mu(s(t+\Delta t)) \leq \mu(s(t))$



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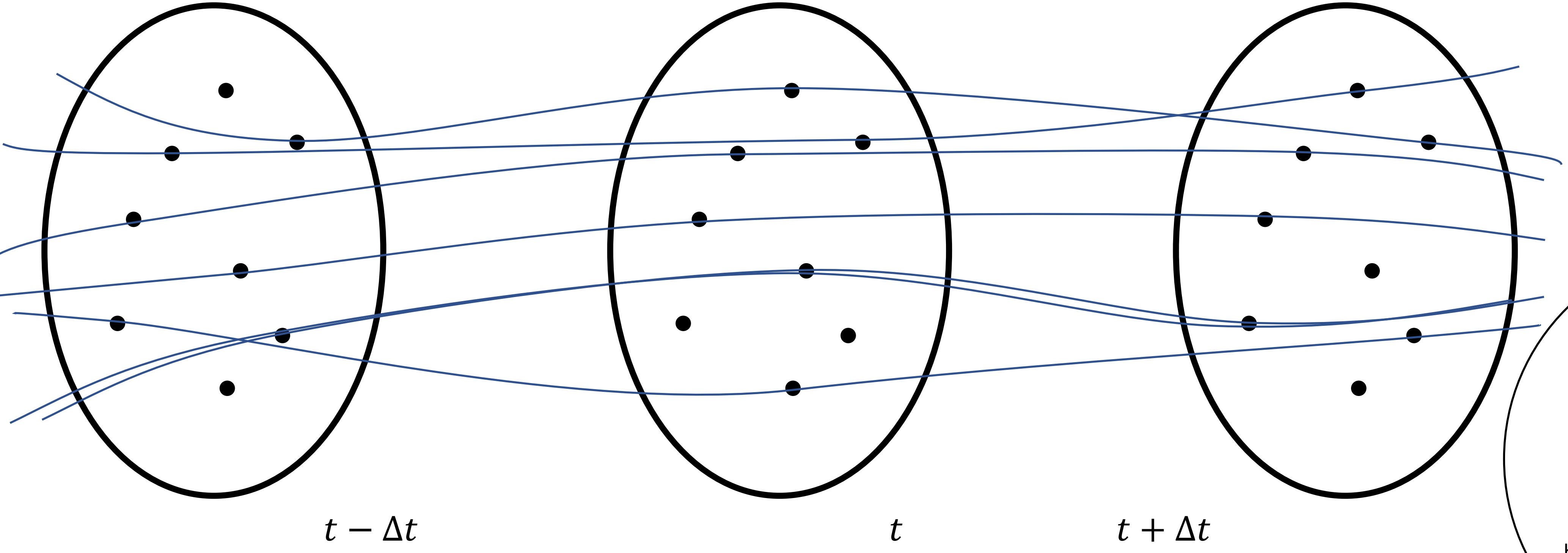
Assumptions of Physics

In a det/rev process, evolutions can never merge nor split

That is, $\mu\big(s(t+\Delta t)\big) = \mu\big(s(t)\big)$



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Assumptions of Physics

For a deterministic process

$$\mu\big(s(t+\Delta t)\big) \geq \mu\big(s(t)\big)$$

(equal if reversible)

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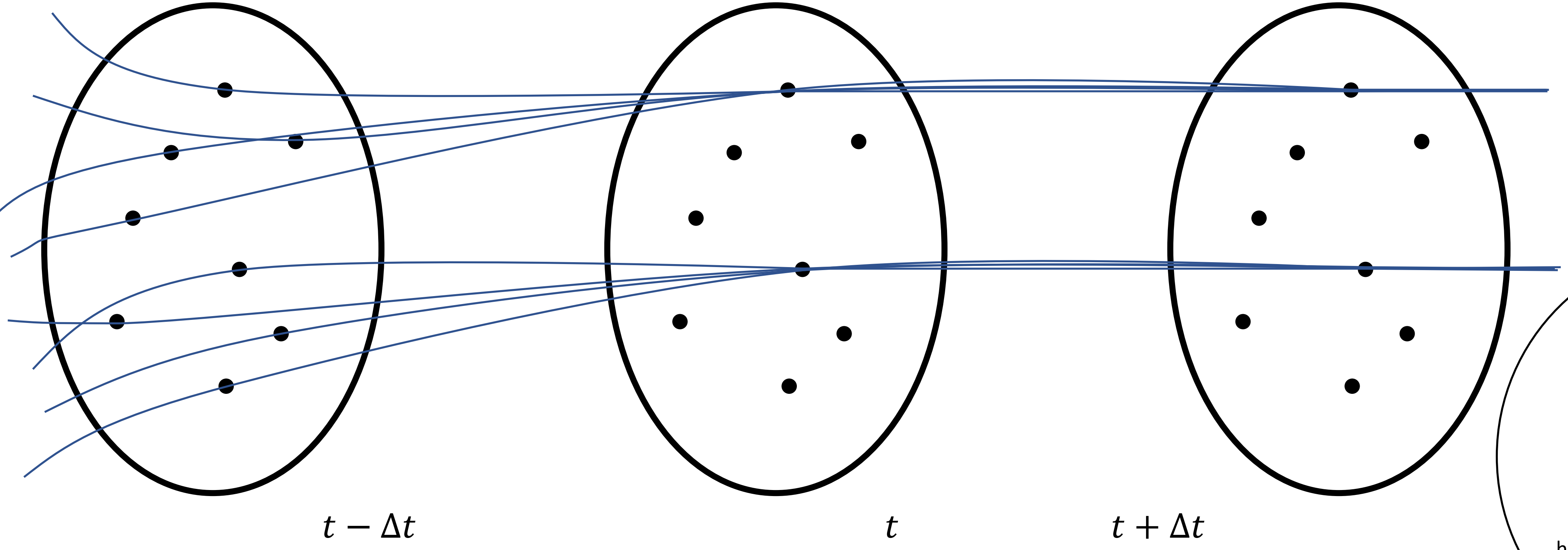
Assumptions of Physics

