Assumptions of Physics
Summer School 2025

Introduction to Reverse Physics

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Main goal of the project

Identify a handful of physical starting points from which the basic laws can be rigorously derived

Assumptions of Physics

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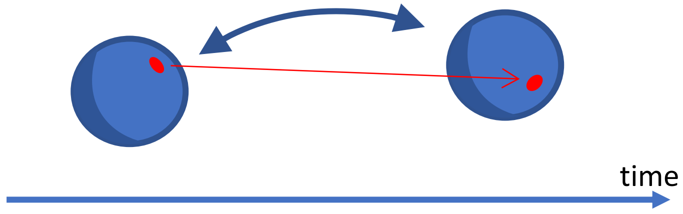
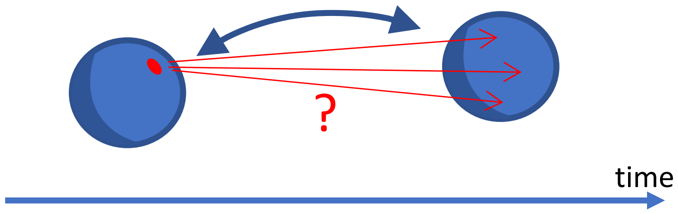
For example:

Infinitesimal reducibility ⇒ Classical state



Irreducibility ⇒ Quantum state



This also requires rederiving all mathematical structures from physical requirements

For example:

Science is evidence based ⇒ scientific theory must be characterized by experimentally verifiable statements ⇒ topologies and σ-algebras

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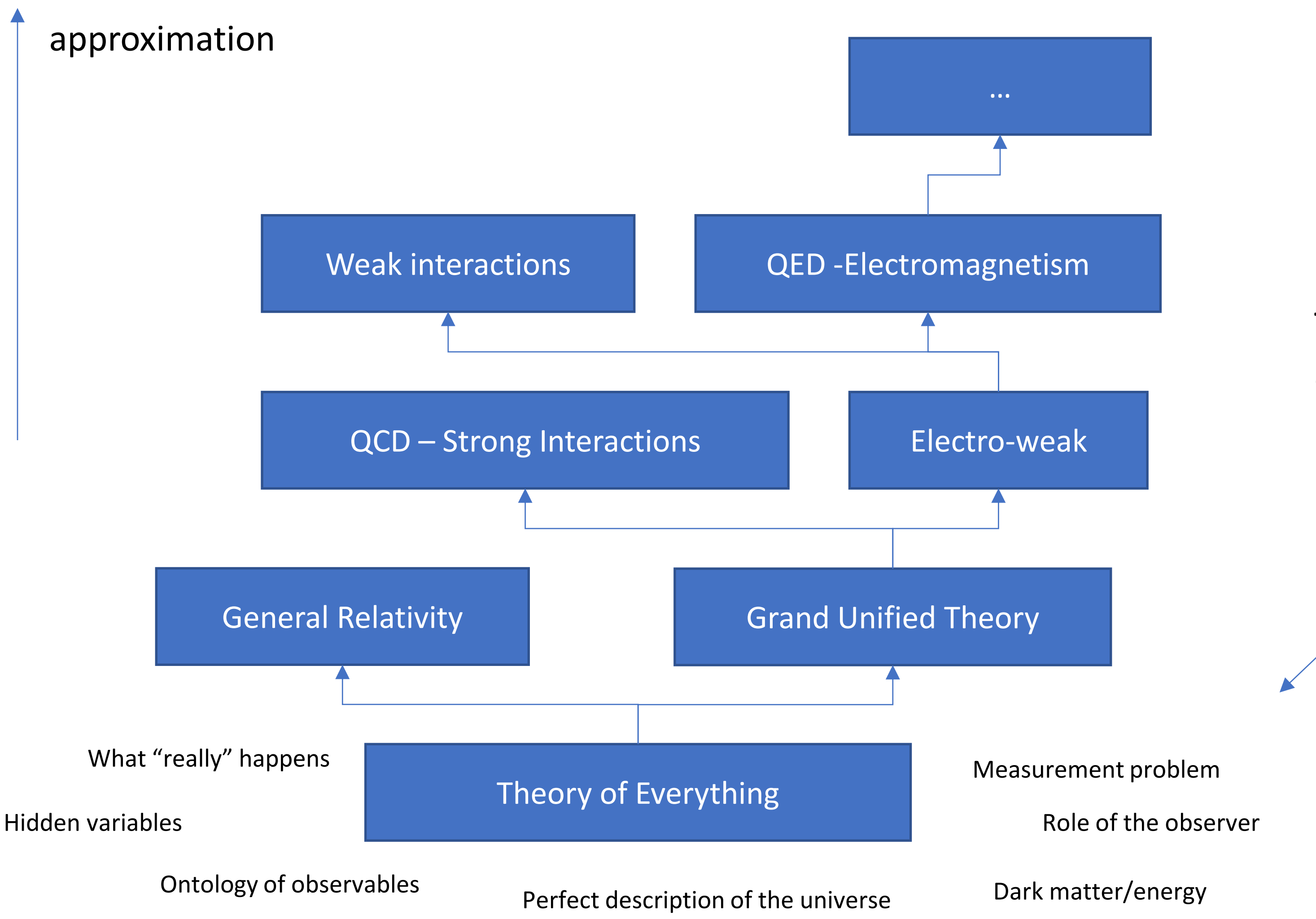
Standard view of the foundations of physics



Goal of physics is to find the true laws of the universe!

The "real" physics!

The foundations of physics!

