Assumptions of Physics
Summer School 2024

Classical Mechanics

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Assumptions of Physics

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Assumptions of Physics

Main goal of the project

Identify a handful of physical starting points from which the basic laws can be rigorously derived

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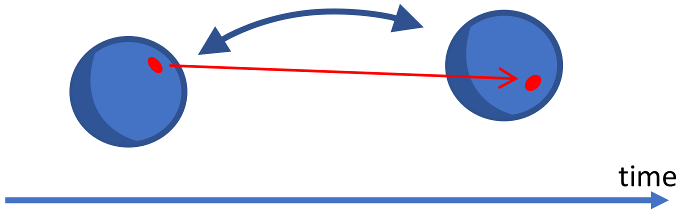
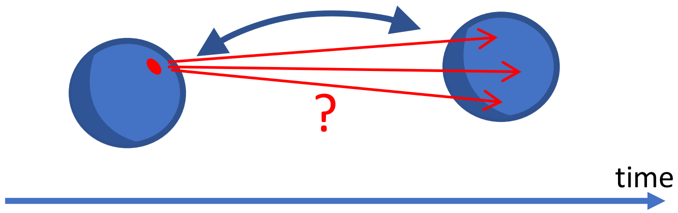
For example:

Infinitesimal reducibility ⇒ Classical state



Irreducibility ⇒ Quantum state



This also requires rederiving all mathematical structures from physical requirements

For example:

Science is evidence based ⇒ scientific theory must be characterized by experimentally verifiable statements ⇒ topology and σ-algebras

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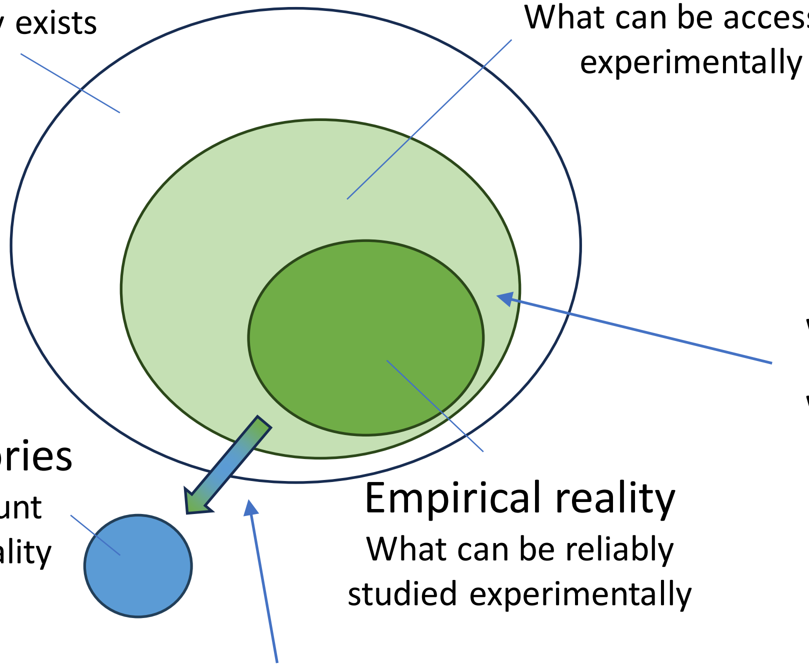
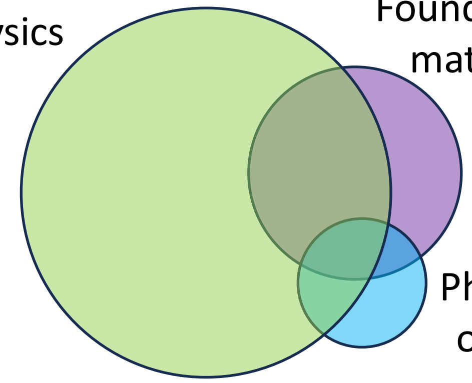
Underlying perspective



What is the boundary?

What are the requirements?

How exactly does the abstraction/idealization process work?



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If physics is about creating models of empirical reality, the foundations of physics should be a theory of models of empirical reality

Requirements of experimental verification, assumptions of each theory, realm of validity of assumptions, …

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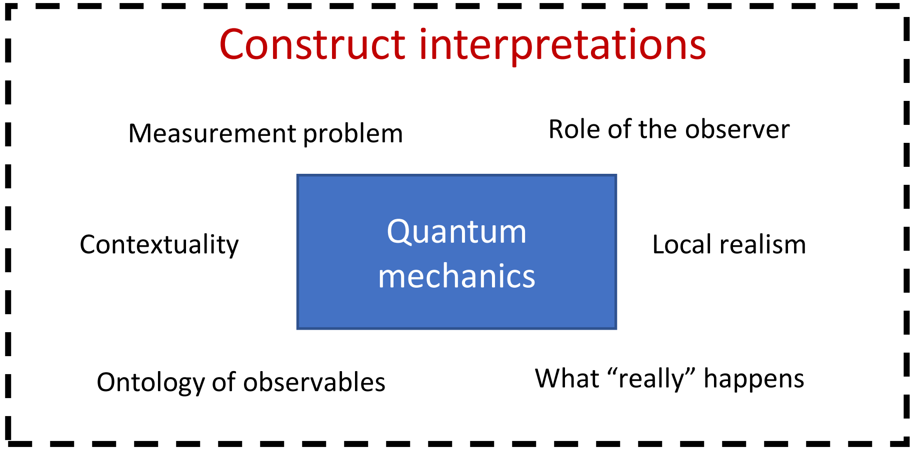
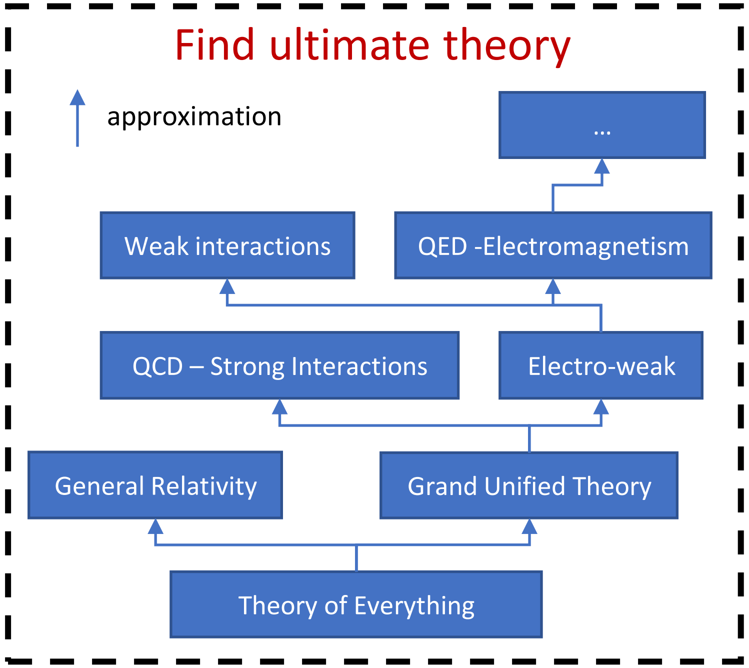
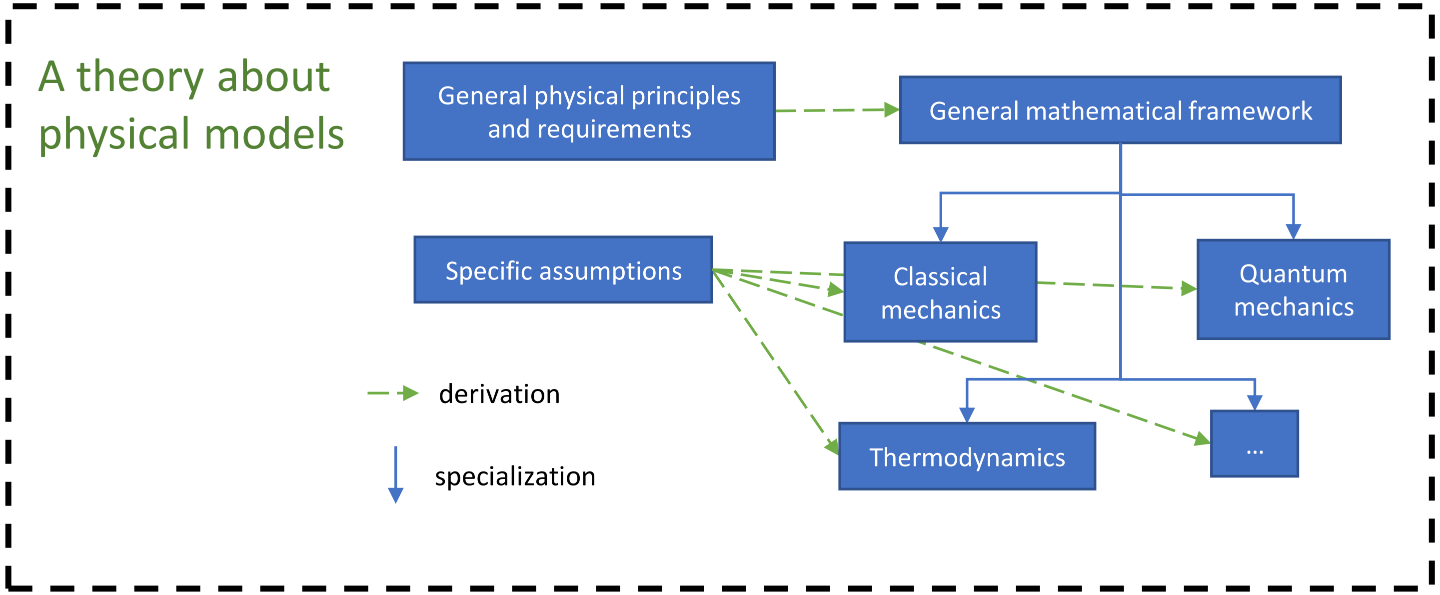
Different approach to the foundations of physics

Typical approaches





Our approach



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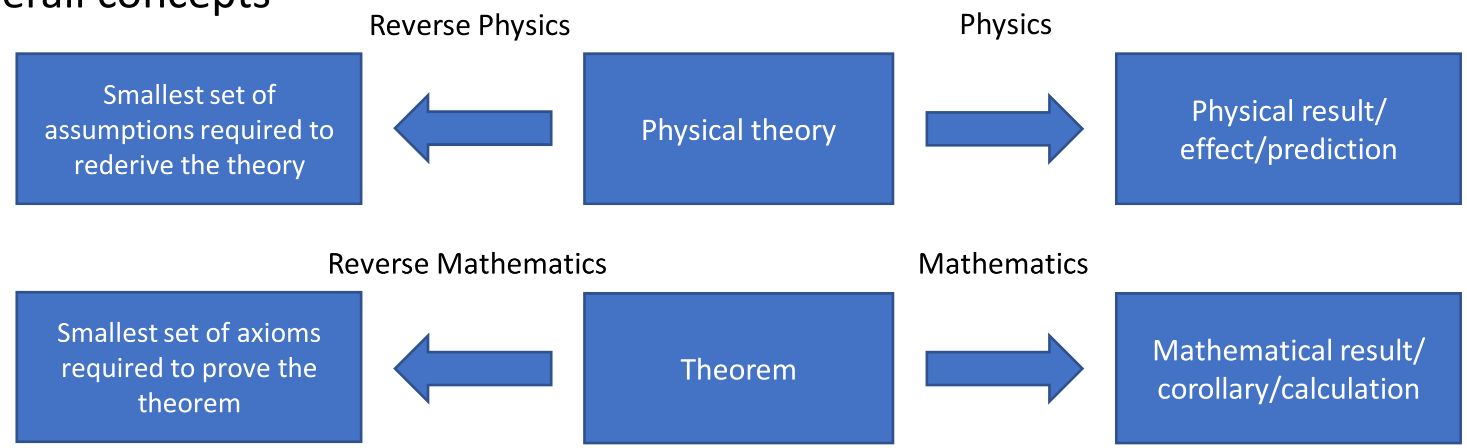
Find the right overall concepts