Consider a process where a system reaches a final equilibrium that can be predicted from the initial state

At equilibrium, evolutions cannot merge anymore



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Assumptions of Physics

For a deterministic process

$$\mu\big(s(t+\Delta t)\big) \geq \mu\big(s(t)\big)$$

(equal if reversible)
(maximum at equilibrium)

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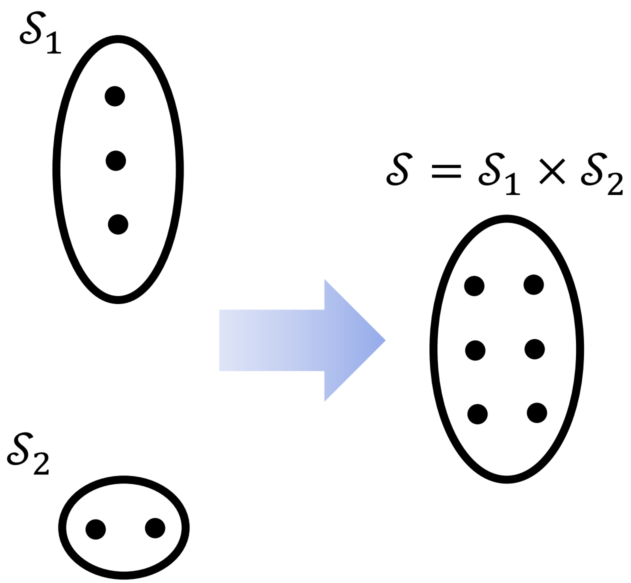
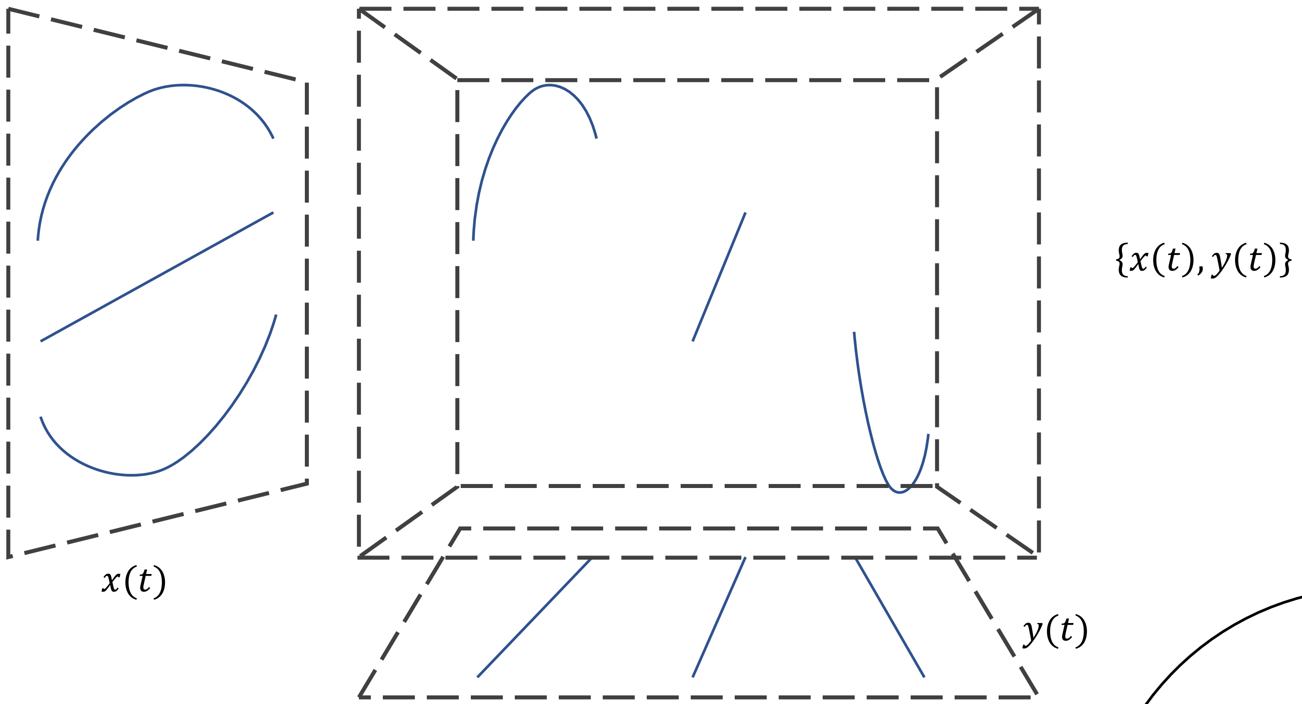
Assumptions of Physics