Everything else is an approximation

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We found:

Experimental verifiability ⇒ topologies and σ-algebras
Geometrical structures ⟺ Entropic structures
Hamiltonian evolution ⟺ det-rev/isolation + DOF independence
Massive particles and potential forces ⟺ + Kinematic eq

Physical requirements and assumptions drive most of the theoretical apparatus

~~Goal of physics is to find the true laws of the universe!~~

Less productive point of view

Goal of physics is to find models that can be empirically tested

More productive point of view

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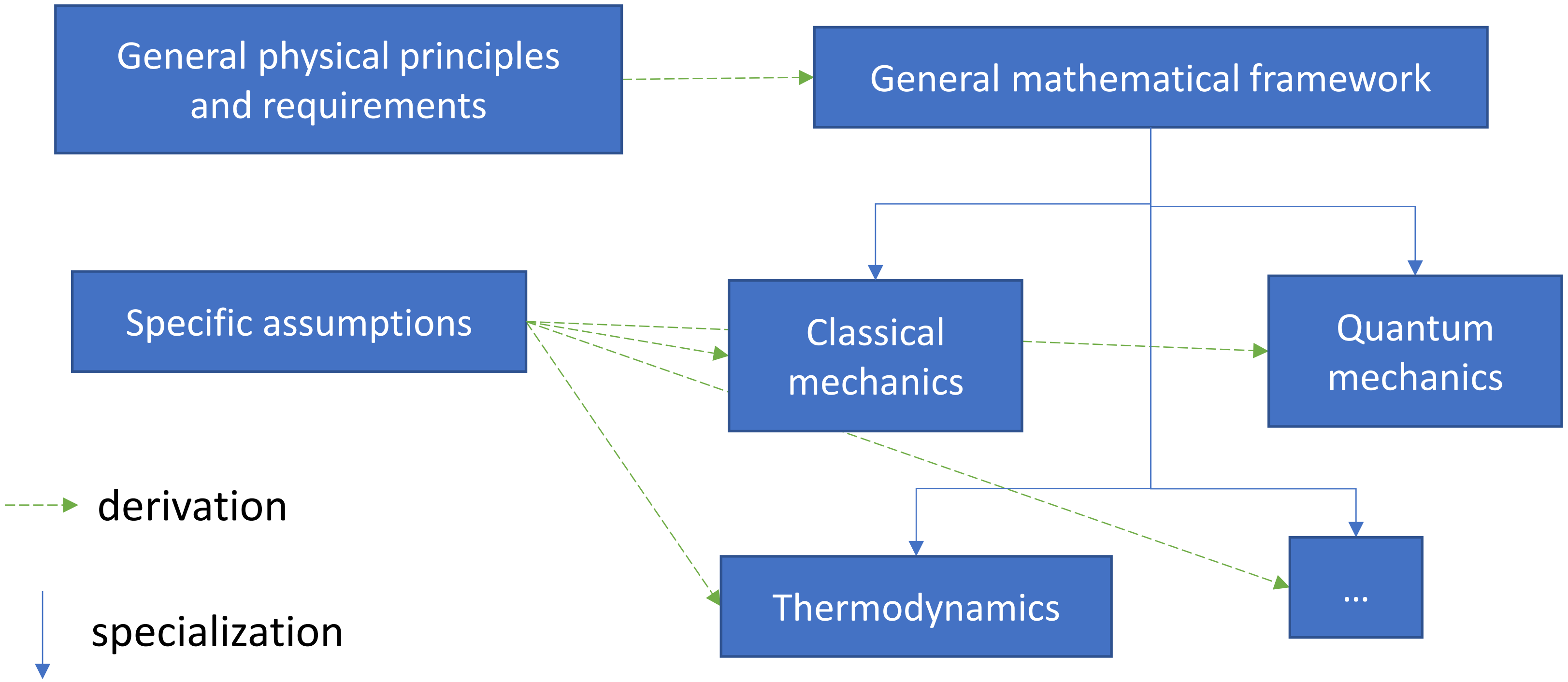
Assumptions of Physics

Our view of the foundations of physics



Foundations of physics
⇕
The theory of physical models

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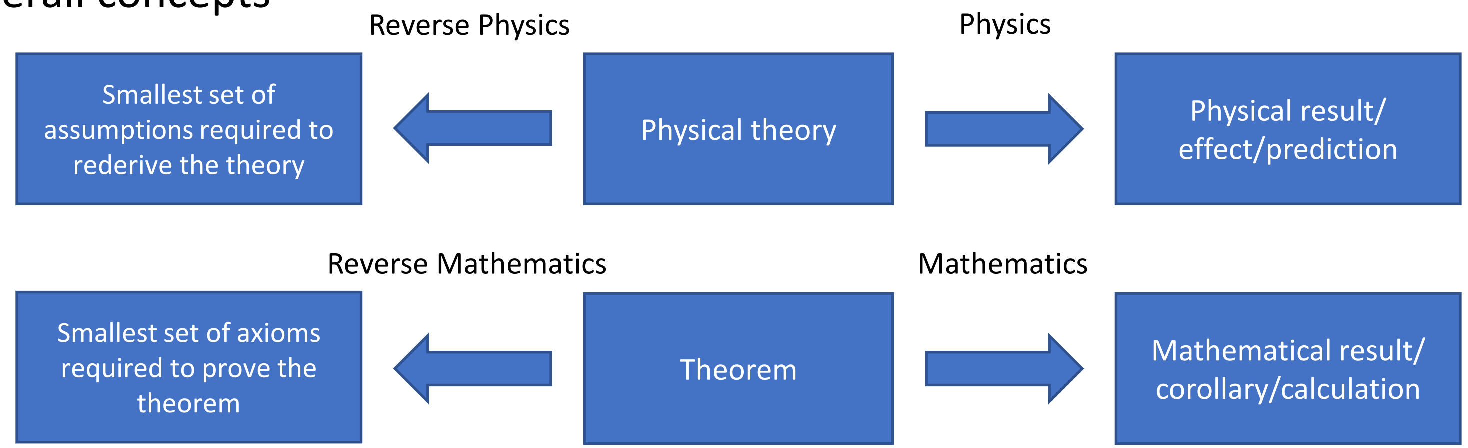
Find the right overall concepts

Reverse physics:

Start with the equations, reverse engineer physical assumptions/principles

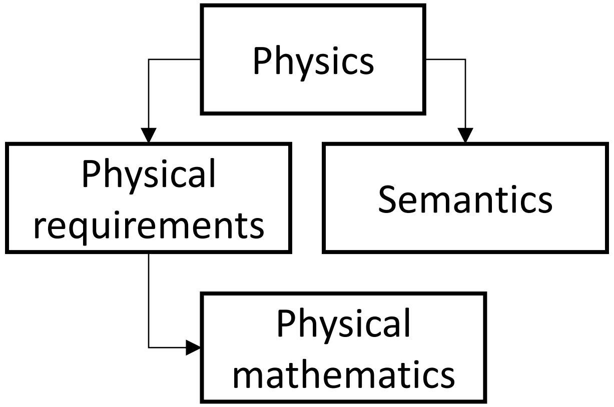
Found Phys **52**, 40 (2022)



Goal: find the right overall physical concepts, “elevate” the discussion from mathematical constructs to physical principles