Reverse physics:

Start with the equations, reverse engineer physical assumptions/principles

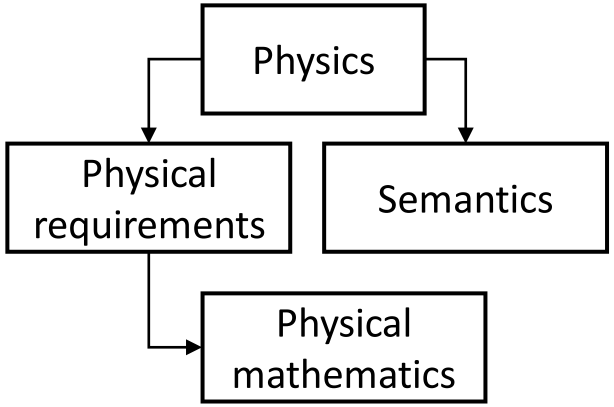
Found Phys **52**, 40 (2022)



Goal: find the right overall physical concepts, "elevate" the discussion from mathematical constructs to physical principles

Physical mathematics:

Start from scratch and rederive all mathematical structures from physical requirements



Goal: get the details right, perfect one-to-one map between mathematical and physical objects

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This session

Reverse Physics:
Classical mechanics

Assumptions of Physics*,*
Michigan Publishing (v2 2023)

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Compare different formulations

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Assumptions of Physics

	State	Dynamics	Evolution
Newtonian Mechanics:	x^i and v^i	$F^i(x^j, v^k, t)$	$F^i = d_t(mv^i)$
	position, velocity		
Lagrangian Mechanics:	x^i and v^i	$L(x^i, v^j, t)$	$\partial_{x^i} L = d_t \partial_{v^i} L$
		$\lvert \partial_{v^i} \partial_{v^j} L \rvert \neq 0$	required for unique solution
Hamiltonian Mechanics:	q^i and p_i	$H(q^i, p_j, t)$	$d_t q^i = \partial_{p_i} H$
	position, conjugate momentum		$d_t p_i = -\partial_{q^i} H$

Are they equivalent?

Newtonian: Three independently chosen functions (i.e. the forces) of position and velocity

Hamiltonian: A single function (i.e. the Hamiltonian) of position and momentum

$$[F^x, F^y, F^z] \leftrightarrow H \longleftrightarrow L$$

Lagrangian: A single function (i.e. the Lagrangian) of position and velocity

There is no continuous one-to-one map between the space of a single function and the space of multiple functions!

No homeomorphism

⇒ Not all Newtonian systems are Lagrangian and/or Hamiltonian

Lagrangian state $(x^i, v^i) \longleftrightarrow (x^i, v^i)$ Newtonian state

Lagrangian EoM $\partial_{x^i} L = d_t \partial_{v^i} L \longrightarrow F^i = m a^i$ Newtonian EoM

$$\partial_{x^i} L = d_t \partial_{v^i} L = \partial_{x^j}\partial_{v^i} L \, d_t x^j + \partial_{v^k}\partial_{v^i} L \, d_t v^k = \partial_{x^j}\partial_{v^i} L \, v^j + \partial_{v^k}\partial_{v^i} L \, a^k$$

$\left|\partial_{v^i}\partial_{v^j} L\right| \neq 0$
$\equiv$ unique solution

$$\partial_{v^k}\partial_{v^i} L \, a^k = \partial_{x^i} L - \partial_{x^j}\partial_{v^i} L \, v^j$$

$$a^k = \left(\partial_{v^k}\partial_{v^i} L\right)^{-1}\left(\partial_{x^i} L - \partial_{x^j}\partial_{v^i} L \, v^j\right)$$

$\Rightarrow$ All Lagrangian systems are Newtonian

Lagrangian state $(x^i, v^i) \longleftrightarrow (q^i, p_i)$ Hamiltonian state

must be invertible

$$v^i = d_t q^i = \partial_{p_i} H \quad \Rightarrow \left|\partial_{p_j}\partial_{p_i} H\right| \neq 0$$

Assumption KE (Kinematic Equivalence). *The kinematics of the system is sufficient to reconstruct its dynamics and vice-versa. That is, specifying the motion of the system is equivalent to specifying its state and evolution.*

$$\left|\partial_{p_i}\partial_{p_j} H\right| = \left|\partial_{p_i} v^j\right| = \left|\partial_{v^i} p_j\right|^{-1} = \left|\partial_{v^i}\partial_{v^j} L\right|^{-1}$$

$p_j = \partial_{v^j} L$

required for unique solution $\left|\partial_{v^i}\partial_{v^j} L\right| \neq 0$

$\Rightarrow$ All Lagrangian systems are Hamiltonian

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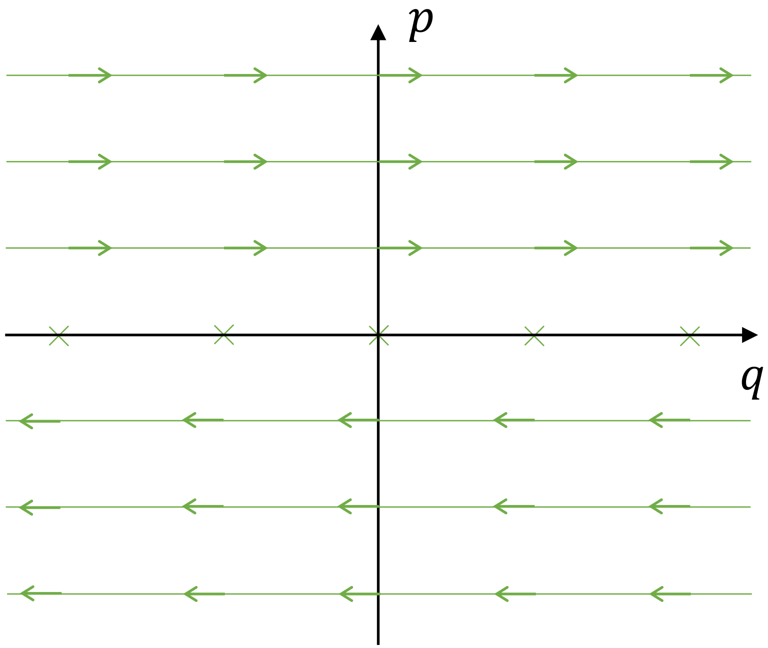
Assumptions of Physics

Photon as a particle

$$d_t q^i = \partial_{p_i} H = c\,\frac{p^i}{|p|}$$

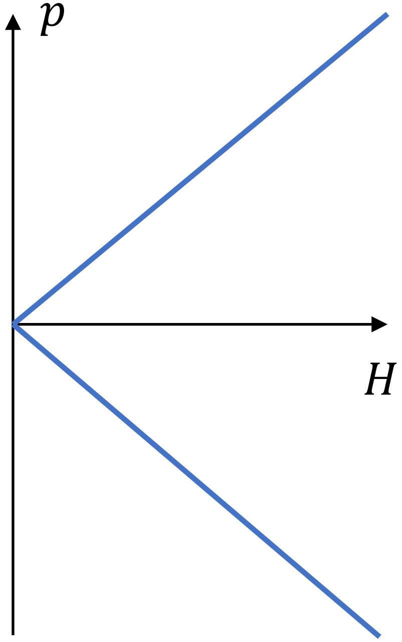
$$H = \hbar|\omega| = c\hbar|k| = c|p|$$

$$d_t p_i = -\partial_{q^i} H = 0$$

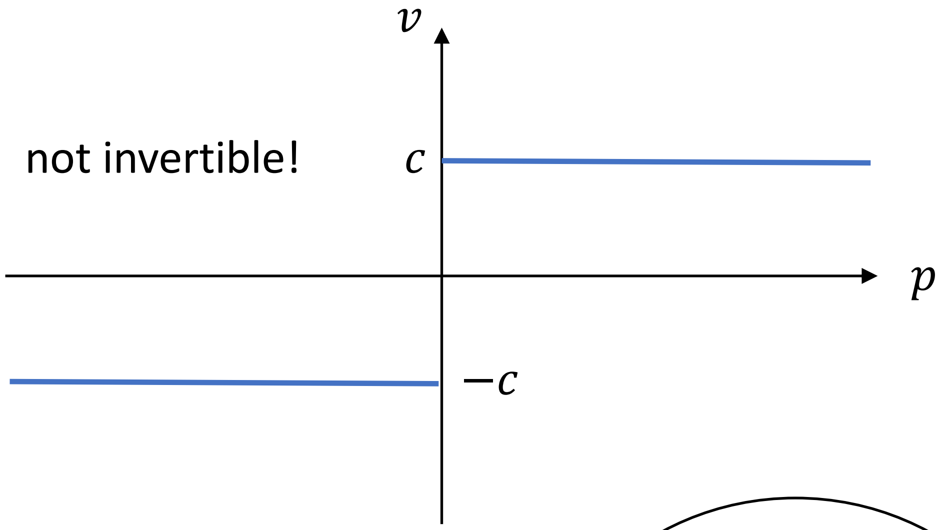


phase-space diagram for a photon

treated as a point particle



plot of the Hamiltonian $H = c|p|$



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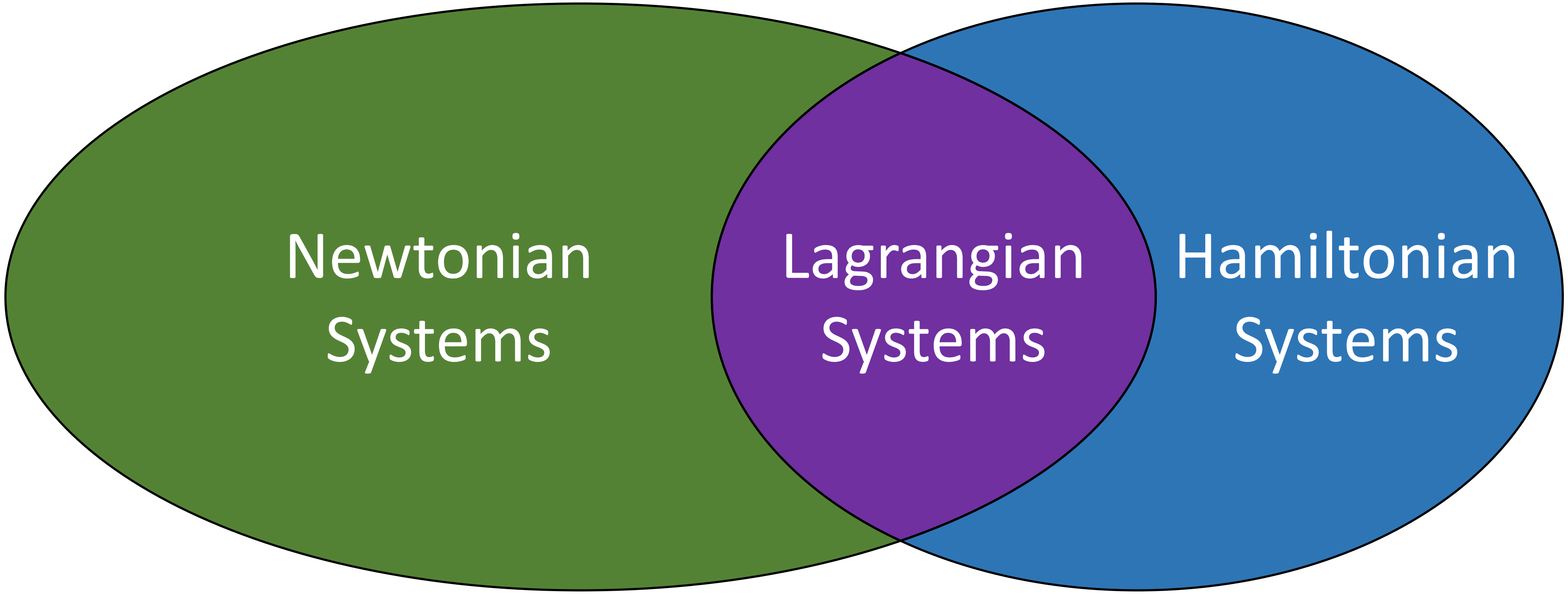
Assumptions of Physics