Suppose we have a composite of two systems



State space
is the product



Evolutions, in general, are
not the product: there may
be correlations

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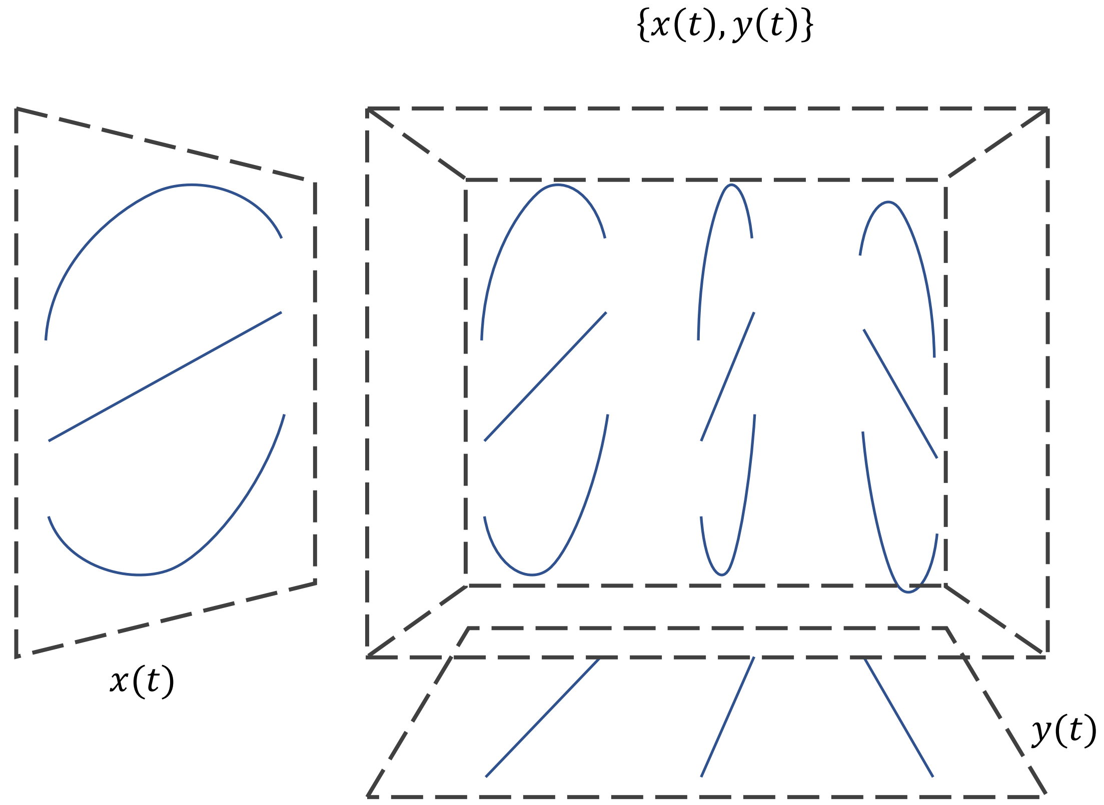
Assumptions of Physics



If the systems are independent, the evolution of one does not constrain the evolution of the other: all pairs are possible

For each pair of states there is a state for the composite system: $\#(\mathcal{S}) = \#(\mathcal{S}_1)\#(\mathcal{S}_2)$

For each pair of evolutions there is an evolution of the composite: $\mu(s_1 \cap s_2) = \mu_1(s_1)\mu_2(s_2)$

