Assumptions of Physics
Summer School 2024

Conceptual and Philosophical Foundations

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Assumptions of Physics

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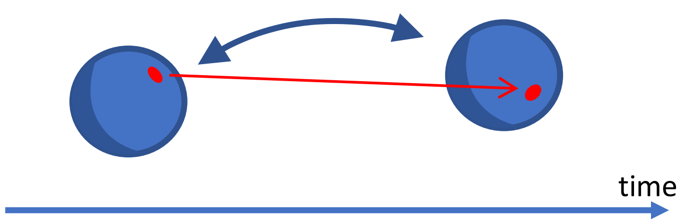
Main goal of the project

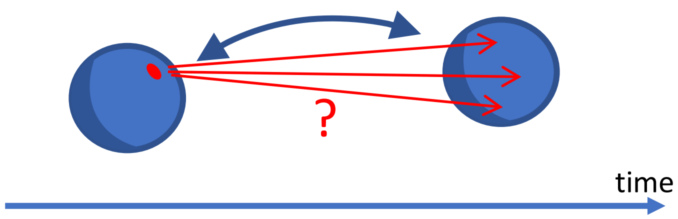
Identify a handful of physical starting points from which the basic laws can be rigorously derived

For example:

Infinitesimal reducibility ⇒ Classical state



Irreducibility ⇒ Quantum state



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This also requires rederiving all mathematical structures from physical requirements

For example:

Science is evidence based ⇒ scientific theory must be characterized by experimentally verifiable statements ⇒ topology and σ-algebras

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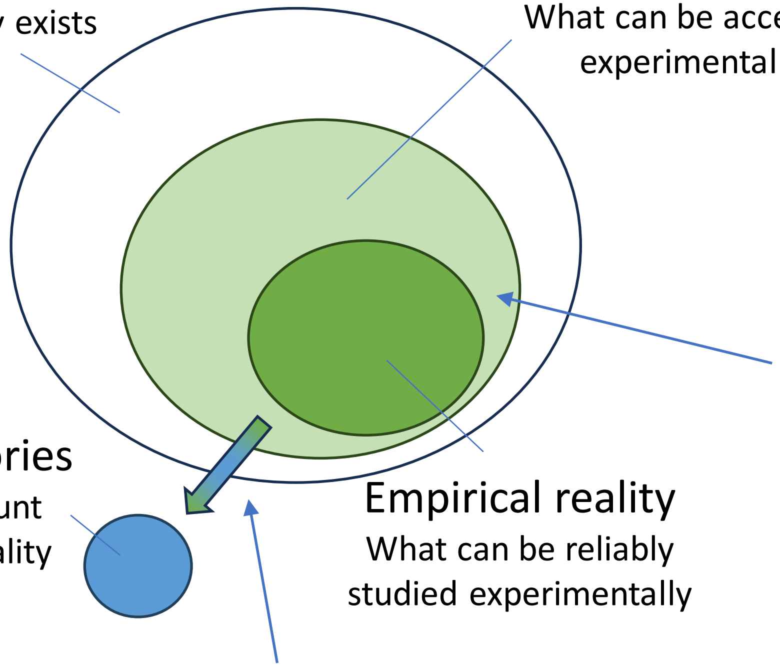
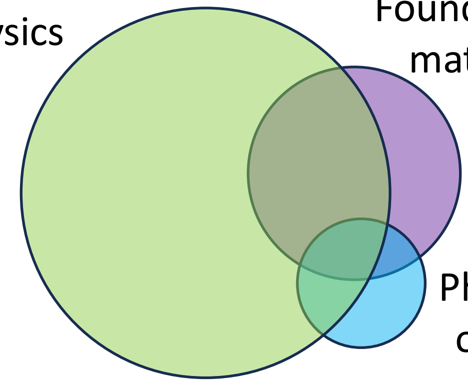
Underlying perspective



What is the boundary?
What are the requirements?

How exactly does the abstraction/idealization process work?



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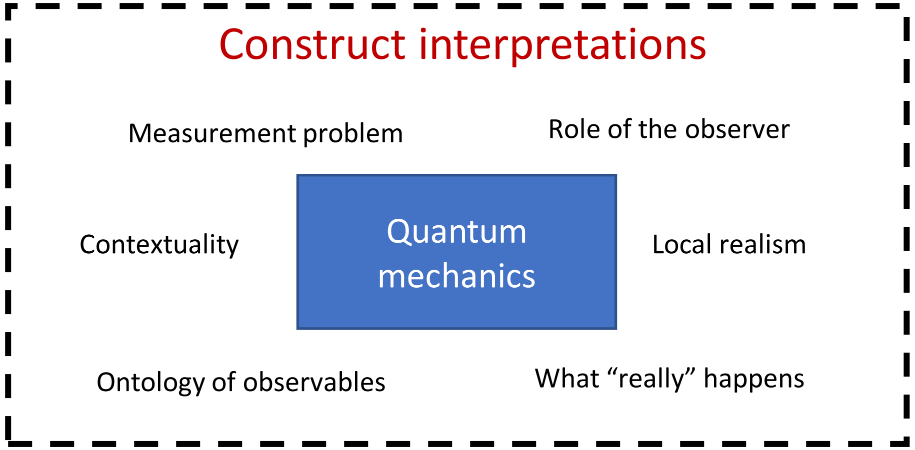
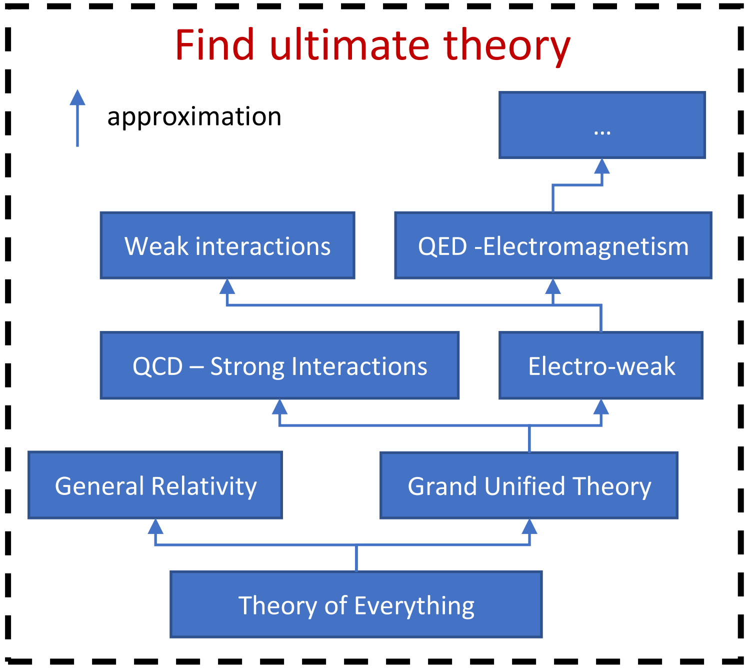
If physics is about creating models of empirical reality, the foundations of physics should be a theory of models of empirical reality

Requirements of experimental verification, assumptions of each theory, realm of validity of assumptions, …