Newtonian Systems
Lagrangian Systems
Hamiltonian Systems

12 equivalent characterizations of Hamiltonian mechanics single DOF

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Assumptions of Physics

Hamilton’s equations

(HM-1D)

$$\frac{dq}{dt} = \frac{\partial H}{\partial p} \qquad d_t q = S^q(q,p) \qquad S^q = \partial_p H$$

$$\frac{dp}{dt} = -\frac{\partial H}{\partial q} \qquad d_t p = S^p(q,p) \qquad S^p = -\partial_q H$$

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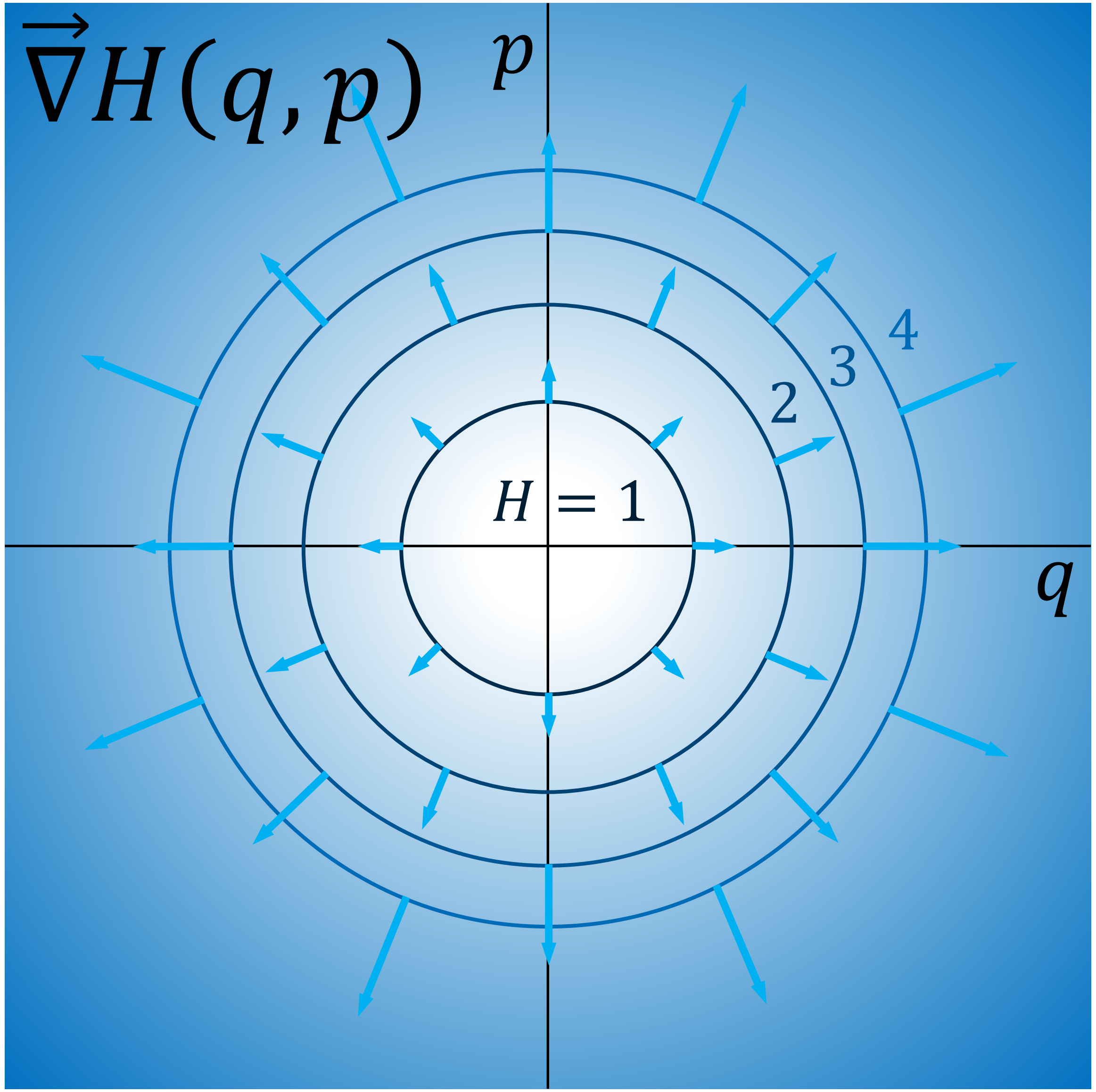
Assumptions of Physics



$$\vec{\nabla} H(q,p) = \begin{bmatrix} \partial_p H \\ \partial_q H \end{bmatrix} = \begin{bmatrix} \frac{p}{m} \\ 2kq \end{bmatrix}$$

$$H = \frac{p^2}{2m} + kq^2$$

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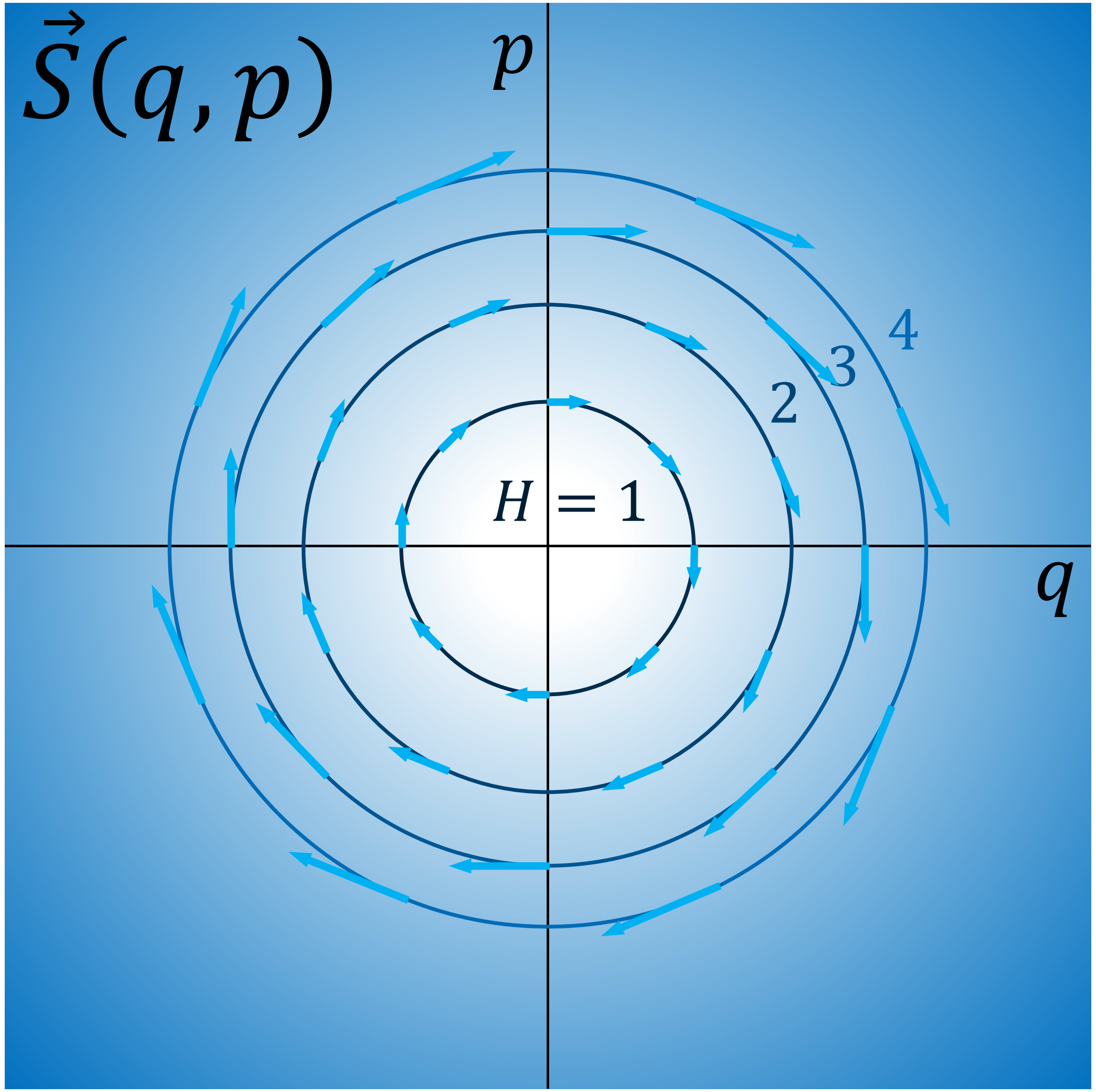
Assumptions of Physics



$$\vec{S}(q,p) = \begin{bmatrix} S^q \\ S^p \end{bmatrix}$$

$$= \begin{bmatrix} d_t q \\ d_t p \end{bmatrix} = \begin{bmatrix} \partial_p H \\ -\partial_q H \end{bmatrix}$$

$$= \circlearrowright^{90°}(\vec{\nabla} H)$$

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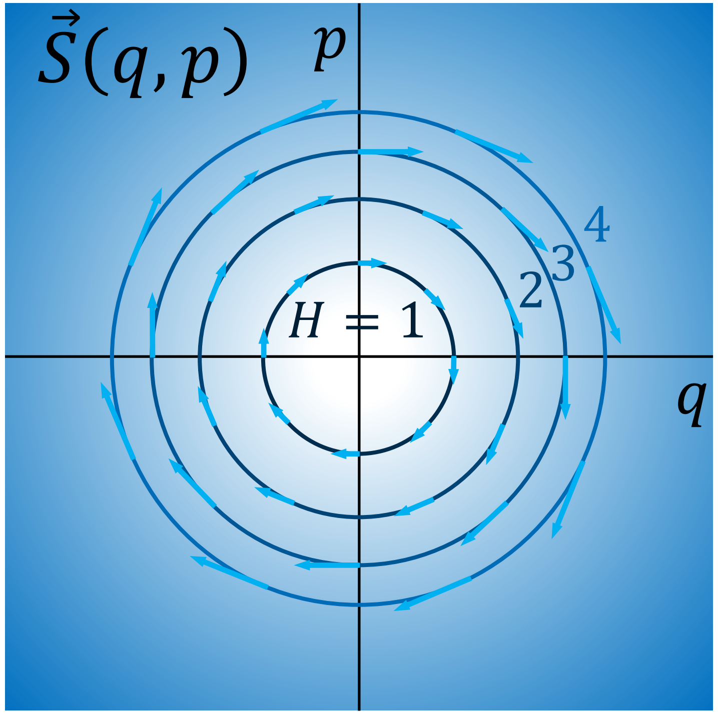
Assumptions of Physics

$$\omega = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$[\omega_{ab}] = \begin{bmatrix} \omega_{qq} & \omega_{qp} \\ \omega_{pq} & \omega_{pp} \end{bmatrix}$$



$$\vec{S} = \omega \, \vec{\nabla} H$$

$$S^b \omega_{ba} = \partial_a H$$

$$\begin{bmatrix} S^q \\ S^p \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \partial_q H \\ \partial_p H \end{bmatrix} = \begin{bmatrix} \partial_p H \\ -\partial_q H \end{bmatrix}$$

$$S^q \omega_{qp} = \partial_p H$$

$$S^p \omega_{pq} = \partial_q H$$

(HM-G)