Physical mathematics:

Start from scratch and rederive all mathematical structures from physical requirements



Goal: get the details right, perfect one-to-one map between mathematical and physical objects

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Assumptions of Physics

Reverse Physics

Assumptions of Physics,
Michigan Publishing (v2 2023)

J. Phys. Commun. **2** 045026 (2018)

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Assumptions of Physics

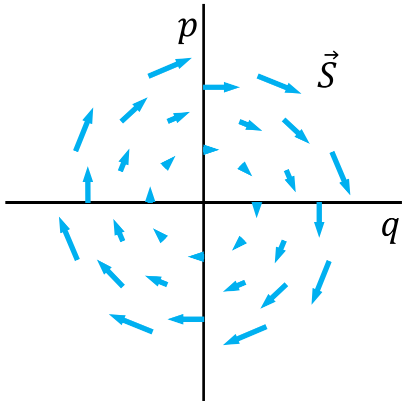
Assumption DR (Determinism and Reversibility). *The system undergoes deterministic and reversible evolution. That is, specifying the state of the system at a particular time is equivalent to specifying the state at a future (determinism) or past (reversibility) time.*

The displacement field is divergenceless: $\partial_a S^a = 0$	(DR-DIV)
The Jacobian of time evolution is unitary: $\left|\partial_b \hat{\xi}^a\right| = 1$	(DR-JAC)
Densities are conserved through the evolution: $\hat{\rho}(\hat{\xi}^a) = \rho(\xi^b)$	(DR-DEN)
Volumes are conserved through the evolution: $d\hat{\xi}^1 \cdots d\hat{\xi}^n = d\xi^1 \cdots d\xi^n$	(DR-VOL)

The evolution is deterministic and reversible.	(DR-EV)
The evolution is deterministic and thermodynamically reversible	(DR-THER)
The evolution conserves information entropy	(DR-INFO)
The evolution conserves the uncertainty of peaked distributions	(DR-UNC)

Assumption IND (Independent DOFs). *The system is decomposable into independent degrees of freedom. That is, the variables that describe the state can be divided into groups that have independent definition, units and count of states.*

The system is decomposable into independent DOFs	(IND-DOF)
The system allows statistically independent distributions over each DOF	(IND-STAT)
The system allows informationally independent distributions over each DOF	(IND-INFO)
The system allows peaked distributions where the uncertainty is the product of the uncertainty on each DOF	(IND-UNC)



The evolution leaves ω_{ab} invariant: $\hat{\omega}_{ab} = \omega_{ab}$	(DI-SYMP)
The evolution leaves the Poisson brackets invariant	(DI-POI)
The rotated displacement field is curl free: $\partial_a S_b - \partial_b S_a = 0$	(DI-CURL)

$$d_t q^i = \partial_{p_i} H$$
$$d_t p_i = -\partial_{q^i} H$$

$$S_a = S^b \omega_{ba} = \partial_a H$$

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Assumptions of Physics

Is the uncertainty principle really a feature of quantum mechanics alone?

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Assumptions of Physics

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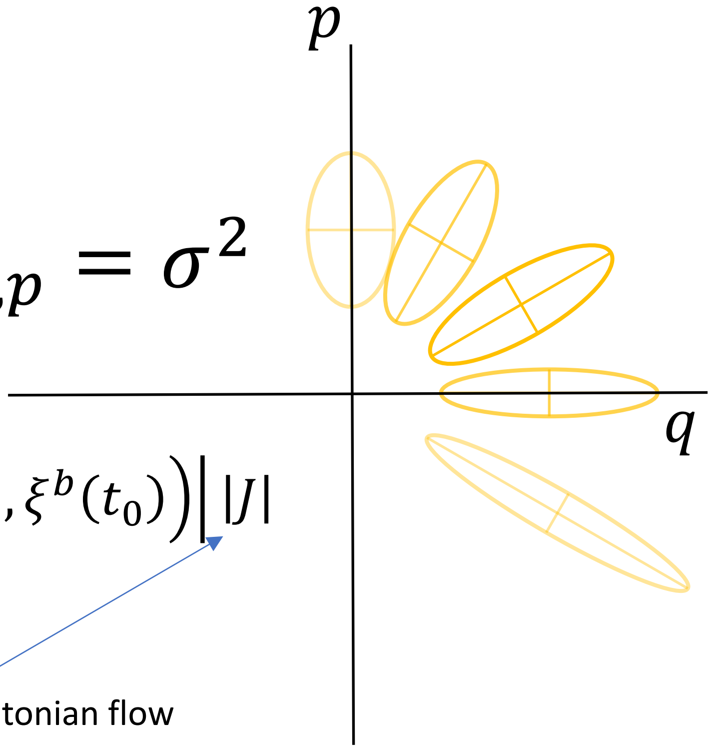
Assumptions of Physics

Determinant of covariance matrix:

$$\left|cov\left(\xi^a, \xi^b\right)\right| = \begin{vmatrix} \sigma_q^2 & cov_{q,p} \\ cov_{p,q} & \sigma_p^2 \end{vmatrix} = \sigma_q^2 \sigma_p^2 - cov_{q,p}^2 = \sigma^2$$



Peaked distribution
⇒ flow is almost linear
⇒ covariance matrix transforms linearly

$$\left|cov\left(\xi^a(t), \xi^b(t)\right)\right| = |J| \left|cov\left(\xi^a(t_0), \xi^b(t_0)\right)\right| |J|$$

1 under Hamiltonian flow

$$\sigma_q^2(t)\sigma_p^2(t) - cov_{q,p}^2(t) = \sigma^2(t_0)$$

$$\sigma_q(t)\sigma_p(t) \geq \sigma(t_0)$$

Uncertainty is bounded during classical evolution

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Assumptions of Physics

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Standard formula for entropy

Logarithm must take a dimensionless quantity

$$S(\rho) = -\int \rho \log \rho \, dqdp$$

In reverse physics, all issues of units must be resolved: if units are not clear, the physics is not clear!!