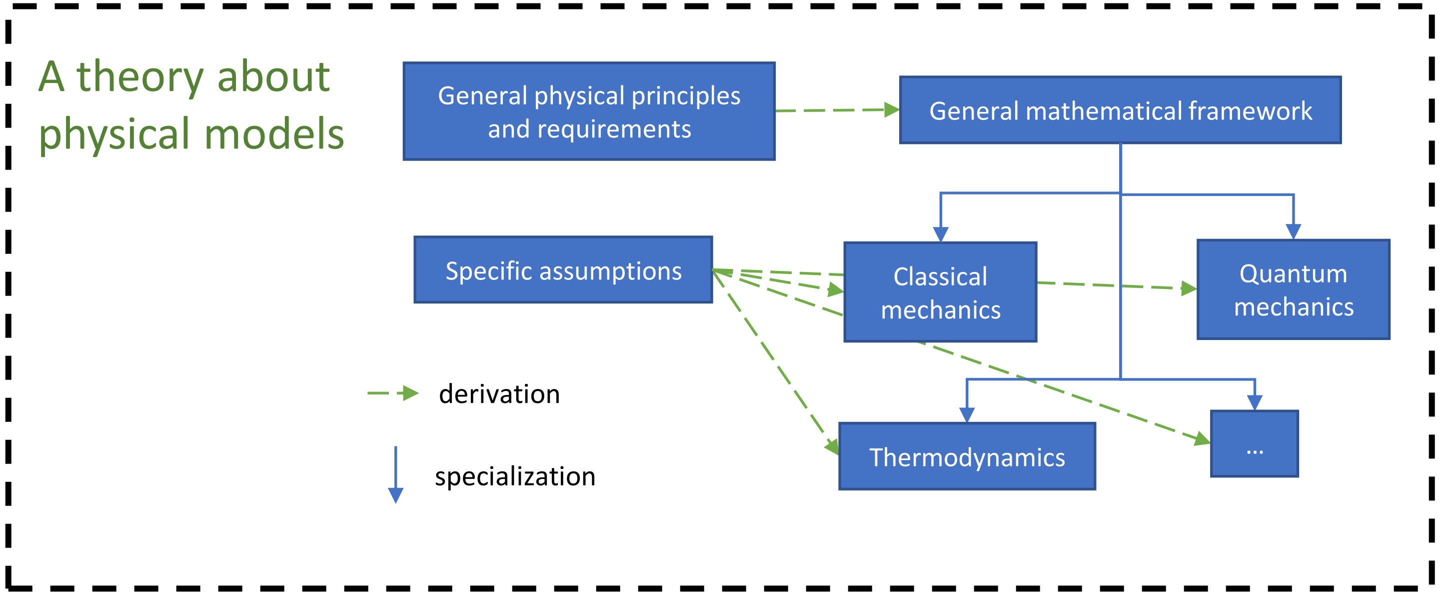
Different approach to the foundations of physics

Typical approaches





Our approach



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Find the right overall concepts

Reverse physics:

Start with the equations, reverse engineer physical assumptions/principles

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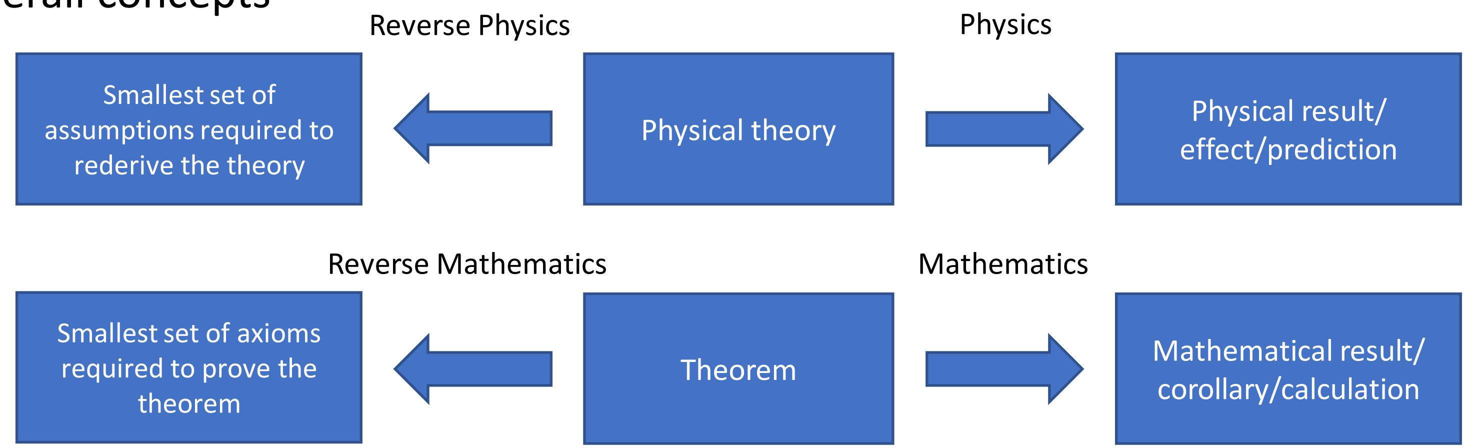
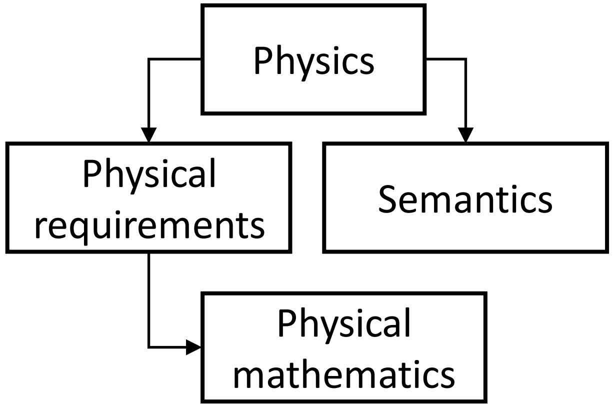


Goal: find the right overall physical concepts, “elevate” the discussion from mathematical constructs to physical principles

Physical mathematics:

Start from scratch and rederive all mathematical structures from physical requirements



Goal: get the details right, perfect one-to-one map between mathematical and physical objects

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This session

Conceptual and
Philosophical Foundations

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Nature of physical theories

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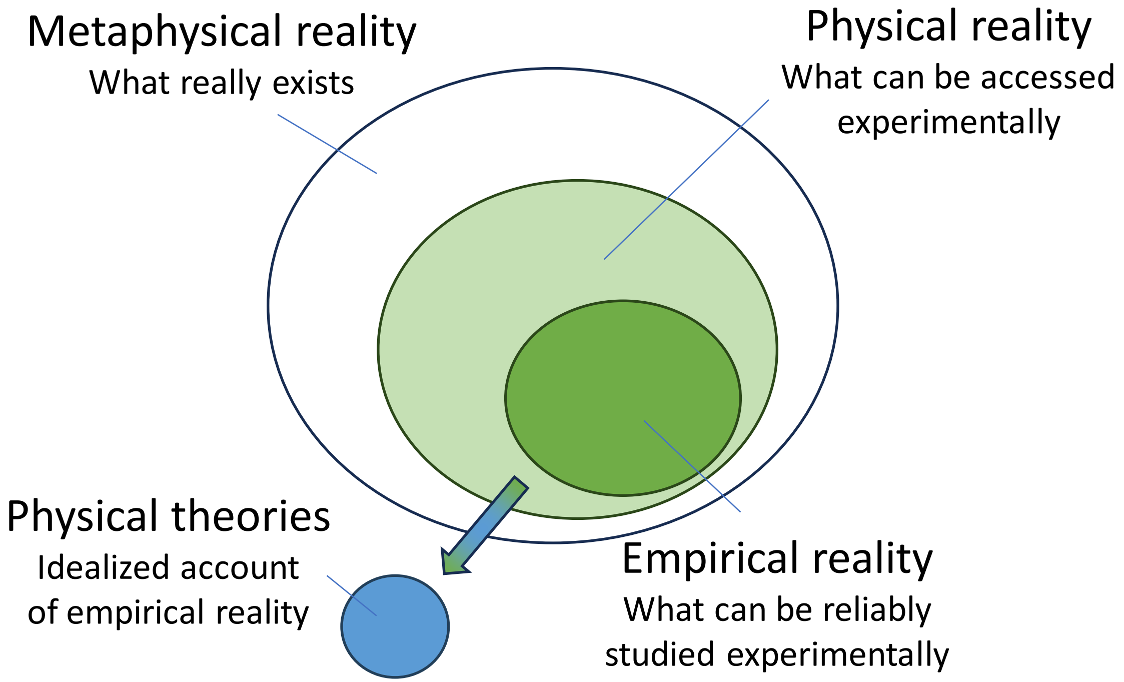
What exactly is the boundary between these areas?