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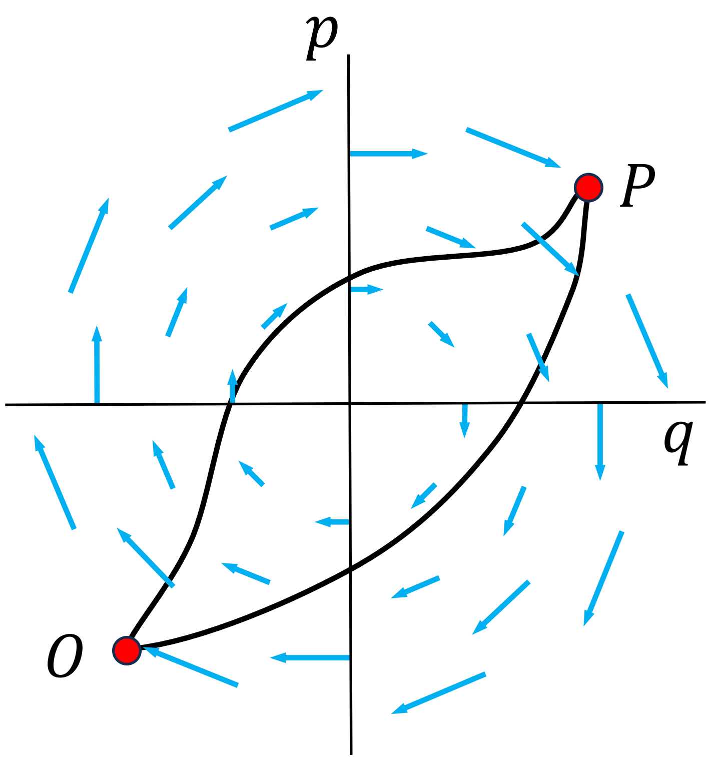
Assumptions of Physics

$$\partial_a S^a = \partial_q S^q + \partial_p S^p$$
$$= \partial_q \partial_p H - \partial_p \partial_q H = 0$$



$$H(P) = \int_{OP} (S^q dp - S^p dq)$$

HM-1D $\Longleftrightarrow$

The displacement field is divergenceless: $\partial_a S^a = 0$ (DR-DIV)

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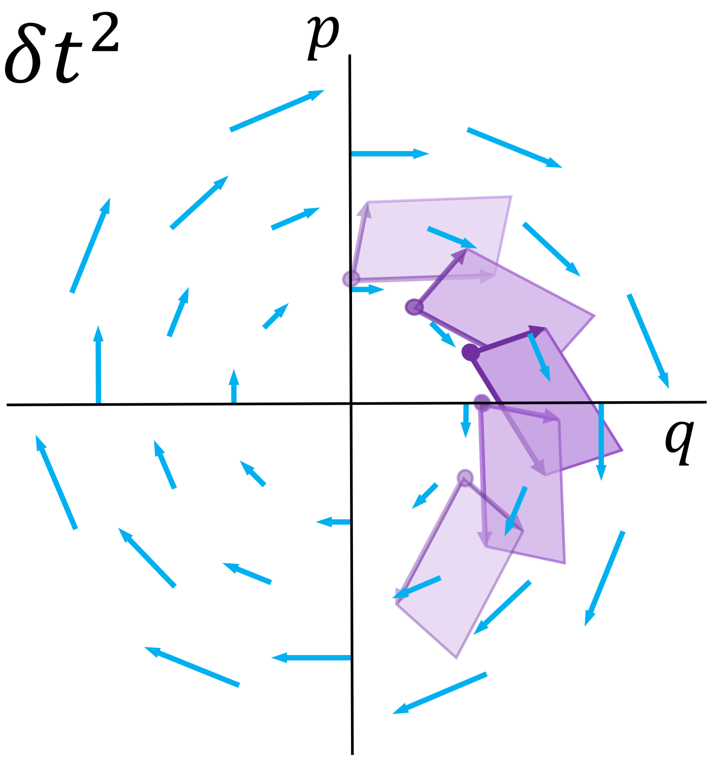
Assumptions of Physics

$$\left|\partial_b \hat{\xi}^a\right| = \left(1 + \partial_q S^q \delta t\right)\left(1 + \partial_p S^p \delta t\right) - \partial_q S^p \partial_q S^q \delta t^2$$
$$= 1 + \left(\partial_q S^q + \partial_p S^p\right)\delta t + O(\delta t^2)$$



$$\left|\partial_b \hat{\xi}^a\right| = 1 \iff \partial_a S^a = 0$$

HM-1D $\iff$ DR-DIV $\iff$

The Jacobian of the time evolution is unitary:

$\left|\partial_b \hat{\xi}^a\right| = 1$ (DR-JAC)

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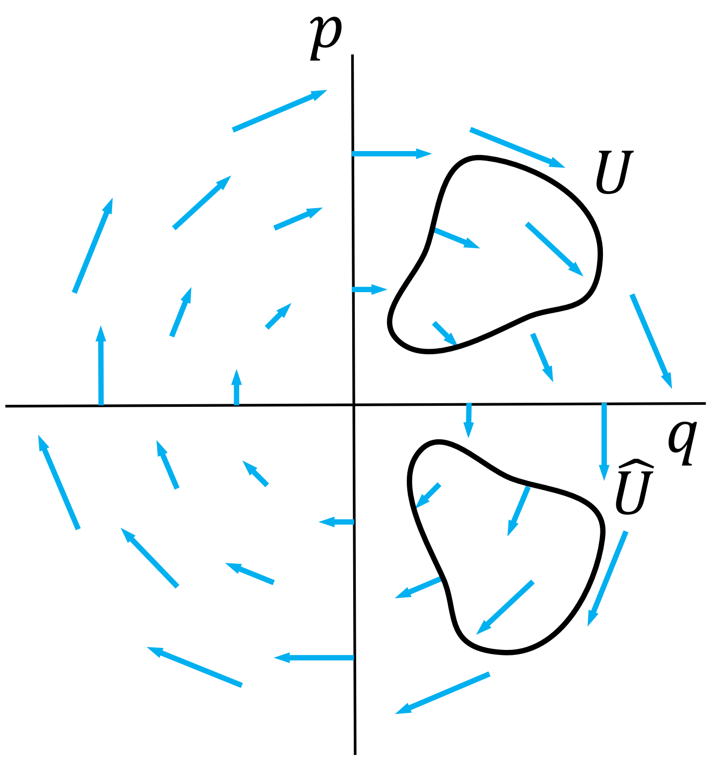
Assumptions of Physics

$$\int_{\hat{U}} d\hat{q}\,d\hat{p} = \int_U \begin{vmatrix} \partial_q \hat{q} & \partial_p \hat{q} \\ \partial_q \hat{p} & \partial_p \hat{p} \end{vmatrix} dq\,dp$$

$$= \int_U dq\,dp$$



DR-JAC $\Leftrightarrow$

Volumes are conserved through the evolution:

$$d\hat{\xi}^1 \ldots d\hat{\xi}^n = d\xi^1 \ldots d\xi^n \text{ (DR-VOL)}$$

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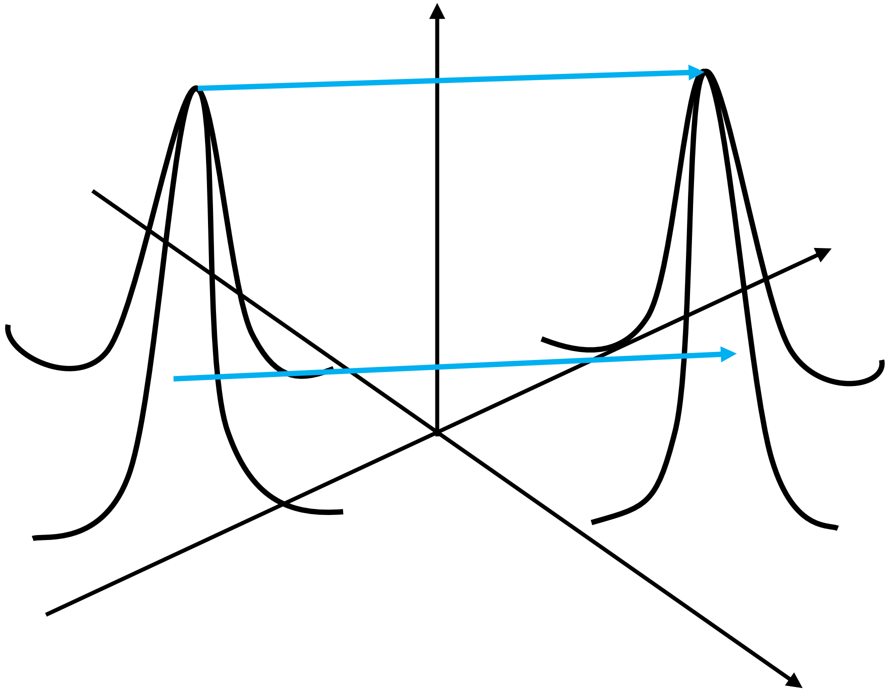
Assumptions of Physics

Transformation of densities

$$\left|\partial_b \hat{\xi}^a\right| \hat{\rho}\left(\hat{\xi}^a\right) = \rho\left(\xi^b\right)$$
$$= \hat{\rho}\left(\hat{\xi}^a\right)$$



DR-JAC $\Longleftrightarrow$

Densities are conserved through the evolution: $\hat{\rho}\left(\hat{\xi}^a\right) = \rho\left(\xi^b\right)$ (DR-DEN)

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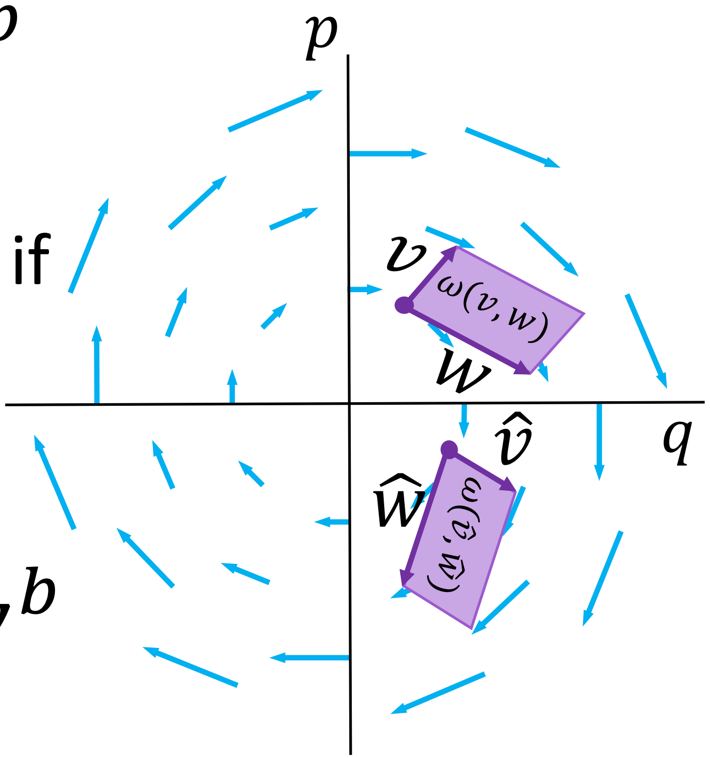
Assumptions of Physics

$$\text{Area}(v^a, w^a) = v^q w^p - v^p w^q = v^a \omega_{ab} w^b$$

Area is conserved if

$$\hat{\omega}_{ab} = \omega_{ab}$$



$$\hat{v}^a = \partial_b \hat{\xi}^a v^b \qquad \hat{w}^a = \partial_b \hat{\xi}^a w^b$$

$$\hat{v}^c \omega_{cd} \hat{w}^d = v^a \partial_a \hat{\xi}^c \omega_{cd} \partial_b \hat{\xi}^d w^b = v^a \hat{\omega}_{ab} w^b$$