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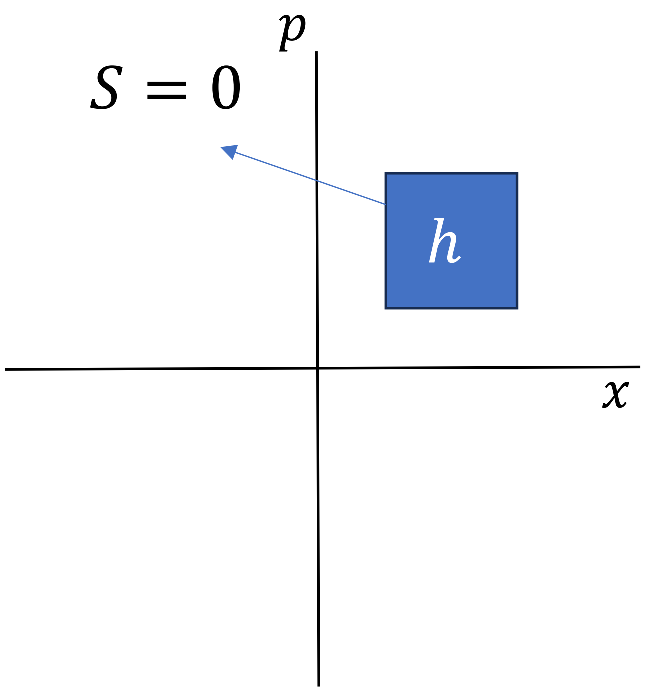
Assumptions of Physics

Formula for entropy with entropy fixed



$$S(\rho) = -\int \rho \log h\rho \, dqdp$$

$$\rho = \frac{1}{h} \Rightarrow S(\rho) = 0$$

h represents the volume of phase space that corresponds to zero entropy for a uniform distribution

No relation to quantum mechanics… just sets the scale for entropy

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Assumptions of Physics

Let’s plot entropy against uncertainty

$$S(\rho) = -\int \rho \log h\rho \, dq dp$$

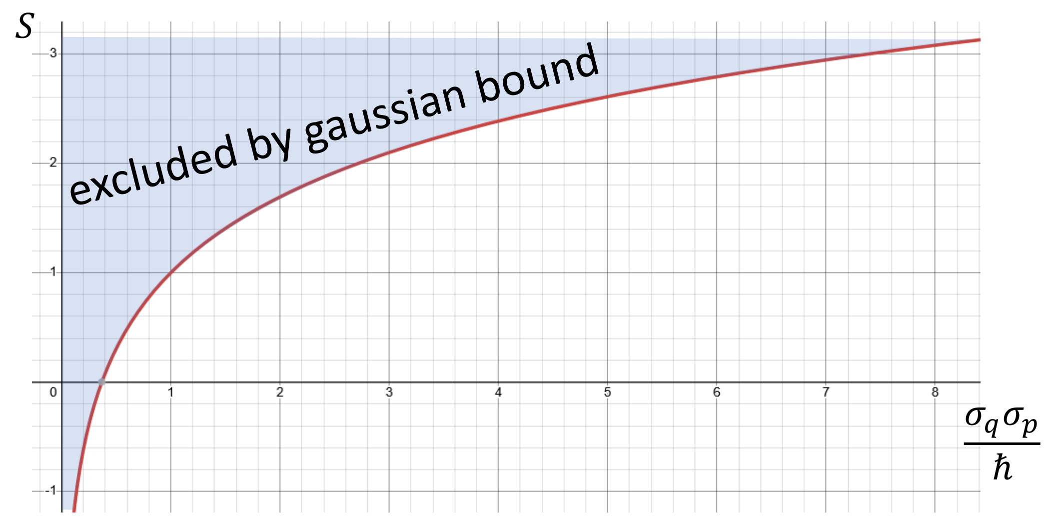
Fixes units

Uniform distribution over volume h has zero entropy

$$S(\rho) \leq \log 2\pi e \frac{\sigma_q \sigma_p}{h}$$

Gaussian maximizes entropy for a given uncertainty

$$\sigma_q \sigma_p \geq \frac{h}{2\pi e} e^{S(\rho)} = \frac{\hbar}{e} e^{S(\rho)}$$



Entropy puts a lower bound on the uncertainty

Hamiltonian evolution conserves entropy

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Assumptions of Physics

Is there anything that puts a lower bound on the entropy?

Every substance has a finite positive entropy, but at the absolute zero of temperature the entropy may become zero, and does so become in the case of perfect crystalline substances.

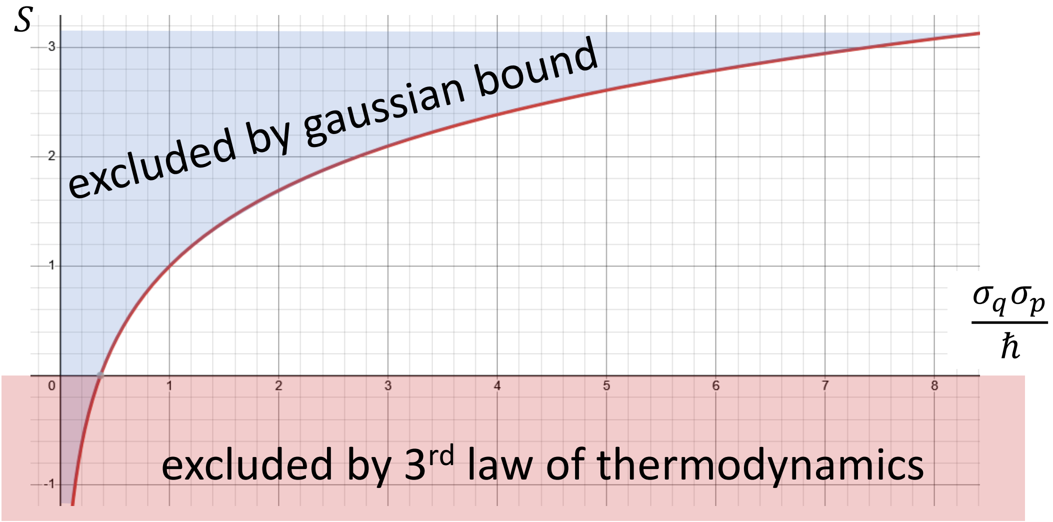
G. N. Lewis and M. Randall, Thermodynamics and the free energy of chemical substances (McGraw-Hill, 1923)

The third law of thermodynamics!

Third law puts a lower bound on the entropy which puts a lower bound on the uncertainty

$$\sigma_q \sigma_p \geq \frac{\hbar}{e} e^0 = \frac{\hbar}{e}$$

Classical uncertainty principle!