Note: $\log \mu = \log \mu_1 \mu_2 = \log \mu_1 + \log \mu_2$

$\log \mu$ is additive for independent systems

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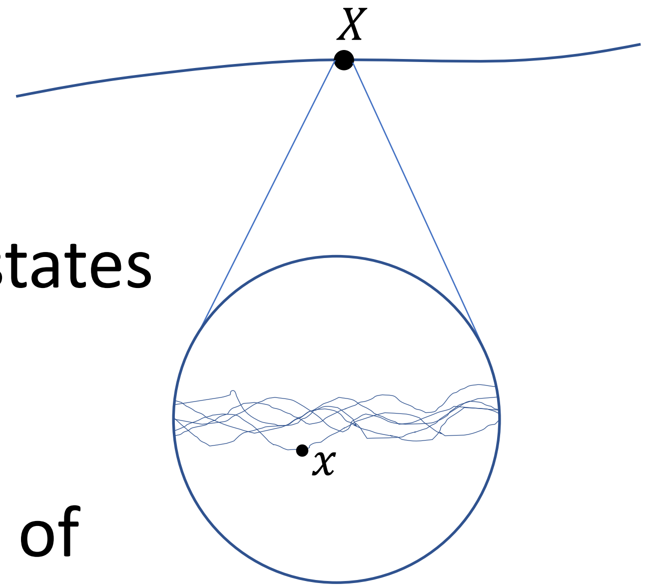
Assumptions of Physics

Define the process entropy as $S = \log \mu$
The log of the count of evolutions per state

It is additive for independent systems
$$S = S_1 + S_2$$

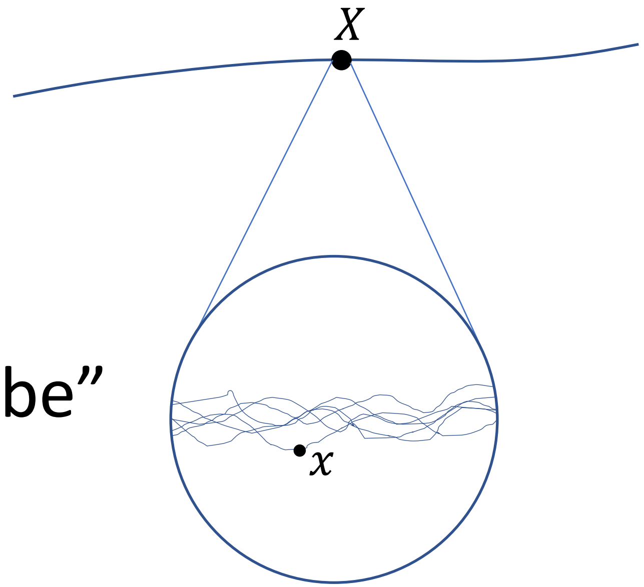
For a deterministic process
$$S\big(s(t + \Delta t)\big) \geq S\big(s(t)\big)$$
(equal if reversible)
(maximum at equilibrium)

Recover information entropy



Suppose macrostate X is a dynamical equilibrium of microstates
⇒ Microstate fluctuations can be characterized by a stable probability distribution $\rho(x)$ in each small Δt
⇒ An evolution $\lambda(t)$ for the microstate is a dense sequence of infinitely many microstates whose recurrence matches $\boldsymbol{\rho}$
⇒ Macrostate measurements cannot be sensitive to permutations of microstates: possible evolutions equals all possible permutations
⇒ Shannon entropy counts all possible permutations of an infinite sequence (dense in Δt) which equals all possible evolutions

Macrostate/equilibrium is a “tube” of evolutions at the lower level



Equilibrium means that no evolution enters or exits the “tube”
(i.e. external agents have equilibrated)

The variables describing the macrostate can still change
(i.e. the “tube” moves around in state space)

We can now study the “tube” as if it were single line
(i.e. internal dynamics and environment are decoupled)

Equilibrium = evolution decoupled from environment and from internal dynamics

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Assumptions of Physics

Physical entropy explained?

- We have not mentioned uncertainty, disorder, statistical distributions, information, lack of information, …
  - Therefore those concepts are not fundamental in this context
- We have not discussed what type of state or system we have (classical, quantum, biological, economic, …)
  - Therefore everything we said is valid independently of the type of system, which would explain the success of thermodynamic ideas outside the realm of physics
- The process entropy increase is explained by the definitions and the settings (processes with equilibria that depend on the initial state)
  - The explanation is straightforward (i.e. does not require a complicated discussion)
  - The explanation is not mechanical (i.e. given by a particular mechanism)