DR-VOL $\Longleftrightarrow$

The evolution leaves ω_{ab} invariant:

$\hat{\omega}_{ab} = \omega_{ab}$ (DI-SYMP)

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Assumptions of Physics

Jacobian transformation to two other variables f and g

Poisson bracket!

$$\begin{vmatrix} \partial_q f & \partial_p f \\ \partial_q g & \partial_p g \end{vmatrix} = \partial_q f \partial_p g - \partial_p f \partial_q g = \{f, g\}$$

We can express the Poisson bracket: $\{f, g\} = -\partial_a f \omega^{ab} \partial_b g = \partial_b g \omega^{ba} \partial_a f$

is the inverse of ω_{ab}

$$\omega_{ab}\omega^{bc} = \delta_a^c = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

DI-SYMP $\Longleftrightarrow$

The evolution leaves the Poisson brackets invariant (DI-POI)

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Assumptions of Physics

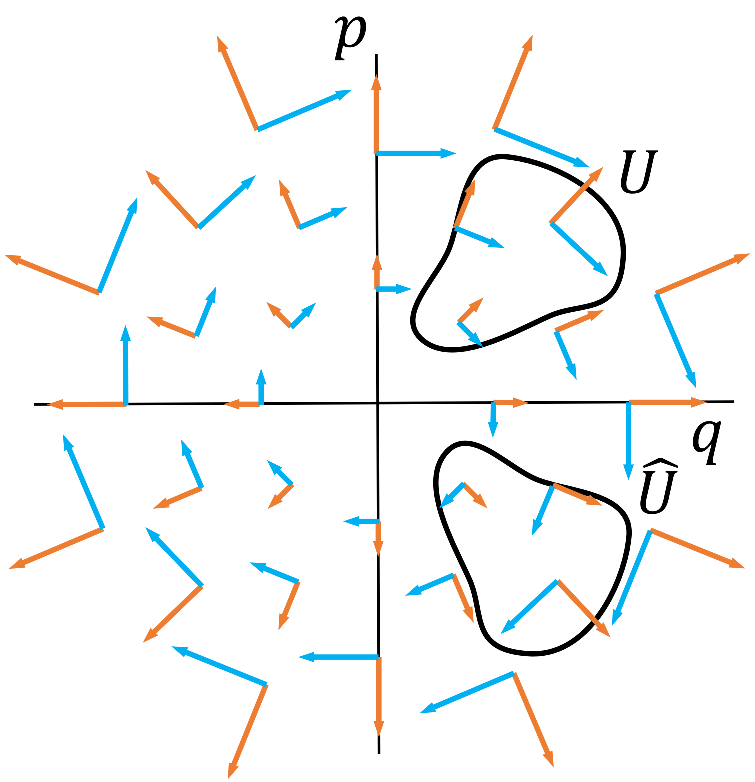
Flow THROUGH the curve

$$\int S^a \times d\xi^b = \int (S^q dp - S^p dq)$$

$$\int S^a \omega_{ab} d\xi^b = \int S_b \cdot d\xi^b$$

90° rotation

Flow ALONG the curve



DR-DIV $\Longleftrightarrow$

The rotated displacement field is curl free: $\partial_a S_b - \partial_b S_a = 0$ (DI-CURL)

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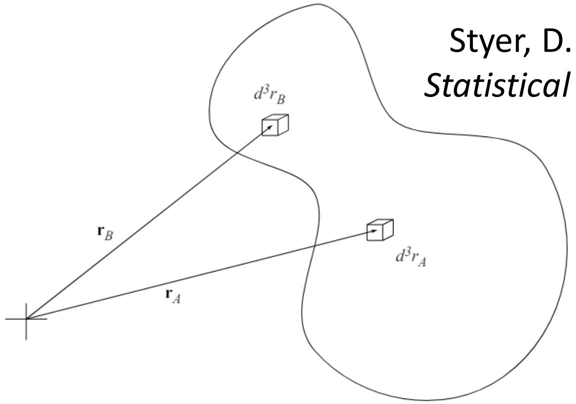
Assumptions of Physics

Note: Hamiltonian mechanics is about transporting areas/densities, not just points!



Styer, D. F. (2023)
Statistical Mechanics

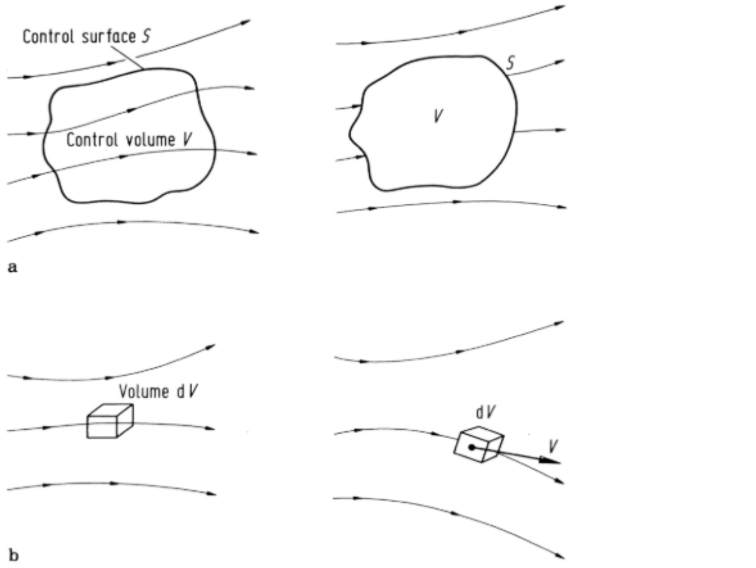


Fig. 2.1 (**a**) Finite control volume approach. (**b**) Infinitesimal fluid element approach

Wendt, J. F. (Ed.). (2008)
Computational Fluid Dynamics

⇒ Classical point particles are better conceived as infinitesimal regions of phase space

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Assumptions of Physics

Stat mech: phase space volumes count the number of microstates

of choice of the potential energy zero.) Then define the microstate count as the dimensionless quantity

$$\Omega(E, \Delta E, V, N) = \frac{1}{h_0^{3N}} \frac{1}{N!} \int_{\sigma(E,\Delta E)} d\Gamma. \qquad \text{(Dimensionless, delabeled volume in phase space.)} \qquad (2.6)$$

Hamiltonian mechanics preserves volumes ⇒ preserves the number of states

⇒ for each initial state there is one and only one final state

DR-VOL ⟺

The evolution is deterministic and reversible (DR-EV)

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Assumptions of Physics

Thermodynamic entropy: $S = k_B \log W$

Since the logarithm is a bijective function, conservation of areas of phase space is equivalent to the conservation of entropy.

DR-VOL $\Longleftrightarrow$

The evolution is deterministic and thermodynamically reversible (DR-THER)

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Assumptions of Physics

Deterministic and reversible evolution

⇒ existence and conservation of energy (Hamiltonian)

Why?

Stronger version of the first law of thermodynamics

Deterministic and reversible evolution

⇒ past and future depend only on the state of the system

⇒ the evolution does not depend on anything else

⇒ the system is isolated

⇒ the system conserves energy

First law of thermodynamics!

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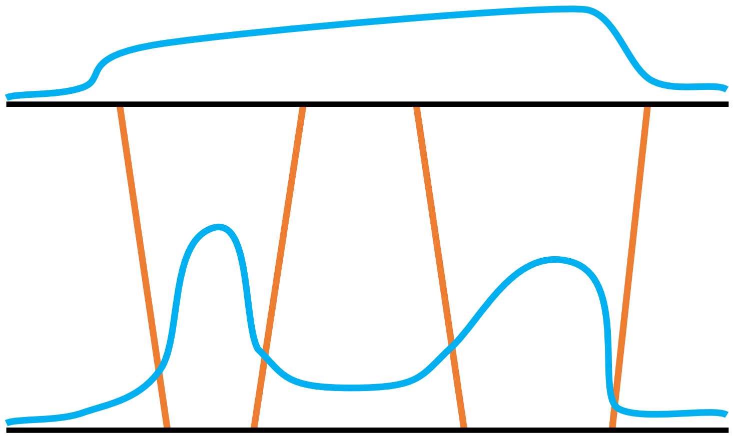
Assumptions of Physics

$$I[\rho(\xi^a)] = I[\hat{\rho}(\hat{\xi}^b)] - \int \hat{\rho}(\hat{\xi}^b) \log |\partial_a \hat{\xi}^b| d\xi^1 \dots d\xi^n$$

Information entropy: $I = -\int \rho \log \rho$



DR-JAC $\Longleftrightarrow$

The evolution conserves information entropy (DR-INFO)

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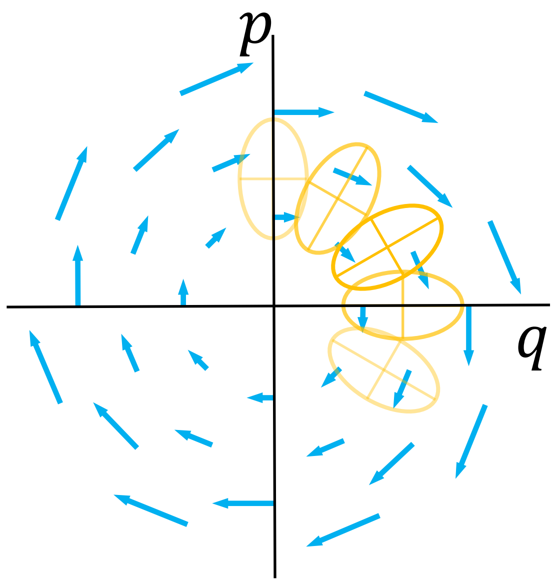
Assumptions of Physics

$$\left|cov\left(\hat{\xi}^c, \hat{\xi}^d\right)\right| = \left|\partial_a \hat{\xi}^c cov\left(\xi^a, \xi^b\right) \partial_b \hat{\xi}^d\right| = \left|\partial_a \hat{\xi}^c\right| \left|cov\left(\xi^a, \xi^b\right)\right| \left|\partial_b \hat{\xi}^d\right|$$

Covariance matrix:

$$cov\left(\xi^a, \xi^b\right) = \begin{bmatrix} \sigma_q^2 & cov_{q,p} \\ cov_{p,q} & \sigma_p^2 \end{bmatrix}$$



DR-JAC $\Longleftrightarrow$

The evolution conserves the uncertainty of peaked distributions (DR-UNC)

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Assumptions of Physics

We have found twelve equivalent characterizations for a single degree of freedom

(HM-1D) $d_t q = \partial_p H, d_t p = -\partial_q H$

(HM-G) $S^b \omega_{ba} = \partial_a H$

(DR-DIV) The displacement field is divergenceless: $\partial_a S^a = 0$

(DR-JAC) The Jacobian of the time evolution is unitary: $\left|\partial_b \hat{\xi}^a\right| = 1$

(DR-VOL) Volumes are conserved through the evolution: $d\hat{\xi}^1 \dots d\hat{\xi}^n = d\xi^1 \dots d\xi^n$