Things that (may) exist: God(s), perfect circles, hidden variables, parallel universes, life after death, …

Things that we can perceive: our own consciousness, our own feelings, historic events, a particular phenomenon at a particular time, …

Things that we can study experimentally: motion of an object falling, repeatable processes under similar conditions, objects that we can probe without disruption, …

Things that we describe in our models: objects that are fully described by finitely many infinitesimally precise quantities, objects under simplified assumptions (i.e. no friction, perfect isolation, linearity, …), …



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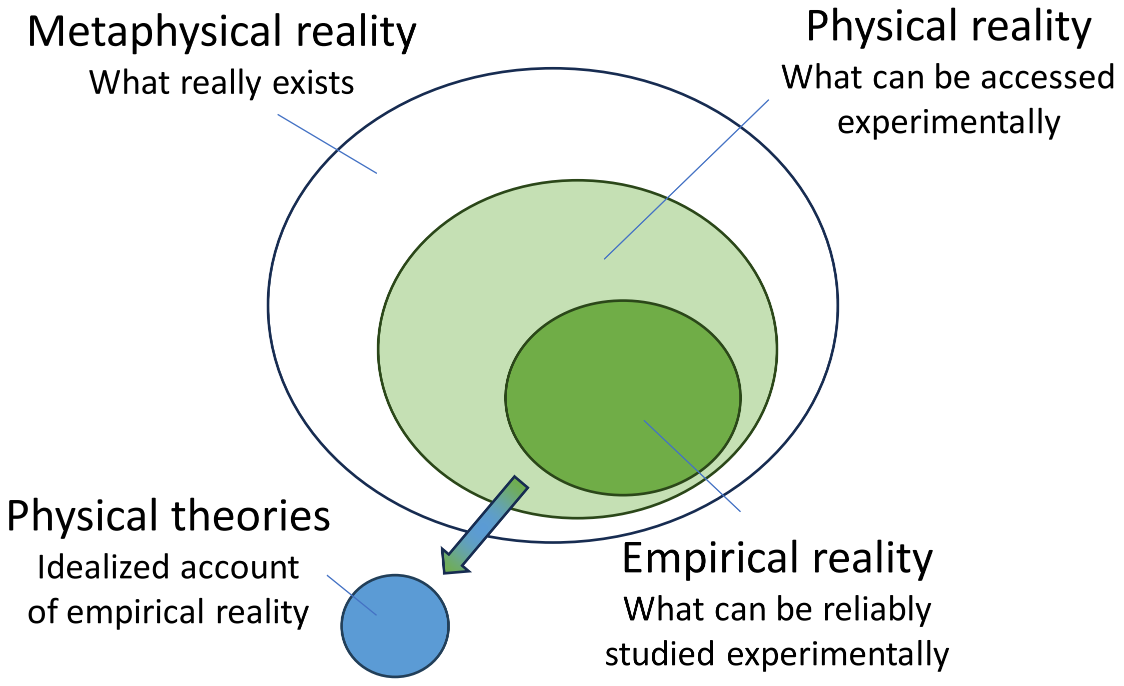
States and laws of evolution do not describe "reality as is", but empirical accessible part that can be reliably studied under suitable idealizations



If topologies and σ-algebras capture experimental verifiability, then they apply only to physical reality: they are unjustified for metaphysical reality

If ensembles capture repeatability of preparations and measurements, then they apply only to empirical reality: they are unjustified for physical reality or metaphysical reality

Mathematical tools have domains of applicability

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Takeaways

- Physical theories are idealized accounts of the experimentally accessible part of reality that can be studied reliably
- A good foundation of physics must understand what the limits and requirements of these idealized accounts are
- If physical theories are models, then the foundations of physics should be a theory of models
- TODOs
  - Refine this approach into a “philosophical position”
  - Can we make an argument that shows how idealization is necessary?

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Developing a formal system for experimental science

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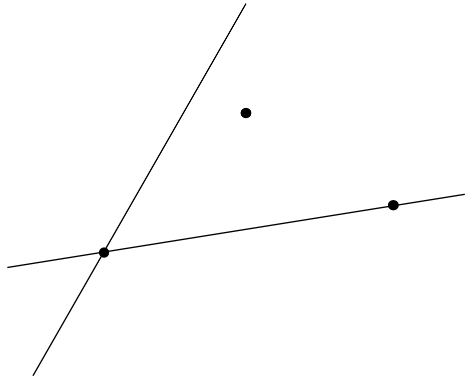
Formal system:

primitive notions

Basic objects that are taken as-is,
without definition in terms of other objects

e.g. Euclidean geometry

E.g. Points and lines



formal language

Symbols and rules to write sentences
in the formal system

E.g. $\mathrm{A}, \mathrm{B}, \mathrm{C}$ for points
$\overline{AB}$ for segment

axioms

Statements about primitive objects that
are to be taken as true

E.g. Given two points,
there is a line that joins them

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Formal system for all of mathematics:

Sets + first-order logic
+ Zermelo–Fraenkel axioms (+ axiom of choice)

Formal system for all of physics:

???

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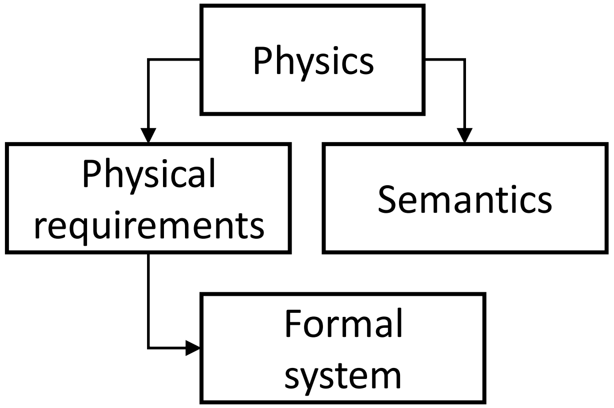
formal language

Not an issue: we can simply use first-order logic

primitive notions

axioms

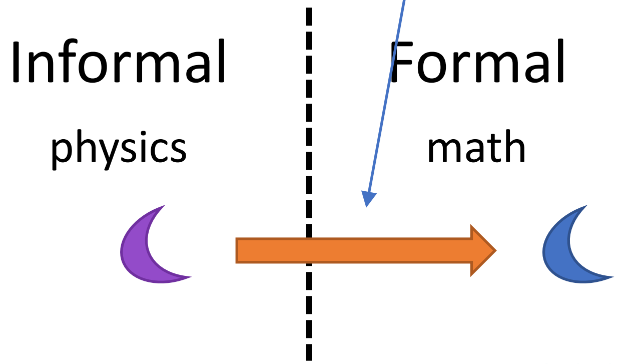
These should be justified by the physics



Choose axioms/primitive notions so that the justification is straightforward

Physical objects live in the physical (informal) world

(e.g. connection to experiment is outside of the formal system)



Mathematical objects are representations of physical objects

What can or cannot be captured by the formal system?

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Problems in formalizing physical concepts

From merriam-webster.com:

tree 1 of 2 noun

ˈtrē

plural **trees**

1 **a** **:** a woody perennial plant having a single usually elongate main stem generally with few or no branches on its lower part

plant 2 of 2 noun

1 **a** **:** a young tree, vine, shrub, or herb planted or suitable for planting

Web of meaning

No primitive concepts: things are always defined in terms of other things

Conceptual cut

If one looks closely, all physical concepts are well-defined only within a restricted realm of applicability

When does an orange become an orange? Can we really define quantities (e.g. distance, time, temperature) with infinite precision? When exactly is an object in thermodynamic equilibrium, such that a temperature is well defined? Since all objects interact gravitationally, does it make sense to talk about isolated systems?