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Assumptions of Physics

“Reversing” thermodynamics

Assume states are equilibria of faster scale processes

Assume states identified by extensive properties

Assume one of these quantities is energy U

$$S(U, x^i)$$

Existence of equation of state

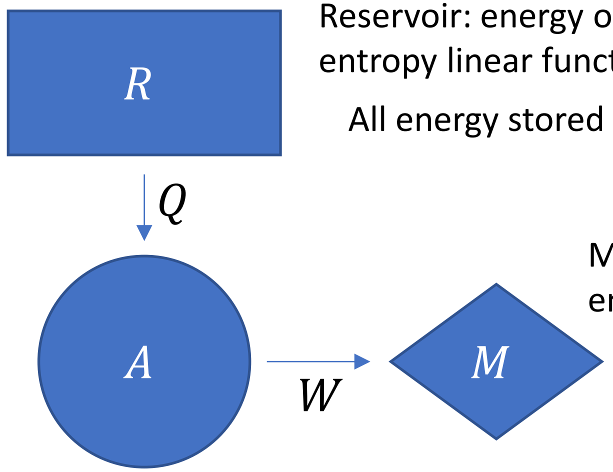
$$\beta = \frac{1}{k_B T} = \frac{\partial S}{\partial U} \quad \text{and} \quad -\beta X_i = \frac{\partial S}{\partial x^i}$$

Define intensive quantities

$$dS = \frac{\partial S}{\partial U} dU + \frac{\partial S}{\partial x^i} dx^i = \beta dU - \beta X_i dx^i$$
$$k_B T dS = dU - X_i dx^i$$
$$dU = T(k_B dS) + X_i dx^i$$

Recover usual relationships

Study interplay of changes of energy and entropy



Reservoir: energy only state variable, entropy linear function of energy

All energy stored in entropy

Mechanical system: same entropy for all states

No energy stored in entropy

$$\beta = \frac{1}{k_B T} = 0$$

$$\Delta U = 0 = \Delta U_A + \Delta U_R + \Delta U_M$$
$$= \Delta U_A - Q + W$$

Recover first law

First law recovered from existence and conservation of Hamiltonian

$$0 \leq \Delta S = \Delta S_A + \Delta S_{\mathrm{R}} + \Delta S_{\mathrm{M}}$$
$$= \Delta S_A + \beta_R \Delta U_R + 0 = \Delta S_A + \frac{-Q}{k_B T_R}$$

Recover second law

Second law recovered from definition of entropy as count of evolutions

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Quantum mechanics

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Assumptions of Physics

Status

- Clear that irreducibility/lower bound on the entropy is the only ingredient that makes quantum mechanics different from classical mechanics
- The goal is to develop a "minimal interpretation" based on what, and only what, can be settled experimentally
  - States are ensembles (pure or mixed) at equilibrium (internal dynamics and environment can be ignored)
  - Projections are processes of equilibration
    - Every state is the output of an equilibration process
    - Measurements are special cases
  - Unitary evolution is deterministic and reversible dynamics
    - Can also be recovered as quasi-static evolution (i.e. series of infinitesimal projections)
- All elements are probably there, scattered in videos/papers/… and need only to be put together

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Assumptions of Physics

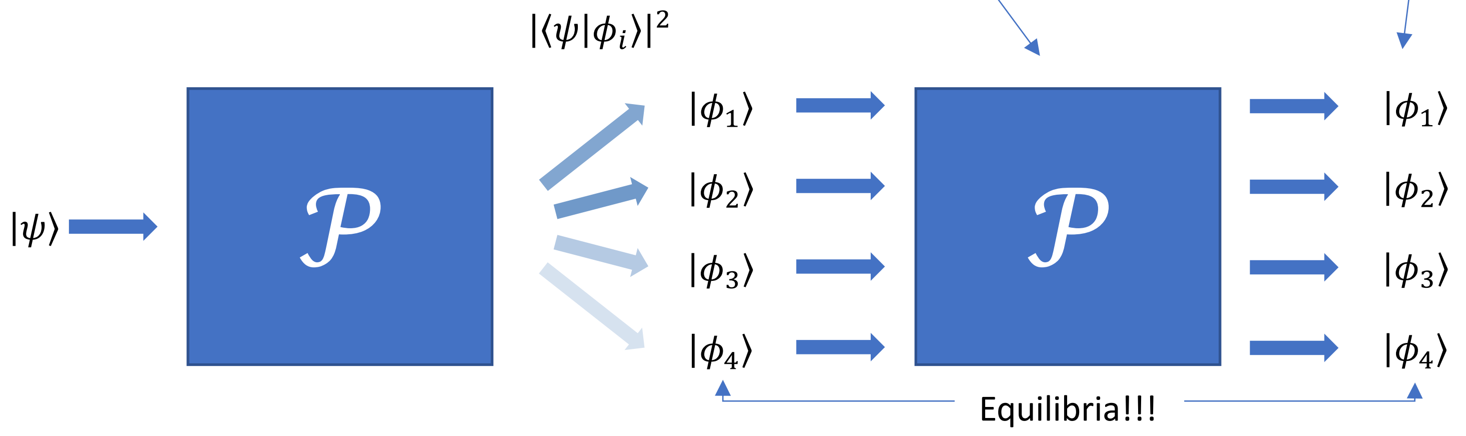
Projections: linear idempotent operations