(DR-DEN) Densities are conserved through the evolution: $\hat{\rho}(\hat{\xi}^a) = \rho(\xi^b)$

(DI-SYMP) The evolution leaves ω_{ab} invariant: $\hat{\omega}_{ab} = \omega_{ab}$

(DI-POI) The evolution leaves the Poisson brackets invariant

(DI-CURL) The rotated displacement field is curl free: $\partial_a S_b - \partial_b S_a = 0$

(DR-EV) The evolution is deterministic and reversible

(DR-THER) The evolution is deterministic and thermodynamically reversible

(DR-INFO) The evolution conserves information entropy

(DR-UNC) The evolution conserves the uncertainty of peaked distributions

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Assumptions of Physics

Takeaways

- Hamiltonian mechanics describes a flow of states that is deterministic and reversible
- There is a deeper way to understand classical mechanics and we should strive to understand all of physics like this
- TODOs:
  - Help popularize these ideas, more pictures/diagrams in the book/etc…

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Assumptions of Physics

Hamiltonian mechanics multiple DOFs

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Assumptions of Physics

HM-G $\Leftrightarrow$ DI-CURL

$\Updownarrow$ $\Updownarrow$

DI-POI $\Leftrightarrow$ DI-SYMP

$\Rightarrow$

DR-JAC $\Leftrightarrow$ DR-DIV

$\Updownarrow$ $\Updownarrow$

DR-DEN $\Leftrightarrow$ DR-VOL

For single DOF, volumes are areas

For multiple DOFs, statements about areas are stronger than statements about volumes

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Assumptions of Physics

$$\omega = \begin{bmatrix} 0 & I_n \\ -I_n & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \otimes I_n \qquad [\omega_{ab}] = \begin{bmatrix} \omega_{q^i q^j} & \omega_{q^i p_j} \\ \omega_{p_i q^j} & \omega_{p_i p_j} \end{bmatrix}$$

Pick $\{\xi, \zeta\}$ from $\{q^i, p_j\}$

$$\omega(d\xi, d\zeta) \neq 0 \qquad \Leftrightarrow \qquad \{\xi, \zeta\} = \{q^i, p_i\} \text{ for some } i$$

ω returns the configuration over a degree of freedom

Orthogonality represents independence

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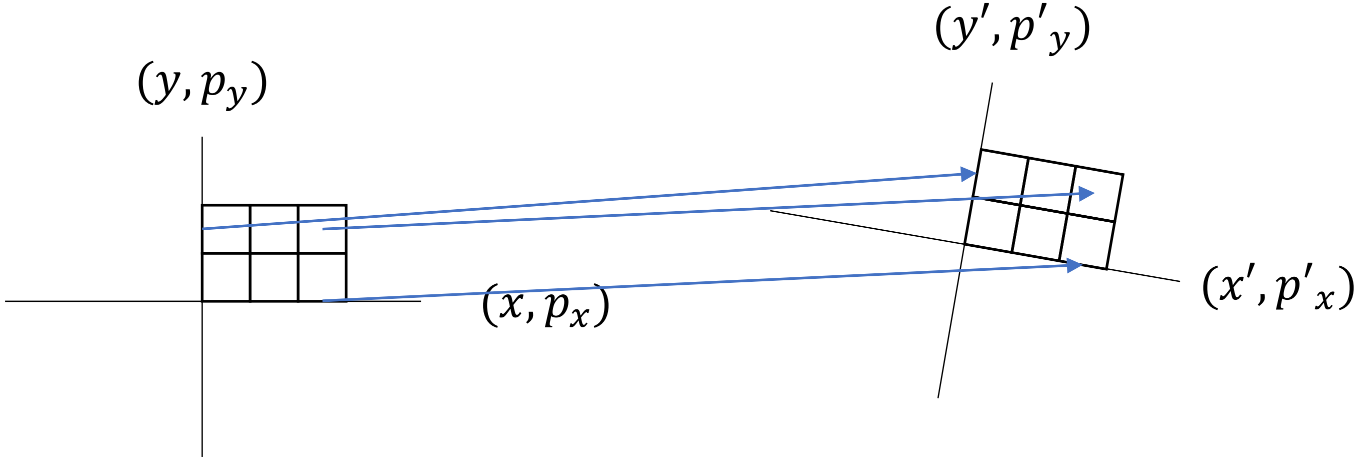
Assumptions of Physics

$$
\begin{array}{ccccccccc}
\text{HM-G} & \Longleftrightarrow & \text{DI-CURL} & & & \text{DR-JAC} & \Longleftrightarrow & \text{DR-DIV} \\
\Updownarrow & & \Updownarrow & \Longrightarrow & & \Updownarrow & & \Updownarrow \\
\text{DI-POI} & \Longleftrightarrow & \text{DI-SYMP} & & & \text{DR-DEN} & \Longleftrightarrow & \text{DR-VOL}
\end{array}
$$



Evolution preserves areas and orthogonality $\Longrightarrow$ Evolution preserves volumes

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Assumptions of Physics

$$d_t q^1 = S^{q^1} = \frac{p_1}{m} \qquad d_t p_1 = S^{p_1} = -b p_1$$ linear drag

$$d_t q^2 = S^{q^2} = \frac{p_2}{m} \qquad d_t p_2 = S^{p_2} = b p_2$$ linear acceleration

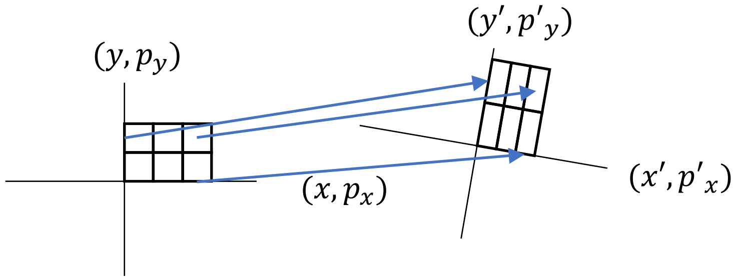
$$\partial_a S^a = \partial_{q^1} \frac{p_1}{m} + \partial_{p_1}(b p_1) + \partial_{q^2} \frac{p_2}{m} + \partial_{p_2}(b p_2) = -b + b = 0$$

DR-DIV and DR-VOL satisfied

$$\partial_{q^1} S_{p_1} - \partial_{p_1} S_{q^1} = \partial_{q^1} S^{q^1} \omega_{q^1 p_1} - \partial_{p_1} S^{p_1} \omega_{p_1 q^1} = \partial_{q^1} \frac{p_1}{m}(1) - \partial_{p_1}(-b p_1)(-1) = -b$$

DI-CURL and DI-SYMP not satisfied

⇒ No Hamiltonian



Evolution preserves areas and orthogonality $\not\Leftarrow$ Evolution preserves volumes

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Assumptions of Physics

Physical characterizations of DOF independence

The system is decomposable into independent DOFs	(IND-DOF)
The system allows statistically independent distributions over each DOF	(IND-STAT)
The system allows informationally independent distributions over each DOF	(IND-INFO)
The system allows peaked distributions where the uncertainty is the product of the uncertainty on each DOF	(IND-UNC)

Similar to physical characterizations of determinism and reversibility

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Assumptions of Physics

HM-G $\Longleftrightarrow$ DI-CURL $\quad$ DR-JAC $\Longleftrightarrow$ DR-DIV

$\Updownarrow$ $\quad$ $\Updownarrow$ $\quad \Longrightarrow \quad$ $\Updownarrow$ $\quad$ $\Updownarrow$

DI-POI $\Longleftrightarrow$ DI-SYMP $\quad$ DR-DEN $\Longleftrightarrow$ DR-VOL

DR+IND $\quad\quad$ DR

Assumption DR (Determinism and Reversibility). *The system undergoes deterministic and reversible evolution. That is, specifying the state of the system at a particular time is equivalent to specifying the state at a future (determinism) or past (reversibility) time.*

Assumption IND (Independent DOFs). *The system is decomposable into independent degrees of freedom. That is, the variables that describe the state can be divided into groups that have independent definition, units and count of states.*

NOTE: in principle we could have IND without DR

Takeaways

- Hamiltonian mechanics is preservation of number of configurations and independence of DOFs
- A single mathematical structure may bundle multiple independent physical assumptions
- TODOs:
    - Help popularize these ideas, more pictures/diagrams in the book/etc…

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Assumptions of Physics

Lagrangian and least action

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Assumptions of Physics

Reversing the principle of least action

DR

$$\nabla \cdot \vec{S} = 0$$

No state is "lost" or "created" as time evolves

$$\vec{S} = -\nabla \times \vec{\theta}$$

(Minus sign to match convention)

KE

$$\mathcal{S}[\gamma] = \int_\gamma L dt = \int_\gamma \vec{\theta} \cdot d\vec{\gamma}$$

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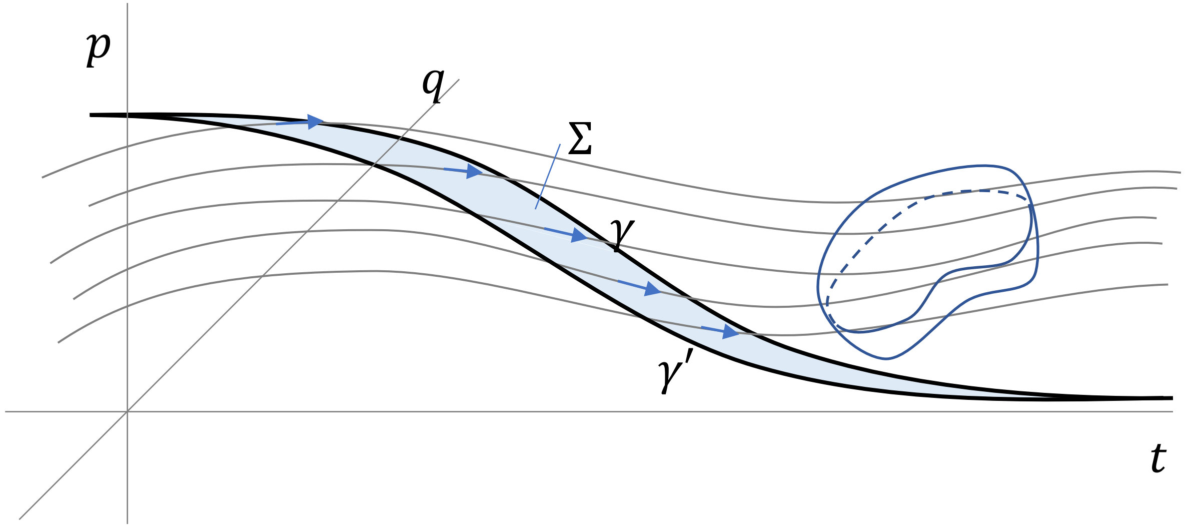
The action is the line integral of the vector potential of the flow of states



Variation of the action

$$\delta\mathcal{S}[\gamma] = \oint_{\partial\Sigma} \vec{\theta} \cdot d\vec{\gamma}$$
$$= -\iint_\Sigma \vec{S} \cdot d\vec{\Sigma}$$

Gauge independent, physical!

Variation of the action measures the flow of states (physical).
Variation = 0 ⇒ flow of states tangent to the path.