dreamstime.com

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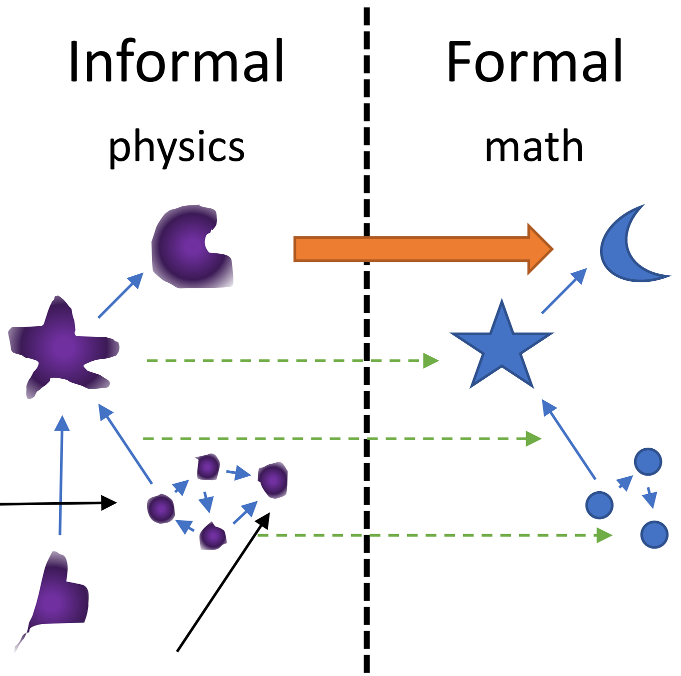
Problems in formalizing physical concepts

Physical concepts are "fuzzy"

Physical concepts may have circular definitions



Mathematical concepts are "crisper idealizations"

Mathematical concepts cannot have circular definitions

Some concepts will have to remain informal

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Guiding principle

What should our primitive "informal" notion be?

Principle of scientific objectivity: science is universal, non-contradictory and evidence based.

Suggest logic as fundamental …

like mathematics!

Universal → same for everybody

Non-contradictory → something is either true or false

Evidence based → truth is determined experimentally

… with some extensions

⇒ Logic of experimentally verifiable statements!

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Not “verifiable statements”

- Chocolate tastes good (not universal)
- It is immoral to kill one person to save ten (not universal and/or evidence based)
- The number 4 is prime (not evidence based)
- This statement is false (not non-contradictory)
- The mass of the photon is exactly 0 eV (not verifiable due to infinite precision)

“Verifiable statements”

- The mass of the photon is less than 10^{-13} eV
- If the height of the mercury column is between 24 and 25 millimeters then its temperature is between 24 and 25 Celsius
- If I take 2 ± 0.01 Kg of Sodium-24 and wait 15 ± 0.01 hours there will be only 1 ± 0.01 Kg left

A scientific theory needs “at least” the concept of a verifiable statement: good primitive notion

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What should be our primitive "formal" notion?

Tempting to try to capture everything into the formal system:

The electron is a fundamental particle and has negative charge

Conversion to formal logic

$$FP(e) \wedge NC(e)$$

constant

predicates

Never going to work!

What is an electron? What is charge? What is a physical object? What is a force? …

Web of meaning

Electron vs β particle, particles vs waves

Different prime elements/definitions in different theories

Conceptual cut

When is an object heavy enough to be "unmovable"?
How do we group objects into the same system?
How do we divide system and environment?

Semantic paradoxes

Berry paradox: the smallest positive integer that cannot be described in fewer than twenty-five words

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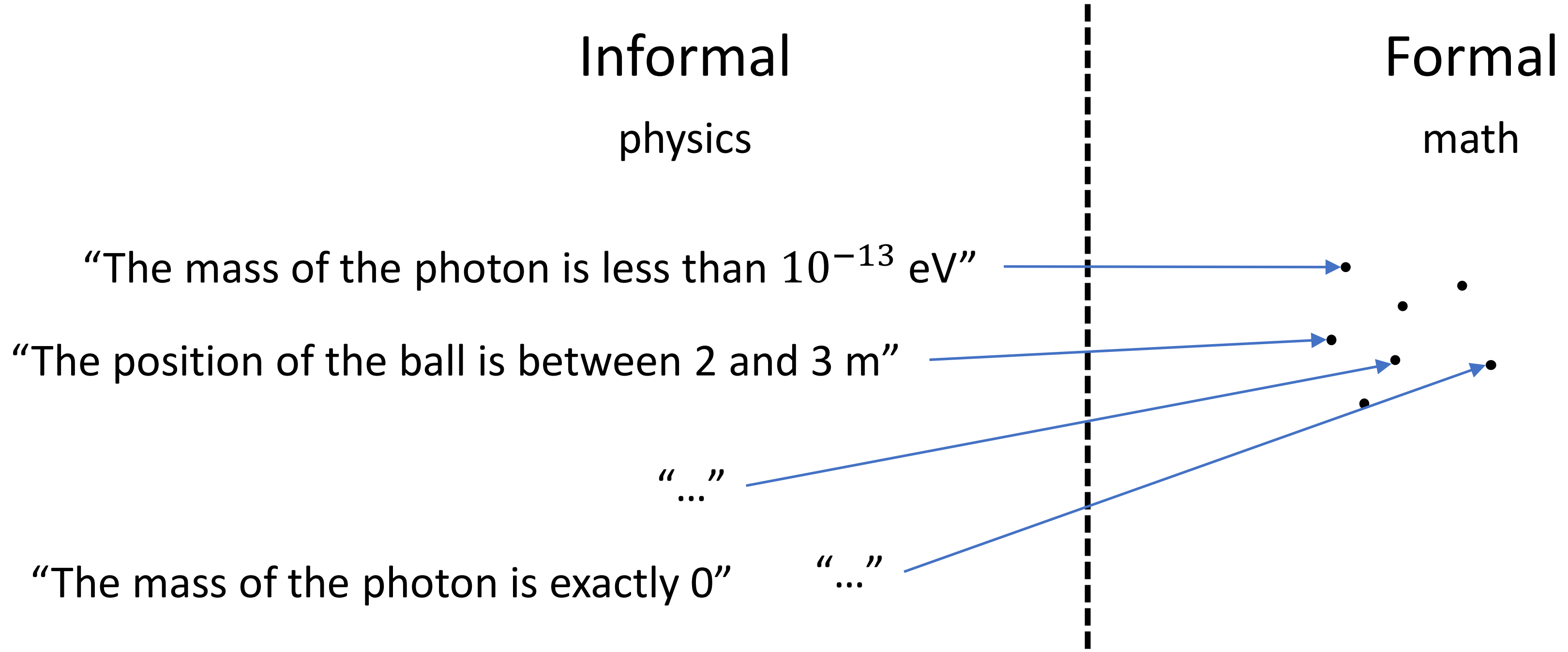
⇒ Statements are primitive notions in the formal system



The formal system does not know or care about the syntax or semantics of the statement

Should statements be the smallest primitive notion?

Can we guarantee universality, non-contradiction and connection to evidence on each statement alone?

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Liar paradox

"This sentence is false"

No-no paradox

"The next sentence is false"
"The previous sentence is true"

Yablo's paradox

S_1: For $i > 1$, S_i is not true.
S_2: For $i > 2$, S_i is not true.
S_3: For $i > 3$, S_i is not true.
…