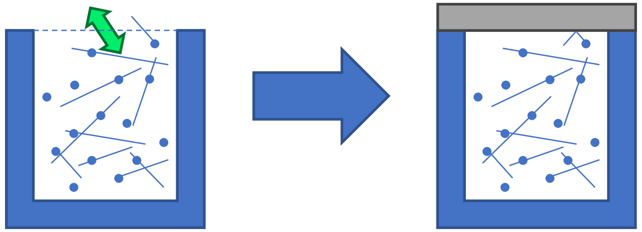
Output for a statistical mixture of inputs is the statistical mixture of outputs
$P(p_1\rho_1 + p_2\rho_2) = p_1 P(\rho_1) + p_2 P(\rho_2)$

Same result if you do it twice

Repeat process

Same result





$[\mu, V, T]$

$[N_1, V, T]$
$[N_2, V, T]$
$[\dots, V, T]$

Different equilibria, different variables

$|x^+\rangle$

$|z^+\rangle$
$|z^-\rangle$

Different contexts, different variables

Projections are equilibration processes

Quantum contexts are different boundary conditions

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Assumptions of Physics

Unitary evolution ⟺ det/rev evolution

$$|\psi(t+dt)\rangle - |\psi(t)\rangle = \mathcal{T}(t)dt|\psi(t)\rangle$$

Change of states depends only on previous state (determinism)

$$\langle\psi(t+dt)|\psi(t+dt)\rangle = 1$$

Final state is normalized (reversibility)

$$= \langle(1+\mathcal{T}(t)dt)\psi(t)|(1+\mathcal{T}(t)dt)\psi(t)\rangle$$

$$= \langle\psi(t)|(1+\mathcal{T}(t)dt)^{\dagger}(1+\mathcal{T}(t)dt)|\psi(t)\rangle$$

$$= \langle\psi(t)|1+\mathcal{T}(t)^{\dagger}dt+\mathcal{T}(t)dt+\mathcal{T}(t)^{\dagger}\mathcal{T}(t)dt^{2}|\psi(t)\rangle$$

$$= 1 + dt\langle\psi(t)|\mathcal{T}(t)^{\dagger}+\mathcal{T}(t)|\psi(t)\rangle + O(dt^{2})$$

$$\Rightarrow \mathcal{T}(t)^{\dagger} = -\mathcal{T}(t)$$

Also recovered from entropy conservation (like in classical mechanics)

Self-adjoint

$$\mathcal{T}(t)dt = -\frac{H(t)dt}{\imath\hbar}$$

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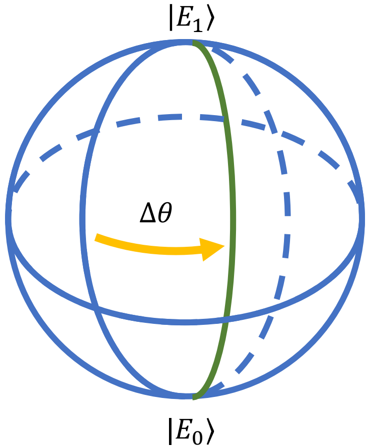
Assumptions of Physics

Any process (deterministic or stochastic) is linear over ensembles

$$\rho_I \longrightarrow \boxed{P} \longrightarrow \rho_O = P(\rho_I)$$

$$P(p_1\rho_1 + p_2\rho_2) = p_1 P(\rho_1) + p_2 P(\rho_2)$$



complete isolation

Deterministic and reversible

$$\rho_I \longrightarrow \boxed{P} \xrightarrow{\rho_O} \boxed{P^{-1}} \longrightarrow \rho_I$$

Conserves entropy/allows an “inverse”

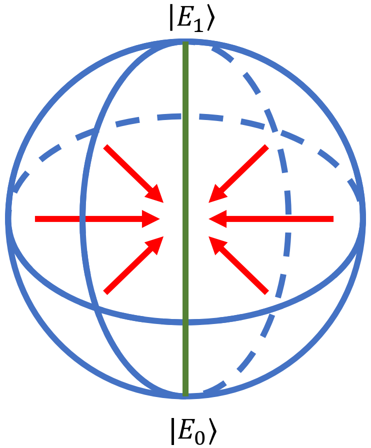
⇒ Unitary operation

Equilibration

$$\rho_I \longrightarrow \boxed{P} \xrightarrow{\rho_O} \boxed{P} \longrightarrow \rho_O$$

Must be repeatable

⇒ Projection



Both are idealizations!