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Assumptions of Physics

$$L = p_i v^i - H$$
$$= p_i d_t q^i - H d_t t$$
$$= \begin{bmatrix} p_i & 0 & -H \end{bmatrix} \begin{bmatrix} d_t q^i \\ d_t p_i \\ d_t t \end{bmatrix}.$$

$$d_t \xi^a = \begin{bmatrix} d_t q^i & d_t p_i & d_t t \end{bmatrix}$$

Phase space extended by time

$$\theta_a = \begin{bmatrix} p_i & 0 & -H \end{bmatrix}$$

$$\mathcal{A}[\gamma] = \int_\gamma L dt = \int_\gamma \theta_a d_t \xi^a dt = \int_\gamma \theta_a d\xi^a = \int_\gamma \theta d\gamma$$

Action is the line integral of θ

$$\omega_{ab} = \begin{bmatrix} \omega_{q^i q^j} & \omega_{q^i p_j} & \omega_{q^i t} \\ \omega_{p_i q^j} & \omega_{p_i p_j} & \omega_{p_i t} \\ \omega_{t q^j} & \omega_{t p_j} & \omega_{tt} \end{bmatrix} = \begin{bmatrix} 0 & \delta^i_j & \partial_{q^i} H \\ -\delta^j_i & 0 & \partial_{p_i} H \\ -\partial_{q^j} H & -\partial_{p_j} H & 0 \end{bmatrix}$$

Encodes both IND and DR: geometry of the flow at equal time and across times

$$S^a \omega_{aq^j} = S^{q^i} \omega_{q^i q^j} + S^{p_i} \omega_{p_i q^j} + S^t \omega_{t q^j}$$
$$= -S^{p_j} - S^t \partial_{q^j} H = -S^{p_j} - \partial_{q^j} H = 0$$
$$S^a \omega_{a p_j} = S^{q^i} \omega_{q^i p_j} + S^{p_i} \omega_{p_i p_j} + S^t \omega_{t p_j}$$
$$= S^{q^j} - S^t \partial_{p_j} H = S^{q^j} - \partial_{p_j} H = 0$$
$$S^a \omega_{at} = S^{q^i} \omega_{q^i t} + S^{p_i} \omega_{p_i t} + S^t \omega_{tt}$$
$$= S^{q^i} \partial_{q^i} H + S^{p_i} \partial_{p_i} H$$
$$= \partial_{p_i} H \partial_{q^i} H - \partial_{q^i} H \partial_{p_i} H = 0.$$

Hamilton's equations in the extended phase space

$$S_a = S^b \omega_{ba} = 0.$$

Flow is described by ω

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Assumptions of Physics

$$\partial_a\theta_b - \partial_b\theta_a = \begin{bmatrix} \partial_{q^i}\theta_{q^j} - \partial_{q^j}\theta_{q^i} & \partial_{q^i}\theta_{p_j} - \partial_{p_j}\theta_{q^i} & \partial_{q^i}\theta_t - \partial_t\theta_{q^i} \\ \partial_{p_i}\theta_{q^j} - \partial_{q^j}\theta_{p_i} & \partial_{p_i}\theta_{p_j} - \partial_{p_j}\theta_{p_i} & \partial_{p_i}\theta_t - \partial_t\theta_{p_i} \\ \partial_t\theta_{q^j} - \partial_{q^j}\theta_t & \partial_t\theta_{p_j} - \partial_{p_j}\theta_t & \partial_t\theta_t - \partial_t\theta_t \end{bmatrix}$$

$$= \begin{bmatrix} \partial_{q^i}p_j - \partial_{q^j}p_i & \partial_{q^i}0 - \partial_{p_j}p_i & \partial_{q^i}(-H) - \partial_t p_i \\ \partial_{p_i}p_j - \partial_{q^j}0 & \partial_{p_i}0 - \partial_{p_j}0 & \partial_{p_i}(-H) - \partial_t 0 \\ \partial_t p_j - \partial_{q^j}(-H) & \partial_t 0 - \partial_{p_j}(-H) & \partial_t(-H) - \partial_t(-H) \end{bmatrix}$$

$$= \begin{bmatrix} 0 - 0 & 0 - \delta_i^j & -\partial_{q^i}H - 0 \\ \delta_j^i - 0 & 0 - 0 & -\partial_{p_i}H - 0 \\ 0 + \partial_{q^j}H & 0 + \partial_{p_j}H & -\partial_t H + \partial_t H \end{bmatrix}$$

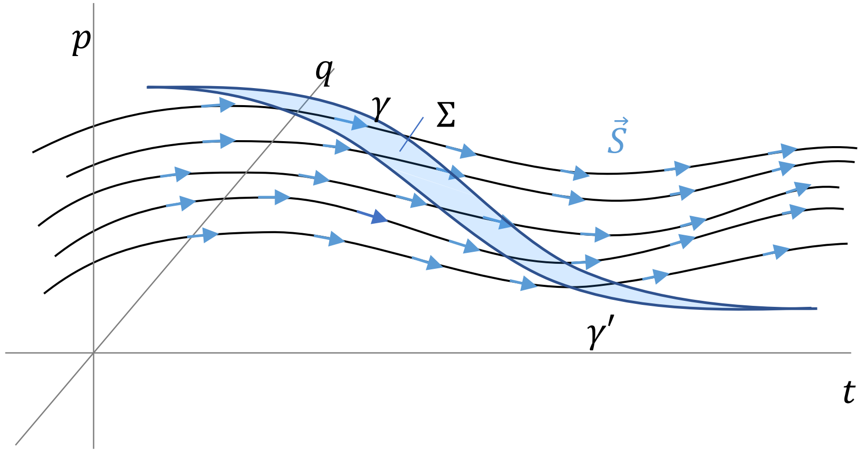
$$= \begin{bmatrix} 0 & -\delta_i^j & -\partial_{q^i}H \\ \delta_j^i & 0 & -\partial_{p_i}H \\ \partial_{q^j}H & \partial_{p_j}H & 0 \end{bmatrix}.$$

$$\omega_{ab} = -(\partial_a\theta_b - \partial_b\theta_a) = -\partial_a \wedge \theta_b$$

θ is the potential of ω

$$\delta\mathcal{A}[\gamma] = \delta\int_\gamma L dt = \delta\int_\gamma \theta_a d\xi^a = \int_\gamma \theta_a d\xi^a - \int_{\gamma'} \theta_a d\xi^a = \oint_{\partial\Sigma} \theta_a d\xi^a = \int_\Sigma \partial_a \wedge \theta_b d\xi^a d\eta^b$$

$$= -\int_\Sigma \omega_{ab} d\xi^a d\eta^b,$$



Variation of the action is the flow through enclosed surface

0 if and only if always tangent to the flow

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Assumptions of Physics

Note: kinematic equivalence is needed only to write the Lagrangian in terms of position and velocity

Principle of least action works in the extended phase space for Hamiltonian systems that are not Lagrangian!

Takeaways

- The principle of least action is just a property of incompressible flows over independent DOFs
- The Lagrangian and the action themselves are not “physical” (i.e. you cannot measure the Lagrangian or the action, defined up to an arbitrary choice of gauge)
- TODOs:
  - Help popularize these ideas, more pictures/diagrams in the book/etc…
  - Generalization to classical field theory
  - Generalization to quantum (path integral)

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Assumptions of Physics

Kinematic equivalence and massive particles

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Assumptions of Physics

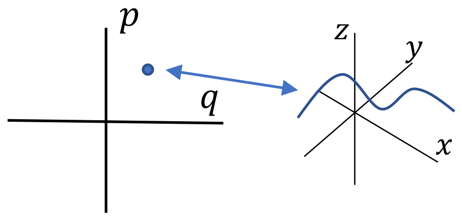
Massive particles under potential forces

Kinematic equivalence assumption:
the state can be recovered from
space-time trajectories

$$\frac{\partial p_i}{\partial \dot{q}^j} \equiv m g_{ij}$$

Must be a linear transformation in terms of coordinates

Fixes the units



Integration of the previous expression

$$p_i = m g_{ij} \dot{q}^j + q A_i(q^k)$$

$$\dot{q} = \frac{dq^i}{dt} = \frac{\partial H}{\partial p_i} = \frac{1}{m} g^{ij} (p_j - q A_j)$$

$$H = \frac{1}{2m} (p_i - q A_i) g^{ij} (p_j - q A_j) + q V(q^k)$$

Hamiltonian for massive particles under potential forces

Mass quantifies number of states per unit of velocity

Higher mass ⇒ more states to go through ⇒ harder to accelerate

BUT

Zero mass ⇒ zero states within finite range of velocity ⇒ velocity is fixed

The laws themselves are highly constrained by simple assumptions

Two flavors of Kinematic Equivalence:

Weak KE: only require invertible relationship

At every position, the relationship between momentum and velocity is invertible and differentiable (WKE-INV)

At every point, the Hessian of the Hamiltonian is non-singular (hyperregularity of H): $|\partial_{p_i}\partial_{p_j}H| \neq 0$ (WKE-HYP)

The Hamiltonian is twice differentiable and concave (or convex) in momentum (WKE-CONC)

The Jacobian of the transformation between state variables and kinematic variables is non-singular. (WKE-NSIN)

Densities over phase space can be expressed in terms of position and velocity: $\rho(x^i, v^j)|J| = \rho(q^i, p_j)$. (WKE-DEN)

Areas and volumes in phase space can be expressed in kinematic variables: $dx^1 \cdots dx^n\, dv^1 \cdots dv^n = |J|\, dq^1 \cdots dq^n\, dp_1 \cdots dp_n$. (WKE-VOL)

The symplectic form ω_{ab} can be expressed in kinematic variables. (WKE-SYMP)

The displacement field S^a can be expressed in kinematic variables. (WKE-DISP)

Jacobian $\partial_{v^i} p_j$ exists and is non-singular

Two flavors of Kinematic Equivalence:

Full KE: linear relationship

There is a linear relationship between conjugate momentum and velocity (FKE-LIN)

The system under study is a massive particle under scalar and vector potential forces (FKE-POT)

The Jacobian of the transformation between state variables and kinematic variables is a non-singular function of position only. (FKE-NSIN)

Densities over phase space can be expressed in terms of position and velocity by rescaling the value at each point: $\rho(x^i, v^j)|J(x^i)| = \rho(q^i, p_j)$. (FKE-DEN)

Areas and volumes in phase space can be expressed in kinematic variables, and the transformation depends on position only: $dx^1 \cdots dx^n \, dv^1 \cdots dv^n = |J(x^i)| \, dq^1 \cdots dq^n \, dp_1 \cdots dp_n$. (FKE-VOL)

The symplectic form ω_{ab} can be expressed in kinematic variables, and its components are a linear function of velocity. (FKE-SYMP)

Jacobian $\partial_{v^i} p_j$ exists and depends only on position

$$\partial_{v^i} p_j = m g_{ij} \qquad v^i = d_t q^i = \partial_{p_i} H = \frac{1}{m} g^{ij}(p_j - \mathfrak{q} A_j)$$

$$p_i = m g_{ij} v^j + \mathfrak{q} A_i \qquad H = \frac{1}{2m}(p_i - \mathfrak{q} A_i) g^{ij} (p_j - \mathfrak{q} A_j) + V$$

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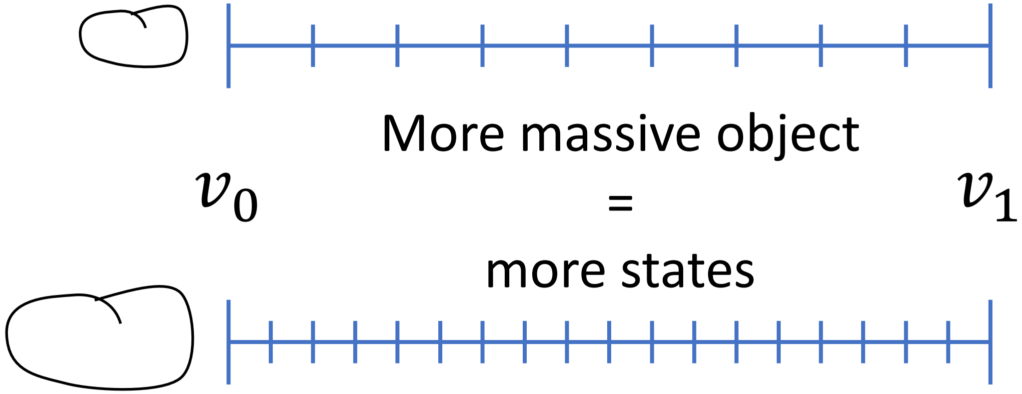
Assumptions of Physics

Inertial mass tells us how many states there are per unit of area of position-velocity in an inertial (Cartesian) frame. (IM)

Harder to accelerate heavier bodies because same change of speed means more states to go through



$$m = \frac{\partial |v|}{\partial |p|} = 0$$

Massless particles are impossible to accelerate (not infinitely easy) because states are not spread over velocity

$$\frac{1}{m} \approx \frac{\partial |p|}{\partial |v|} = 0$$

Zero mass is “kind of the same” as infinite mass

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Assumptions of Physics

Is full Kinematic Equivalence required?