⇒ Logical consistency is a property of groups of statements

⇒ Logical relationships are well-defined only with consistent semantics

The pope is the bishop of Rome

The bishop can only move diagonally

~~⇒ The pope can only move diagonally~~

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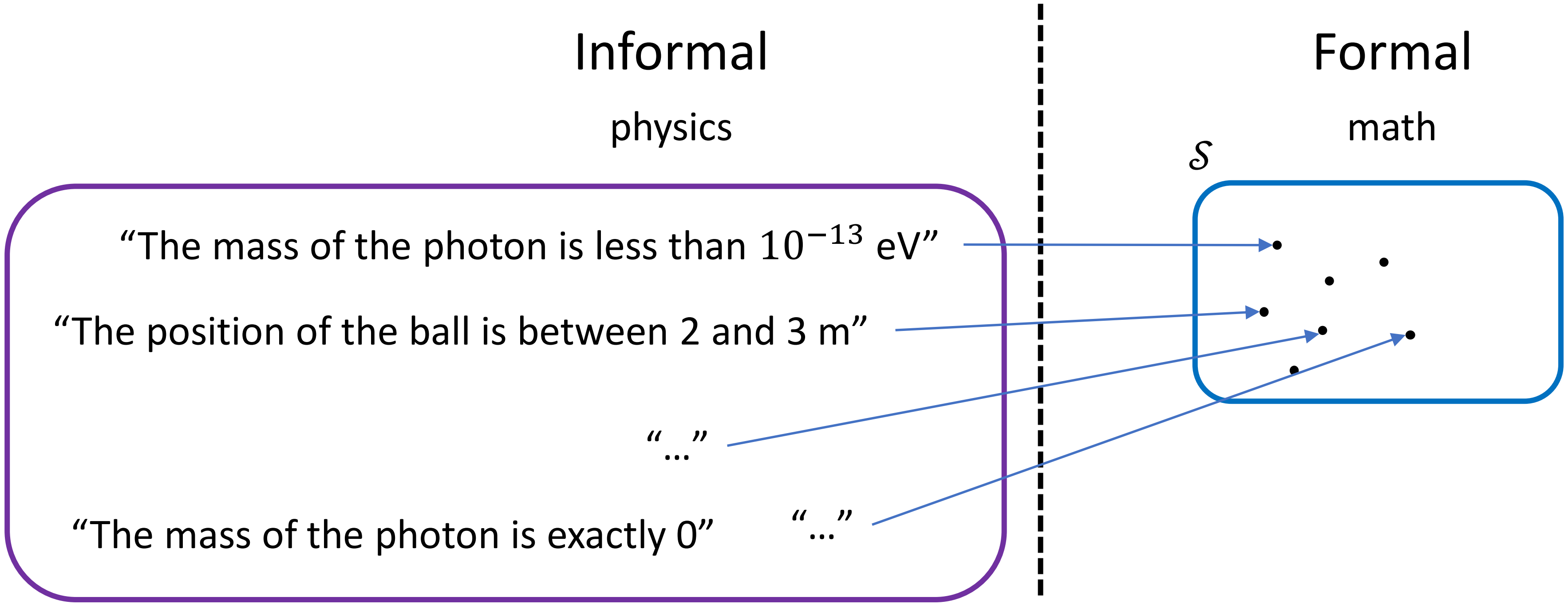
⇒ Statements must be grouped into logical contexts



Logical context contains all statements that are logically and semantically related

Still need connection to experimental evidence!

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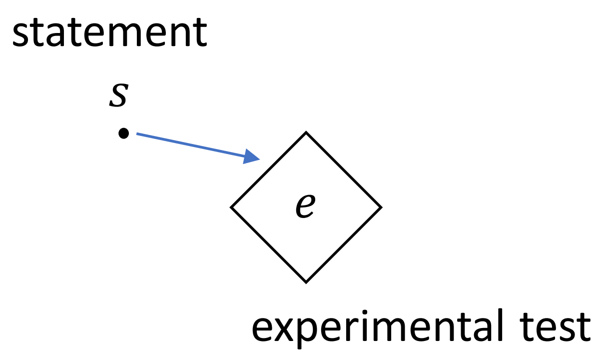
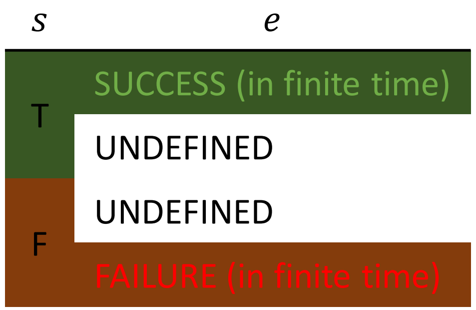
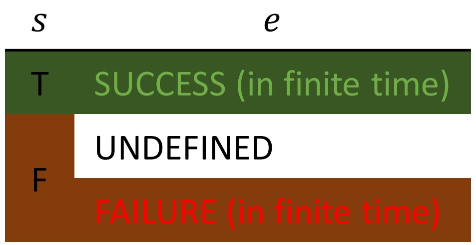




Don’t necessarily have a test that always finishes in all cases

There exists extra terrestrial-life
The mass of the photon is exactly zero
The ratio between the mass of the electron and proton in eV is a rational number
…



Verifiable statement: test guaranteed to finish successfully if and only if the statement is true

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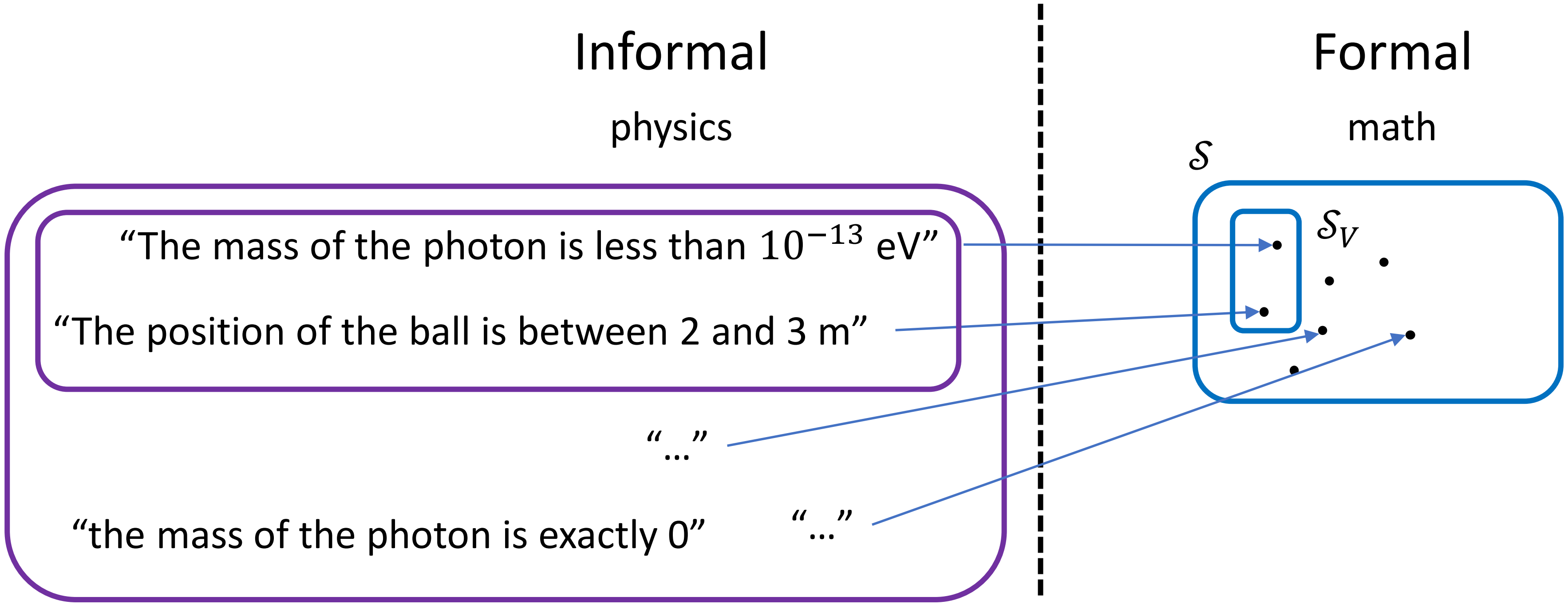
⇒ Contexts should keep track of verifiable statements



Is this primitive enough?