No measurement problem, since unitary evolution is not more “real” than projection

However, “states as equilibria” implies existence of equilibration processes (mathematically, Hilbert spaces are Banach spaces + projectors on each subspace): projections are more “fundamental” as they need to be assumed to exist

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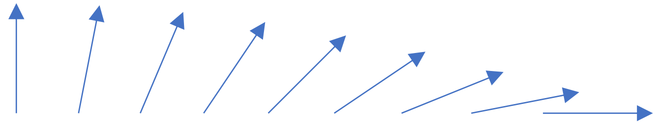
Unitary evolution $\Longleftrightarrow$ quasi-static evolution

Unitary evolution
$\Longleftrightarrow$
sequence of infinitesimal projections



$$|\langle\psi(t+dt)|\psi(t)\rangle|^2 = 1$$

$$= \langle\psi(t+dt)|\psi(t)\rangle\langle\psi(t)|\psi(t+dt)\rangle$$
$$= (1 + \langle d\psi(t)|\psi(t)\rangle)(1 + \langle\psi(t)|d\psi(t)\rangle)$$
$$= 1 + (\langle d\psi(t)|\psi(t)\rangle + \langle\psi(t)|d\psi(t)\rangle) + O(dt^2)$$
$$= 1 + dt(\langle\mathcal{T}(t)\psi(t)|\psi(t)\rangle + \langle\psi(t)|\mathcal{T}(t)\psi(t)\rangle) + O(dt^2)$$

$$\Rightarrow \mathcal{T}(t)^\dagger = -\mathcal{T}(t)$$

Projections can be seen as fundamental "black box" processes

Solves the "multilevel problem"

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Assumptions of Physics

Deterministic and reversible evolution

Black-box process with equilibria

Quasi-static evolution

Unitary evolution

Projection

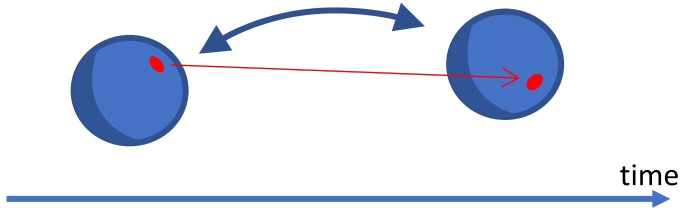
Every preparation is a measurement

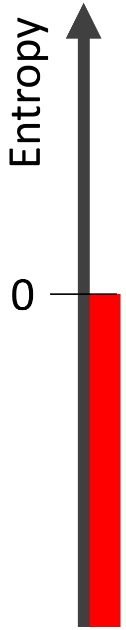
Time evolution prepares the system at each time

⇒ Time evolution is a series of measurements

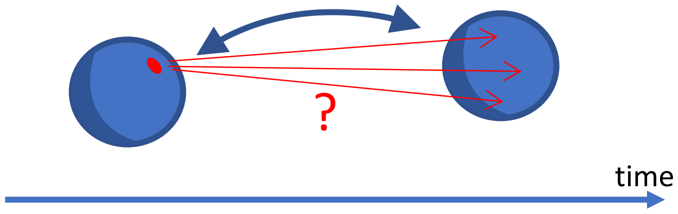
Quantum mechanics as irreducibility

Classical





Quantum



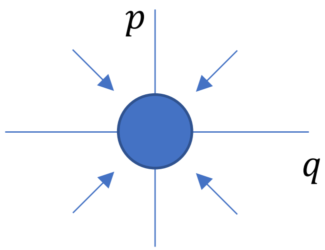
Can prepare ensembles at arbitrarily low entropy: we can study arbitrarily small parts

We always have access to the internal dynamics

Entropy is bounded at zero: we cannot study parts

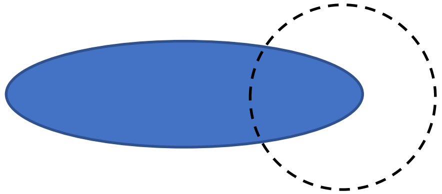
No access to the internal dynamics

Minimum uncertainty



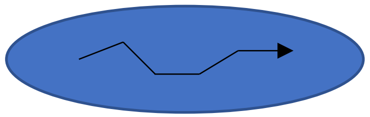
Can't squeeze ensemble arbitrarily

Non-locality



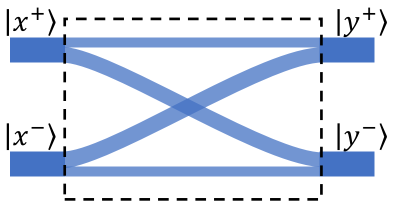
Can't refine ensembles ⇒
Can't interact with parts

Superluminar effects
that can't carry information



Can't refine ensembles ⇒
Can't extract information

Probability of transition



$$p(x^+|y^-) = p(y^-|x^+)$$

Symmetry of the inner product

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Assumptions of Physics

Let’s wait until both classical field theory and quantum mechanics are done!