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Assumptions of Physics

Comparing theories

classical

$$\sigma_q \sigma_p \geq \frac{\hbar}{e}$$

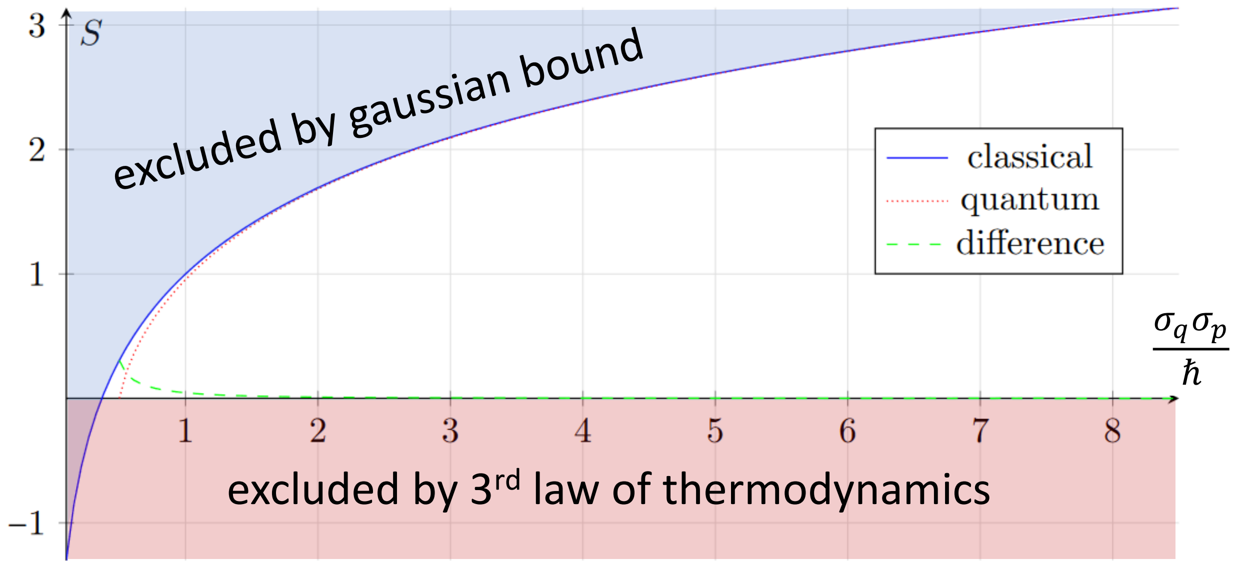
2.71828...

quantum

$$\sigma_q \sigma_p \geq \frac{\hbar}{2}$$

Entropy of quantum states is already non-negative

The gaussian bound quickly becomes very similar across theories



$$S_C = \ln e\sigma$$

$$S_Q = \left(\sigma + \tfrac{1}{2}\right)\ln\left(\sigma + \tfrac{1}{2}\right) - \left(\sigma - \tfrac{1}{2}\right)\ln\left(\sigma - \tfrac{1}{2}\right)$$

Quantum mechanics incorporates the third law

Classical mechanics does not

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Assumptions of Physics

Comparing theories

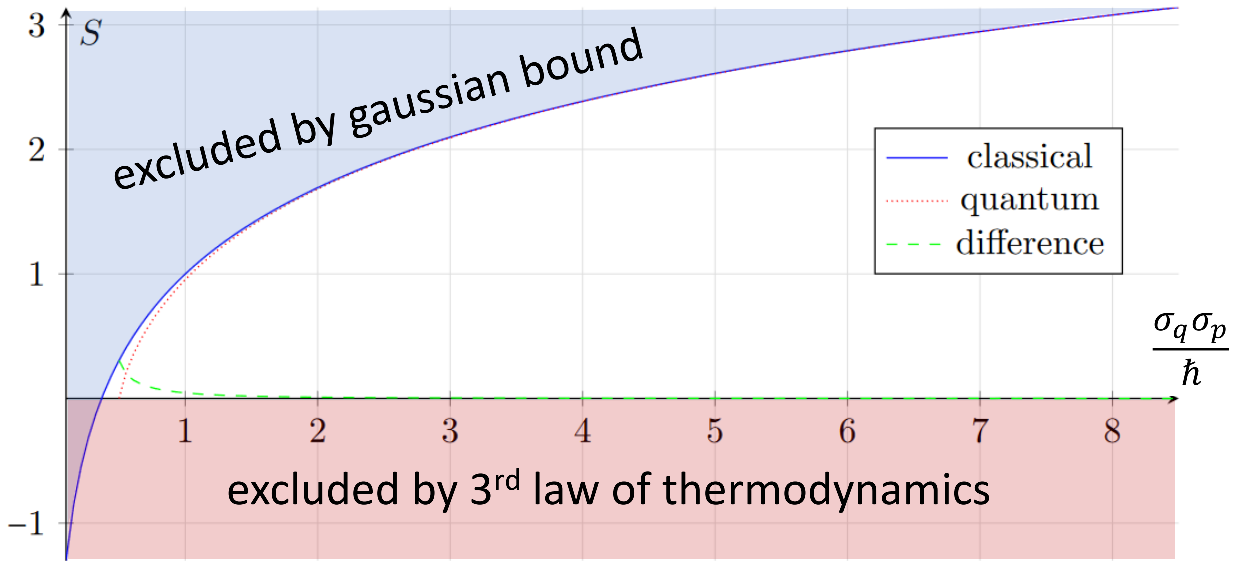
classical

$$\sigma_q \sigma_p \geq \frac{\hbar}{e}$$

2.71828…

quantum

$$\sigma_q \sigma_p \geq \frac{\hbar}{2}$$



$$S_C = \ln e\sigma$$

$$S_Q = \left(\sigma + \tfrac{1}{2}\right) \ln\left(\sigma + \tfrac{1}{2}\right) - \left(\sigma - \tfrac{1}{2}\right) \ln\left(\sigma - \tfrac{1}{2}\right)$$

The uncertainty principle is a consequence of the lower bound of the entropy imposed by the third law of thermodynamics

In quantum mechanics, $L_q L_p \geq \pi e \hbar$ is a stronger inequality

$e^{-\int \rho(q) \log \rho(q) dq}$

exponential of entropy for position

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Suppose the lower bound on the entropy is the only difference, then in the limit of high entropy of quantum mechanics we should recover classical mechanics

Can we?

Classical mechanics as high entropy limit?

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greetings

Manuele Landini <manulando@gmail.com>
To carcassi@umich.edu

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Wed 7/10/2024 6:15 AM

You replied to this message on 7/10/2024 10:00 AM.