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$\mathbb{R}$

“The acceleration is $a \pm \Delta a$ m/s²”

$$F = ma$$

“The force is $F \pm \Delta$F N” “The mass is $m \pm \Delta m$ Kg”

“The mass is $m \pm \Delta m$ Kg”$\wedge$“The acceleration is $a \pm \Delta a$ m/s²”

$$\Rightarrow$$

“The force is $ma \pm \Delta F$ N”

The equation is really expressing relationships between experimentally verifiable statements

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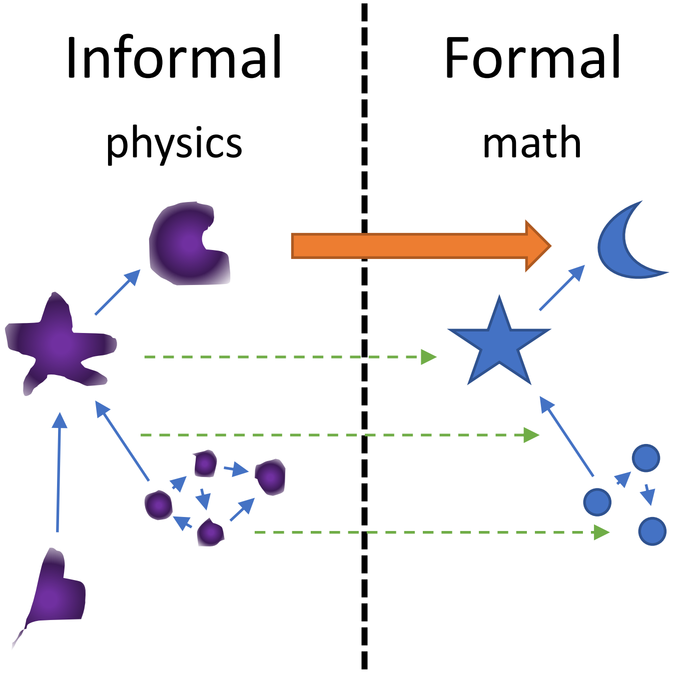
We can construct a formal system for experimental science provided we understand that



The physics will always live in the informal system

The formal system is made “precise” by removing all things that can’t be captured in a precise way

The logic system needs to be “augmented” to keep track of experimental verifiability (is it enough?)

Primitive notions should be specifically chosen to expose only necessary complexity

Axioms/definitions should be specifically chosen to have straightforward physical justifications

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Takeaways

- It is possible to construct a formal system for experimental science but…
- The physics will always live in the informal system
- The formal system is made “precise” by removing all things that can’t be captured in a precise way
- Primitive notions should be specifically chosen to expose only necessary complexity
- Axioms/definitions should be specifically chosen to have straightforward physical justifications
- TODOs
  - Refine this approach into a “philosophical position”

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Logic of experimental verifiability

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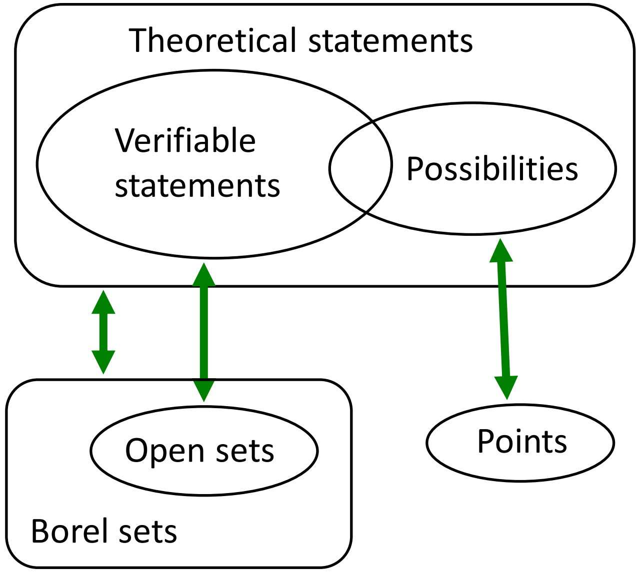
Topology and σ-algebra

Experimental verifiability ⇒ topology and σ-algebras (foundation of geometry, probability, …)

Perfect map between math and physics



s_1	Test Result
T	SUCCESS (in finite time)
T	UNDEFINED
F	UNDEFINED
F	FAILURE (in finite time)

$int(A)$ corresponds to the verifiable part of a statement

∂A corresponds to the undecidable part of a statement

$ext(A)$ corresponds to the falsifiable part of a statement

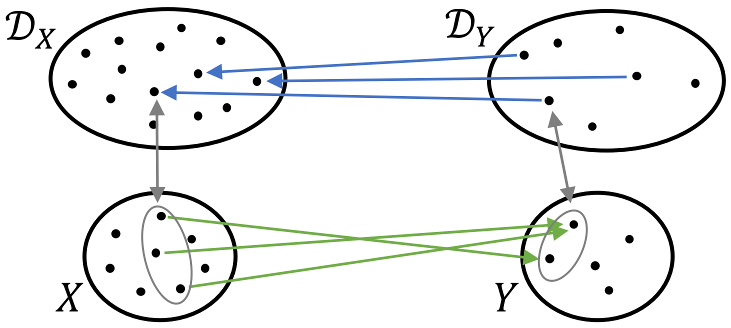
NB: in physics, topology and σ-algebra are parts of the **same** logic structure

Open set $(509.5, 510.5)$ ⟺ Verifiable "the mass of the electron is 510 ± 0.5 KeV"

Closed set $[510]$ ⟺ Falsifiable "the mass of the electron is exactly 510 KeV"

Borel set $\mathbb{Q}$ $(int(\mathbb{Q}) \cup ext(\mathbb{Q}) = \emptyset)$ ⟺ Theoretical "the mass of the electron in KeV is a rational number" (undecidable)

Inference relationship $r: \mathcal{D}_Y \to \mathcal{D}_X$ such that $r(s) \equiv s$

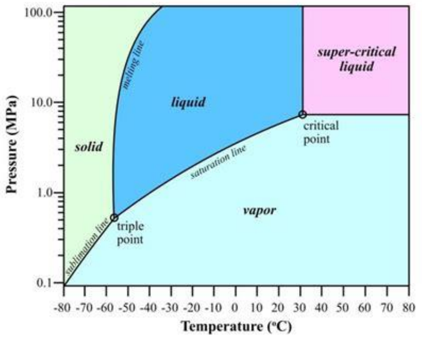


Causal relationship $f: X \to Y$ such that $x \preccurlyeq f(x)$

Inference relationship
⟺
Causal relationship

Relationships must be topologically continuous

Topologically continuous consistent with analytic discontinuity on isolated points



Phase transition ⟺ Topologically isolated regions

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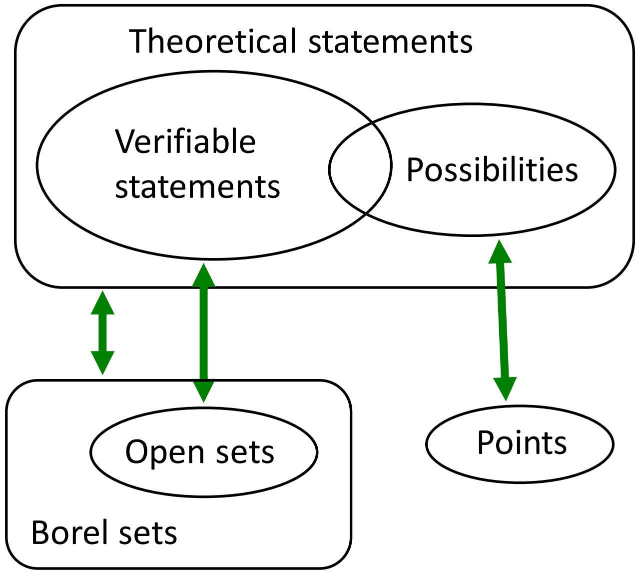
Physical theories are logical structures generated by countably many verifiable statements



The topological structure (i.e. closure under finite intersection and arbitrary union) stems from requirement of finite time termination (i.e. closure under finite AND and countable OR)

The σ-algebra (i.e. closure under countable union and complement) of the Borel sets, instead, stems from requirement of tests, regardless of termination (i.e. closure under countable OR and NOT)

Note: nothing "proves" that verifiable tests "exist" strictly speaking

(Conceptual cut, Descartes' evil demon, problem of inference, …)

But, if they exist, they must follow that structure

and it's the exact mathematical structure we use for proofs and calculations

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Takeaways

- Topology and σ-algebras capture experimental verifiability
- This link provides a perfect map between physics and math
- It supports the idea that physical theories are, in the end, logical structures
- Given that the structure is generated by countably many verifiable statements, the structure is fully grounded in experimental science
- TODOs
  - Philosophical consequences interesting for current debates?
  - Better connection to work from Kevin Kelly et al.

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Ensembles as fundamental concept

Typical view: first define “pure states”, then define ensembles as probability distribution over pure states

This is backwards compared to experimental practice

In practice, we can only prepare ensembles, and imagine making them more and more refined.