Quantum field theory

Should be simply putting a lower bound on the entropy of fields

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Assumptions of Physics

Quantum gravity?

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Assumptions of Physics

Status

- Not really aiming to solve this… just connecting some trivial bits
- What is quantum mechanics?
  - According to Reverse Physics, it is putting a lower bound on the entropy (lower bound on the count of states)
- What is gravity?
  - In Reverse Physics, it is conjectured to be the proper handling of the count of DOFs and configurations (entropy) over a continuum of DOFs
- Arguably, a lower bound on the entropy/count of states cannot be achieved without imposing a lower bound on the count of DOFs as well
  - Is this what "quantum gravity" is?

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Assumptions of Physics

The problem with counting on the continuum

We'd like to say:

1. Every state is a single case (i.e. $\mu(\{\psi\}) = 1$)
2. Finite continuous range carries finite information (i.e. $\mu(U) < \infty$)
3. Count is additive for disjoint sets (i.e. $\mu(\cup U_i) = \sum \mu(U_i)$)

Incompatible!

Pick two!

Discard 1 ⇒ Lebesgue measure

Discard 2 ⇒ counting measure

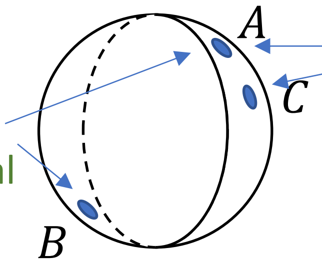
Discard 3 ⇒ "Quantum measure"

$$\mu(U) = 2^{\sup\left(S(\mathrm{hull}(U))\right)}$$

Exponential of the maximum entropy reachable with convex combinations (statistical mixtures) of U (reduces to counting/Liouville measure)

Orthogonal states: different states all else equal

additive



Non-orthogonal states: different states but in different contexts

sub-additive

$\mu(\{A\}) = 2^0 = 1$

$\mu(\{A, B\}) = 2^1 = 2$

$\mu(\{A, C\}) < 2 = \mu(\{A\}) + \mu(\{C\})$

Quantum mechanics ⇒ lower bound on #conf (entropy) on continuous DOF

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Assumptions of Physics

Conjecture: quantum gravity ⇒ lower bound on DOF count

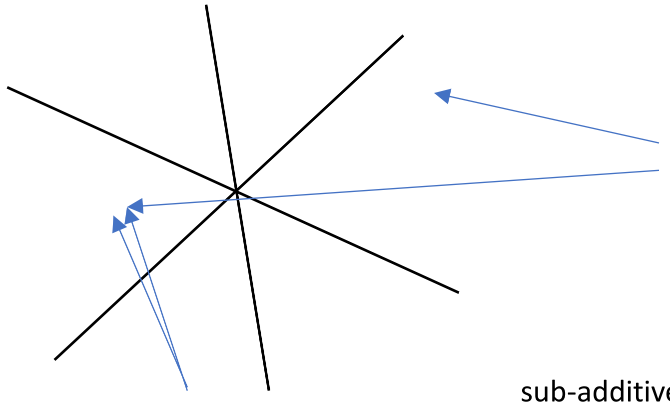
#conf=#DOFs $\dfrac{\#\text{conf}}{\#\text{DOFs}}$

Lower bound on this…

…requires a lower bound on this

Same problem!

1. Every point is a single DOF (i.e. $\mu(\{x\}) = 1$)
2. Finite volume carries finitely many DOFs (i.e. $\mu(U) < \infty$)
3. Count is additive for disjoint regions (i.e. $\mu(\cup\, U_i) = \sum \mu(U_i)$)



Distant points: independent DOFs — additive

sub-additive

Close points: DOFs not independent

From QM: Lower bound on state count requires a severe revisitation of particle state space

Does lower bound on DOF count require an equally severe revisitation of space-time?

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Assumptions of Physics

How to participate and contribute to Reverse Physics

- Prerequisite: share the interest in building a conceptual framework that treats physical theories as models of physical systems and nothing else
  - No theories of everything, interpretations that posit “how the universe works,” …
- Passive contribution: make yourself available as a “consultant”
  - Don’t have to follow the project, called only if there is something relevant that matches your background/expertise
  - Occasional discussion/review of material to make sure things make sense from multiple perspectives
- Active contribution: case-by-case
  - Need to have sufficient background or be able to get it independently
  - Much better chance of success if already working/expert in a research area

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Assumptions of Physics

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Assumptions of Physics