Caro Gabriele,

Mi chiamo Manuele Landini e lavoro a Innsbruck (Austria) come senior scientist in un gruppo di fisica atomica sperimentale. Puoi vedere di cosa ci occupiamo sul nostro sito: https://quantummatter.at.

Ho visto un po' dei tuoi video su youtube. Mi sembra un progetto molto ambizioso, ma promettente. Mi farebbe piacere riuscire a spiegare agli studenti in futuro in termini piu' fisici concetti come le sovrapposizioni o il teorema spin-statistica.

Per la storia della metrica, da quel che ho capito hai bisogno di una metrica che non sia basata sull'entropia, visto che vuoi definire una distanza a entropia costante. Ci sono varie opzioni, ma la trace distance Trace distance - Wikipedia funziona perche' ha una propireta' fondamentale che puoi usare. Chiamala: T(rho,sigma)

Se parti da stati puri, si riduce a (1-<psi|phi>)^(1/2). Quindi per massimizzarla, scegli due stati ortogonali (non importa quali). Il massimo e' T_0=1. Una volta che hai questi stati, che hanno entropia 0, li puoi trasformare in stati con entropia finita (in particolare quelli con massima distanza) tramite una trace preserving map M.

Siccome T si contrae, hai che T(M(rho),M(sigma))<=T(rho,sigma). L'uguale vale se la mappa e' unitaria. Cosi' definisci un serie di step in cui la distanza massima decresce T_n+1<T_n, fino ad arrivare a 0 per stati fully mixed.

arXiv > quant-ph > arXiv:2411.00972

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Quantum Physics

[Submitted on 1 Nov 2024 (v1), last revised 3 Dec 2024 (this version, v2)]

Classical mechanics as the high-entropy limit of quantum mechanics

Gabriele Carcassi, Manuele Landini, Christine A. Aidala

We show that classical mechanics can be recovered as the high-entropy limit of quantum mechanics. That is, the high entropy masks quantum effects, and mixed states of high enough entropy can be approximated with classical distributions. The mathematical limit $\hbar \to 0$ can be reinterpreted as setting the zero entropy of pure states to $-\infty$, in the same way that non-relativistic mechanics can be recovered mathematically with $c \to \infty$. Physically, these limits are more appropriately defined as $S \gg 0$ and $v \ll c$. Both limits can then be understood as approximations independently of what circumstances allow those approximations to be valid. Consequently, the limit presented is independent of possible underlying mechanisms and of what interpretation is chosen for both quantum states and entropy.

Comments: 14 pages, 3 figures
Subjects: **Quantum Physics (quant-ph)**
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(or arXiv:2411.00972v2 **[quant-ph]** for this version)
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References & Citations
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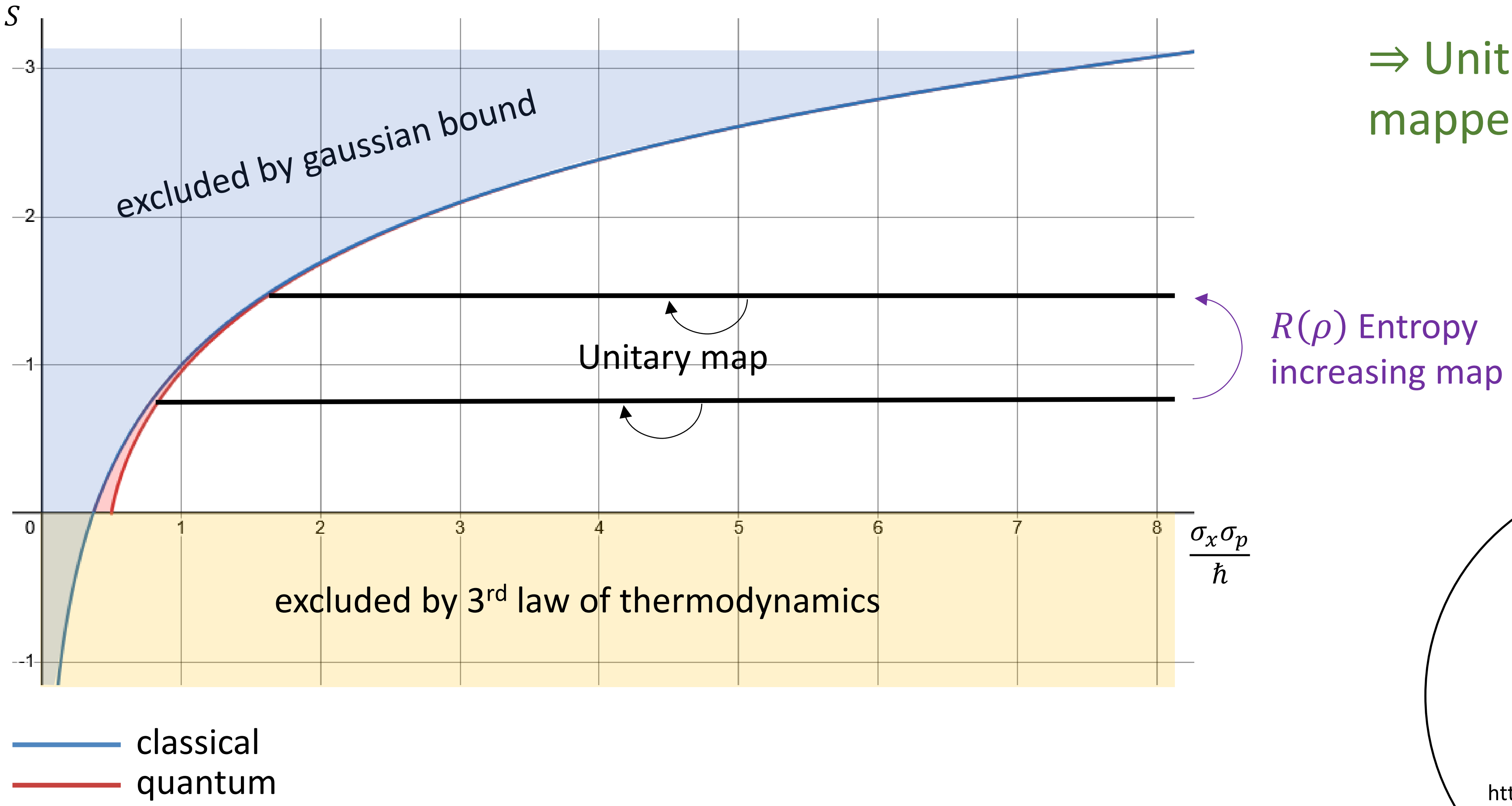
Assumptions of Physics