If we want a theory based on experimental practice, the ensembles are the primary object, and the pure states are particular ensembles that correspond to “best possible preparation”.

Basic arguments:

In practice, one never prepares or measures a pure state. There is always uncertainty. But we can imagine each preparation as the statistical mixture of better preparations.
⇒ each preparation is a mixture of “ideal” preparations (pure states of classical and/or quantum mechanics)

Physical laws represent “if-then” relationships. These rules apply not to a single instance of preparation/measurement, but to all similar preparations/measurements. Therefore, laws relate collections of identically prepared systems: physical laws are about ensembles.

Basic arguments:

Repeatability is intrinsic to science. Repeatability always implies that "we can always do it one more time." Saying that the electron has a particular value of mass means that we can always produce an electron and repeat the measurement. The value of mass, then, is a property of the collection of all electrons: it is a property of an ensemble.

Statistical mixing ⇒ Convex structure

$$(e_1, e_2, p) \Rightarrow pe_1 + (1-p)e_2$$

Processes ⇒ Linear transformations $\mathcal{T}: \mathcal{E} \to \mathcal{E}$

$$\mathcal{T}(pe_1 + (1-p)e_2) = p\mathcal{T}(e_1) + (1-p)\mathcal{T}(e_2)$$

Quantities ⇒ Linear operators $\mathcal{O}: \mathcal{E} \to \mathbb{R}$

$$\mathcal{O}(pe_1 + (1-p)e_2) = p\mathcal{O}(e_1) + (1-p)\mathcal{O}(e_2)$$

all?

Ensembles ⇒ Linear structures in physics

A classical measurement "refines" the ensemble

Mere "update" to more precise ensemble

Preparation → e $(= \sum p_i e_i)$ → Measurement / selection → e_k

e_k part of the original decomposition

Final ensemble has lower entropy

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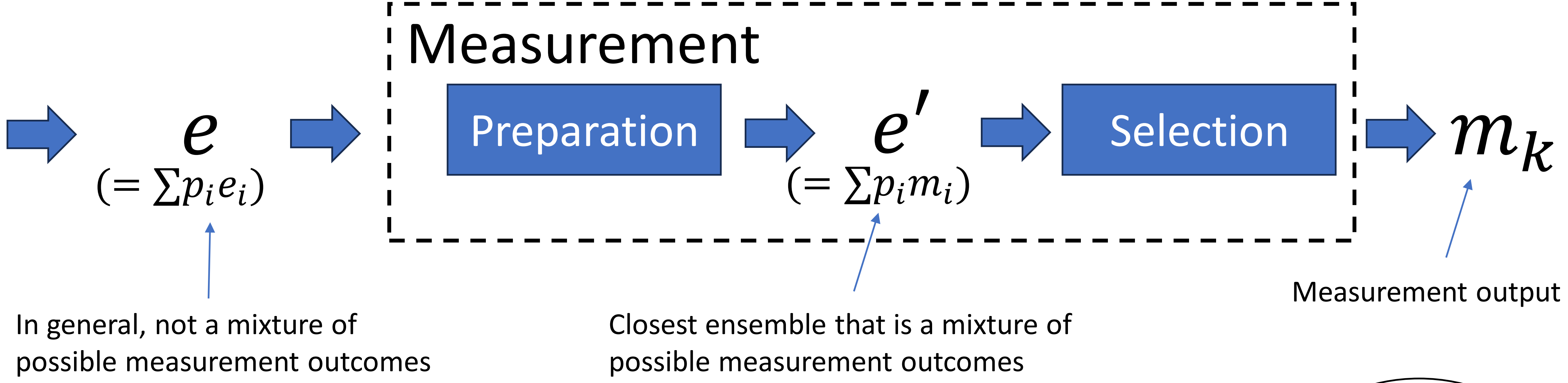
A quantum measurement may also reprepare the system



First part can increase entropy

Second part can decrease entropy

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Takeaways

- Ensembles are necessary fundamental objects in physics
- All processes are linear maps from ensembles to ensembles
- TODOs
  - Cleanup and refine

System definition

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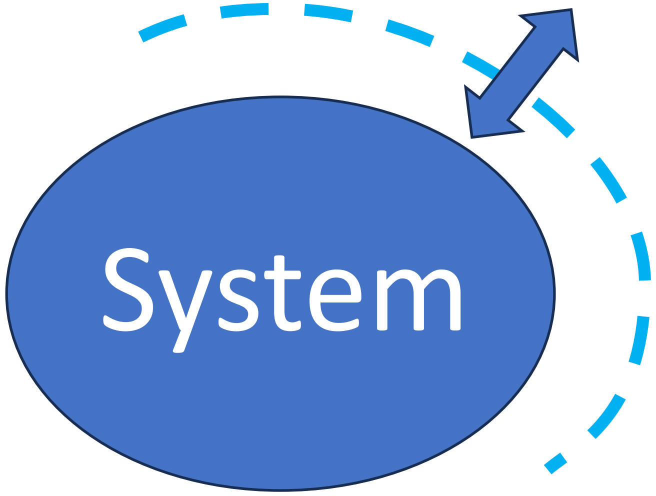
What are basic requirements to define a system?

Establish a system/environment boundary

Independence from environment,
internal dynamics and other systems

System/environment boundary

Environment



Needs to define what is in the system and what is not

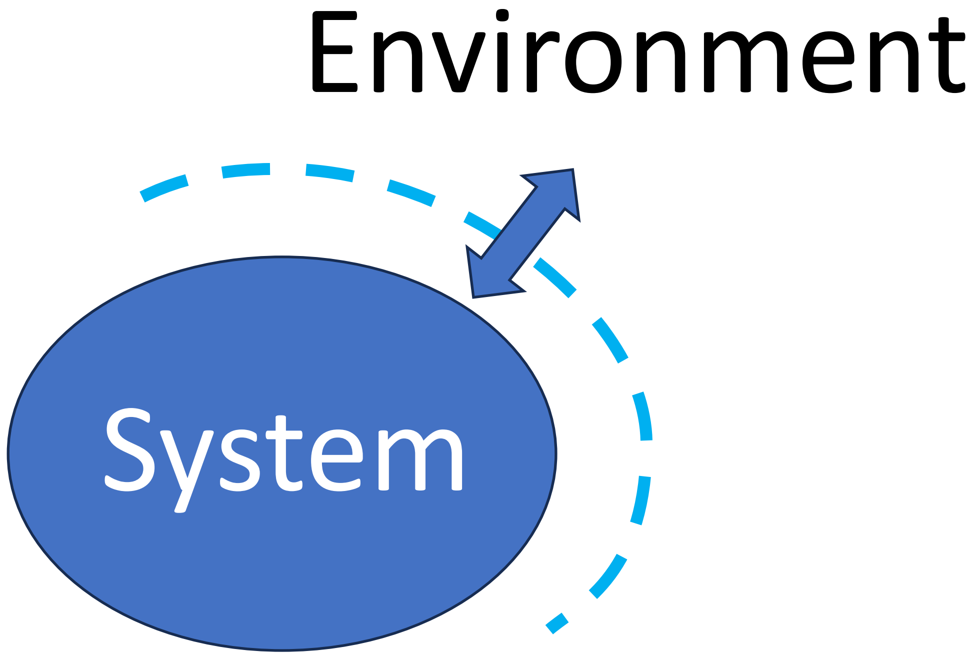
- This includes which properties are relevant

Interaction at the boundary determines the system and its properties

- System can only exist in equilibrium with environment
- System property is well-defined only if unchanged by the environment

All physics (science) is contextual

System independence



Systems can be defined only if, in some circumstances, they can be described and studied independently

- From environment, other systems and internal dynamics

Description of system must decouple from all the rest

- Clearly idealized assumption
- Similar condition of equilibria as system decoupling we saw in thermodynamics