Looking for a map $R(\rho)$ that increases entropy of all mixed states, such that every level set of entropy maps to another level set



⇒ Unitary must be mapped to unitary

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Assumptions of Physics

In classical mechanics

$$S\big(R(\rho)\big) = S(\rho) + \log\lambda \Leftrightarrow \{R(x), R(p)\} = \lambda\{x, p\}$$

Jacobian is a constant:
all volumes rescaled
by the same factor

In quantum mechanics

Stretching map $\quad [R(X), R(P)] = \lambda[X, P]$

Pure stretching map $\quad T(X) = \sqrt{\lambda}\, X \quad T(P) = \sqrt{\lambda}\, P$

Need to take care of operator ordering!!!

Infinitesimal pure stretching map

$$\frac{dX}{dt} = \frac{\imath}{\hbar}[H, X] + \gamma\left(L^\dagger X L - \frac{1}{2}\{L^\dagger L, X\}\right)$$

Lindblad eq (open quantum system)

$$L = a^\dagger = \sqrt{\frac{m\omega}{2\hbar}}\left(X + \frac{\imath}{m\omega}P\right) \qquad \gamma = \lambda$$

Anti-normal ordering and Husimi Q are preferred

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Assumptions of Physics

Another perspective: move the pure states to minus infinite entropy

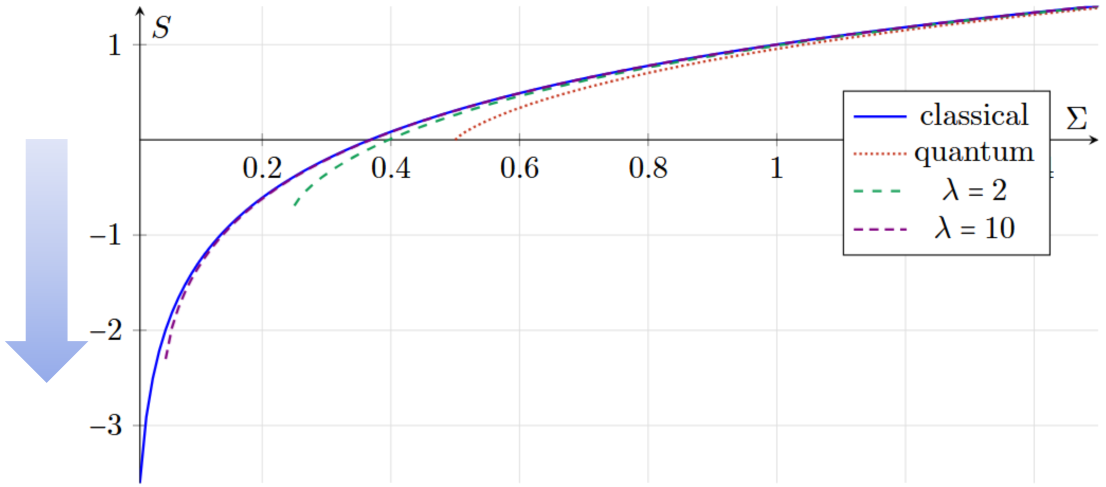
Instead of

$$[X,P] = \imath\hbar \qquad [T(X),T(P)] = \lambda\imath\hbar$$

Redefine original space such that

$$[\hat{X},\hat{P}] = \frac{\imath\hbar}{\lambda} \qquad [T(\hat{X}),T(\hat{P})] = \imath\hbar$$

$$\lambda \to \infty \;\Rightarrow\; \frac{\hbar}{\lambda} \to 0$$



Mathematically equivalent to lowering the entropy of a pure state to $-\infty$, or $\hbar \to 0$ (group contraction)

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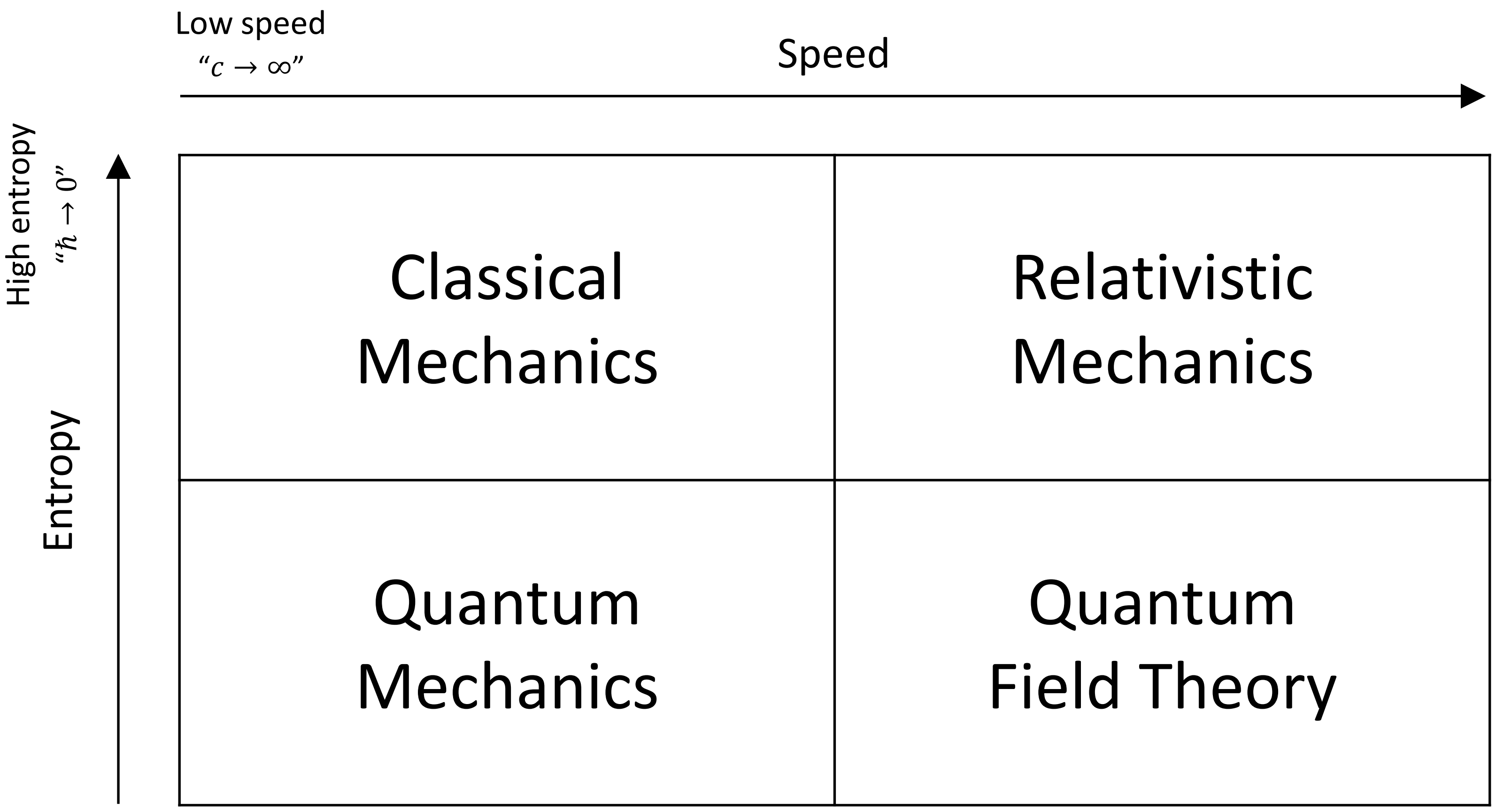