State are ensembles in equilibrium

Contexts are different sys/env equilibria

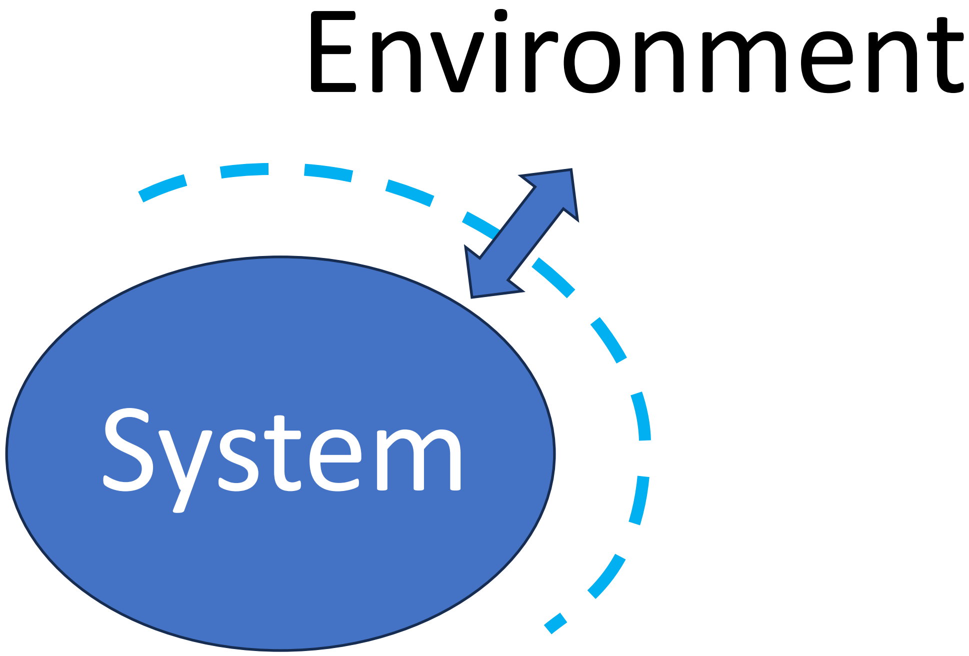
Evolution of system is also evolution of context

Immutable fundamental objects do not necessarily exist

Same “energy/matter” takes different form in different contexts

Measurement apparatus is part of the environment

“State of the whole universe” is problematic



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Takeaways

- Definition and description of a system is also description of sys/env boundary
- States are ensembles in equilibrium
- TODOs
  - Cleanup and refine

Foundations of probability

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Quantum mechanics does not follow classical Kolmogorov probability

⇒ None of the interpretations of classical probability will work for Quantum Mechanics

Need a generalization of probability that makes physical sense and recovers classical probability

Probability space $(\Omega, \Sigma, p: \Sigma \to [0,1])$

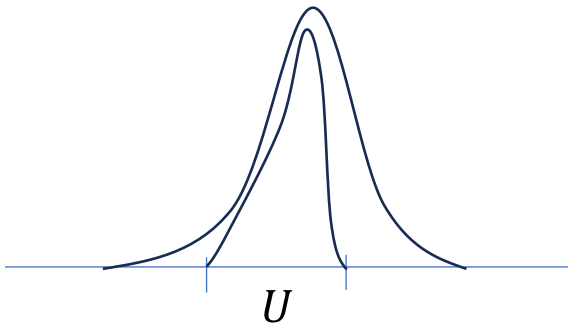
additive

$p(A \cup B) = p(A) + p(B)$ if $A \cap B = \emptyset$

How to generalize?

Probability of a subset: weight for the biggest part that has support in that subset

$$p(x) = p(x|U)p(U) + p(x|U^C)p(U^C)$$



$$p(U|\rho) = \sup(\{p \in [0,1] | \rho = p\rho_1 + (1-p)\rho_2, \rho_1: U \to \mathbb{R}, \rho_2: X \to \mathbb{R}\})$$

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Assumptions of Physics

mixing

entropy

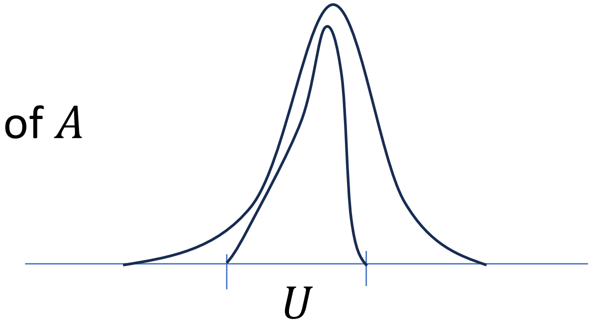
Ensemble space $(\mathcal{E}, +, S: \mathcal{E} \to \mathbb{R})$

$A \in \Sigma_{\mathcal{E}}$ ← Set of ensembles

$\mathrm{hull}(A)$ ← All mixtures of A

$$p(A|e) = \sup(\{p \in [0,1] |$$
$$e = p e_1 + (1-p) e_2, e_1 \in \mathrm{hull}(A), e_2 \in \mathcal{E}\})$$

Mixing coefficient of the biggest component of e reachable with mixtures of A



Disjunct ensembles: when mixing provide the maximum entropy increase

Correspond to orthogonal elements

Can be discriminated with an experiment

p is sub-additive in general but additive for disjunct sets

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Assumptions of Physics

Primitive notion is not frequency, propensity, …

It’s ensemble mixing

Survives in quantum mechanics as well

Entropy is what defines classical contexts

Lattice of subspaces where two disjoint subspaces are also disjunct (i.e. maximize entropy during mixing)

Takeaways

- We need an approach to probability that works for both classical and quantum mechanics
- We can construct one where ensembles are primitives
- It also requires a new interpretation of probability in terms of ensemble mixing
- TODOs
  - Construct a full philosophical approach

Physical theories as assumptions

Infinitesimal reducibility
+
Determinism/reversibility
+
Independence of DOFs

⇔ Classical Hamiltonian Mechanics

Necessary and sufficient assumptions ⇔ Physical theory

Assumptions not only identify the realm of applicability of the theory, but they are exactly the theory

⇒ Studying the nature of the assumptions means studying the nature of physical theories

Some assumptions come from requirements of experimental science

Theory must be generated by countably many verifiable statements

Ensembles are the fundamental physical objects

← Not about nature

Some assumptions are specific to the system at hand

Infinitesimal reducibility, deterministic and reversible evolution, …

Some assumptions are somewhat in between

System isolation

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Assumptions of Physics

Incompatibility between theories is, in the end, incompatibility of assumptions

Projections during measurements are change of equilibrium with environment

Einstein's equations come from a Lagrangian, assume det/rev evolution, no abrupt change in sys/env boundary (?)

⇒ Projections and general relativity are incompatible

A “theory of everything” would need ALWAYS valid assumptions

Is that possible?

All scientific explanations are “reduction to assumptions”

Reducing everything to the same assumptions may not provide the best explanation

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Assumptions of Physics

Thus my first answer to the implied question about the unreasonable effectiveness of mathematics is that we approach the situations with an intellectual apparatus so that we can only find what we do in many cases. It is both that simple, and that awful. What we were taught about the basis of science being experiments in the real world is only partially true. Eddington went further than this; he claimed that a sufficiently wise mind could deduce all of physics. I am only suggesting that a surprising amount can be so deduced. Eddington gave a lovely parable to illustrate this point. He said, “Some men went fishing in the sea with a net, and upon examining what they caught they concluded that there was a minimum size to the fish in the sea.”

- Richard Hamming

THE UNREASONABLE EFFECTIVENESS OF MATHEMATICS

Takeaways

- Scientific theories are, in the end, equivalent to the assumptions that define them
- Scientific explanations, in the end, can only say why a given assumption leads to a particular phenomenon
- TODOs
  - Probably many connections to current philosophy of math and physics

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Assumptions of Physics

Wrapping it up

- Providing more solid foundations for physics means also fixing its philosophical foundations
- The goal is not to do philosophy for philosophy’s sake, but to clearly specify the starting point from which to rederive the math and the physics
- Problems that are not critical to derive the math and the physics, or where people can disagree, are not in scope
- Ideally, we would find people in the philosophy community who would help us make progress

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