Dirac's correspondence principle: putting an entropic lower bound on classical theory

Only one way to do it

No-mechanism limit
(same as non-relativistic limit)

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Entropy seems to play a very important role in both classical and quantum mechanics

How far can we push this role?

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Assumptions of Physics

In classical mechanics

Uniform probability densities are inverse of volumes

Geometry gives us areas/volumes

$\mu(U) = \int_U \omega^n$

Symplectic manifold

Standard probability

$\rho_U(x) = \dfrac{1}{\mu(U)}$

Entropy of uniform distributions is the logarithm of volumes

ρ uniform over U

Information theory

$S_U(\rho) = \log \mu(U)$

You can either provide the volume of each region or, equivalently, the entropy of all uniform distributions

Geometry is entropy!

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Assumptions of Physics

In quantum mechanics

Geometry given by inner product

$\langle \psi | \phi \rangle$

Projective Hilbert space

$$\rho = \frac{1}{2}\rho_\psi + \frac{1}{2}\rho_\phi$$

Quantum probability

Quantum information theory

Probability given by inner product

$$p(\psi|\phi) = |\langle \psi | \phi \rangle|^2$$

Entropy of mixtures of pairs given by the probability

$$S(\rho) = S\left(\frac{1+\sqrt{p}}{2}, \frac{1-\sqrt{p}}{2}\right)$$

You can either provide the inner product between each pair of states, of the entropy of the mixtures of all pairs

Geometry is entropy!

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Assumptions of Physics

Geometry is entropy!

The geometric structures of both classical and quantum mechanics are equivalent to the entropic structure

$\mu(U) = \int_U \omega^n$

Symplectic manifold

ρ uniform over U

Standard probability

$\rho(x) = \frac{1}{\mu(U)}$

Information theory

$S(\rho) = \log \mu(U)$

Thermodynamics/Statistical mechanics are not built on top of mechanics

Mechanics is the ideal case of thermodynamics/statistical mechanics

$\langle\psi|\phi\rangle$

Projective Hilbert space

$\rho = \frac{1}{2}\rho_\psi + \frac{1}{2}\rho_\phi$

Quantum probability

$p(\psi|\phi) = |\langle\psi|\phi\rangle|^2$

Quantum information theory

$S(\rho) = S\left(\frac{1+\sqrt{p}}{2}, \frac{1-\sqrt{p}}{2}\right)$

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Assumptions of Physics

Extracting principles/assumptions behind the laws gives us solid intuition that cuts across fields and leads to new insights/results

Strive for multiple equivalent starting points

Hamiltonian mechanics/entropy conservation/uncertainty conservation

Every detail counts

dimensional analysis (e.g. $-\int \rho \log \rho \Rightarrow -\int \rho \log h\rho$)

Always be ready to reinterpret/recombine results

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Assumptions of Physics

Many insights from reverse physics started as “preposterous tricks”

It took time to see there was truth to them

You can’t judge arguments you are unfamiliar with

yours or others’

A superficial understanding of an argument does not allow you to see potential connections to other things

Conversely, entrenched ideas prevent you from seeing the merit of the unfamiliar

It’s essential to remain playful and not to be too attached to “your ideas”

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Assumptions of Physics

Wrapping it up

- Assumptions of Physics: different approach to the foundations of physics
  - No interpretations, no theories of everything: physically meaningful starting points from which we can rederive the laws and the mathematical frameworks they need
  - Physical theories are models
    - Need to clarify exactly what the realm of applicability of each model is
- Reverse physics: reverse engineer principles from the known laws
  - Current laws are given through mathematical structures, which make it hard to understand the physics
  - We have to find physical principles and assumptions from which to rederive the laws
  - These starting points must have a clear intuitive meaning for the majority of physicists and engineers
  - Investigate physics as a whole (no theory is “more fundamental”)

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Assumptions of Physics

Assumption of Physics is an open project

Our main output is an open access book: https://assumptionsofphysics.org/book/

All our material is developed on GitHub: https://github.com/assumptionsofphysics

One YouTube channel dedicated to publicize results: https://www.youtube.com/user/gcarcassi

Livestream discussions

Another YouTube channel dedicated to research: https://www.youtube.com/@AssumptionsofPhysicsResearch

Activities coordinated through a Discord server (contact me for an invite)

Always looking for experts to gain insights and/or help

Always looking for collaborations

Always looking for editors/journals/conferences that are sympathetic to the mission

https://assumptionsofphysics.org/

Assumptions of Physics

To learn more

- Project website
  - https://assumptionsofphysics.org for papers, presentations, …
  - https://assumptionsofphysics.org/book for our open access book (updated every few years with new results)
- YouTube channels
  - https://www.youtube.com/@gcarcassi
    Videos with results and insights from the research
  - https://www.youtube.com/@AssumptionsofPhysicsResearch
    Research channel, with open questions and livestreamed work sessions
- GitHub
  - https://github.com/assumptionsofphysics
    Book, research papers, slides for videos...

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