Density expressed in velocity at the same position is proportional to the density over states (FKE-PROP)

Uniform distributions along momentum correspond to uniform distributions along velocity (FKE-UNIF)

At each position, there exists a local inertial frame (FKE-INER)

The position fully defines the units of all state variables, therefore an invertible transformation between momentum and velocity (FKE-UNIT)

Full kinematic equivalence is connected to the existence of inertia, the way densities behave in velocity, and the unit system

Takeaways

- Weak Kinematic Equivalence recovers Lagrangian mechanics and Full Kinematic Equivalence recovers massive particles under potential forces
- Mass is a geometric property of the state space when charted by position and velocity
- The basic laws are more constrained than one may think at first
- TODOs:
  - Help popularize these ideas, more pictures/diagrams in the book/etc…
  - Deeper understanding of FKE: what does a purely WKE system look like?
  - Inertial mass vs gravitational mass?
  - With multiple particles, what does it mean that the metric tensor is the same for all? Can be articulated as a property of the unit system? (i.e. we must be able to choose the same units for all particles?)

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Assumptions of Physics

Relativistic mechanics

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Assumptions of Physics

Relativistic mechanics

Relativistic aspects without space-time and in Newtonian mechanics

potential of the displacement

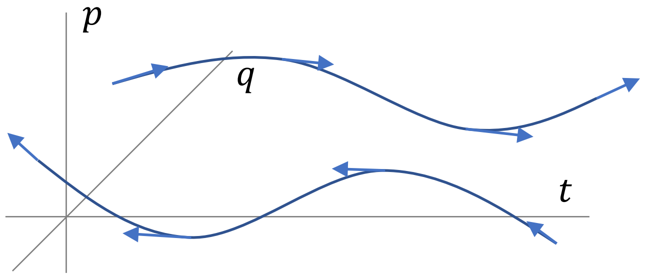
$$\theta = [p^i, -E, 0, 0]$$

energy-momentum co-vector

$$F = \frac{d\hat{t}}{dt}\hat{F} = \frac{d}{dt}\left(\hat{m}\frac{dt}{d\hat{t}}\frac{dx}{dt}\right) = \frac{d}{dt}\left(m\frac{dx}{dt}\right)$$

rest mass scaled by time dilation

Classical antiparticles



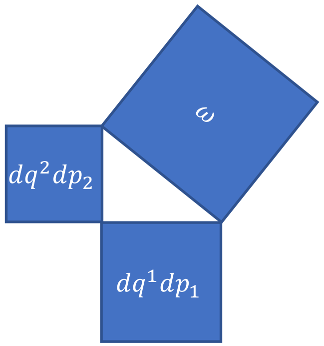
$$\frac{dt}{ds} = \frac{\partial \mathcal{H}}{\partial E}$$

Affine parameter anti-aligned with time: parameterization "goes back" in time

Lorentzian relativity is the only "correct" one

Minkowski signature appears on the extended phase space

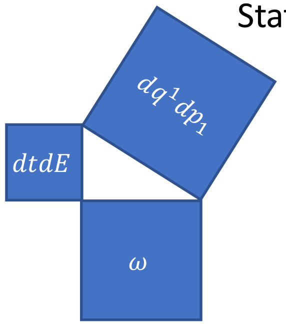
$$\omega = dq^1dp_1 + dq^2dp_2$$



Indep DOFs are orthogonal

$$\omega = dq^1dp_1 - dtdE$$

$$dq^1dp_1 = \omega + dtdE$$

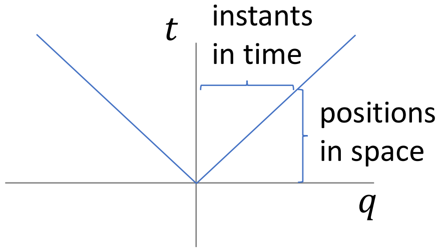


States are counted at equal time: temporal DOF orthogonal to ω

No clear idea what $G_{\alpha\beta\gamma}$ is…
Inertial forces?

$$\omega_{ab} = \begin{bmatrix} -mG_{\alpha\beta\gamma}u^\gamma + qF_{\alpha\beta} & g_{\alpha\beta} \\ -g_{\alpha\beta} & 0 \end{bmatrix}$$

Metric tensor quantifies states charted by position and velocity



Constant c converts state count between space and time

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Assumptions of Physics

Want to extend Hamiltonian mechanics to handle changes of time variable

Recall $\theta_a = \begin{bmatrix} p_i & 0 & -H \end{bmatrix}$ $\quad q^\alpha = [t, q^i] \quad p_\alpha = [-E, p_i]$

$$\xi^a = [q^\alpha \; p_\alpha] \qquad \theta_a = [p_\alpha \; 0]$$

$$S^a = d_s \xi^a = [d_s q^\alpha \; d_s p_\alpha]$$

$\xi^a(s)$ ← Evolution in s (affine parameter)!

$$\omega_{ab} = \partial_a \wedge \theta_b = \begin{bmatrix} \omega_{q^\alpha q^\beta} & \omega_{q^\alpha p_\beta} \\ \omega_{p_\alpha q^\beta} & \omega_{p_\alpha p_\beta} \end{bmatrix} = \begin{bmatrix} 0 & I_n \\ -I_n & 0 \end{bmatrix}$$

$\mathcal{H} = 0 \qquad S^a \omega_{ab} = \partial_b \mathcal{H}$ ← Hamilton's equations

↑ Hamiltonian constraint

FKE in phase space extended by time and energy

$$\partial_{u^\alpha} p_\beta = m g_{\alpha\beta}$$

$$p_\alpha = m g_{\alpha\beta} u^\beta + \mathfrak{q} A_\alpha$$

$$u^\alpha = d_s q^\alpha = \partial_{p_\alpha} \mathcal{H} = \frac{1}{m} g^{\alpha\beta} (p_\beta - \mathfrak{q} A_\beta)$$

$$\mathcal{H} = \frac{1}{2m} (p_\alpha - \mathfrak{q} A_\alpha)\, g^{\alpha\beta} \left(p_\beta - \mathfrak{q} A_\beta\right) + U$$

Recovers EM Hamiltonian for massive particle

Recovers geodesic equation

$$d_s u^\alpha = \{u^\alpha, \mathcal{H}\} = -u^\beta u^\gamma g^{\alpha\delta} \Gamma_{\delta\beta\gamma} + \frac{q}{m} F^{\alpha\gamma} g_{\gamma\beta} u^\beta$$

$$q^\alpha = x^\alpha$$

$$p_\alpha = m g_{\alpha\beta} u^\beta + \mathfrak{q} A_\alpha$$

$$x^\alpha = q^\alpha$$

$$u^\beta = \frac{1}{m} g^{\beta\alpha} (p_\alpha - \mathfrak{q} A_\alpha)$$

$$\mathcal{H} = \frac{1}{2} m u^\alpha g_{\alpha\beta} u^\beta + \frac{1}{2} m c^2$$

Hamiltonian constraint fixes the rest mass

ω in kinematic variables

$$\xi^a = [\, q^\alpha, u^\alpha \,]$$

???

$$\partial_\alpha g_{\beta\gamma} - \partial_\beta g_{\alpha\gamma}$$

$$\partial_\alpha A_\beta - \partial_\beta A_\alpha$$

$$\omega_{ab} = \begin{bmatrix} -mG_{\alpha\beta\gamma}u^\gamma - \mathfrak{q}F_{\alpha\beta} & mg_{\alpha\beta} \\ -mg_{\alpha\beta} & 0 \end{bmatrix}$$

Conservative forces define state density over position/position?

Metric tensor defines state density over position/velocity

Must be encoding some geometric information!

$$\nabla_\alpha A_\beta - \nabla_\beta A_\alpha = \partial_\alpha A_\beta - \partial_\beta A_\alpha$$

$$\nabla^\alpha A^\beta - \nabla^\beta A^\alpha = \partial^\alpha A^\beta - \partial^\beta A^\alpha + G^{\alpha\beta\gamma} A_\gamma$$

$$q^\alpha = x^\alpha$$

$$p_\alpha = mg_{\alpha\beta}u^\beta + \mathfrak{q}A_\alpha$$

$$x^\alpha = q^\alpha$$

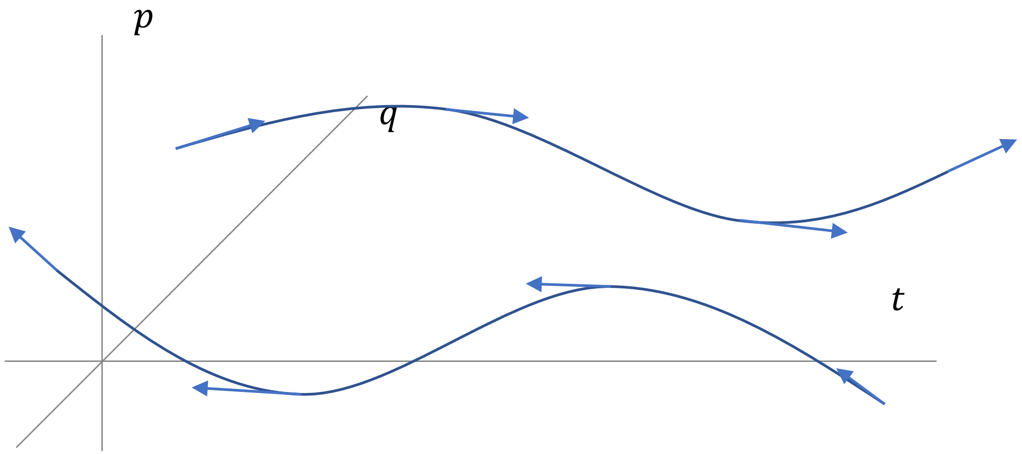
$$u^\beta = \frac{1}{m} g^{\beta\alpha}(p_\alpha - \mathfrak{q}A_\alpha)$$

$$\mathcal{H} = \frac{1}{2} m u^\alpha g_{\alpha\beta} u^\beta + \frac{1}{2} mc^2$$

Classical anti-particles

This can be negative

$$d_s t = \partial_{-E}\mathcal{H} \qquad d_s(-E) = \partial_t \mathcal{H}$$
$$d_s q^i = \partial_{p_i}\mathcal{H} \qquad d_s p_i = \partial_{q^i}\mathcal{H}$$



Particle: affine parameter aligned with time

Anti-particle: affine parameter anti-aligned with time

Parametrization flows backwards with respect to time

An evolution cannot change time alignment

Free particle

$$d_s t = \frac{E}{mc^2}$$

Anti-particle = negative energy

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Assumptions of Physics

Recover standard Hamiltonian

Over valid states $\mathcal{H} = 0$ and $E = H$

$$\mathcal{H} = \lambda(H - E)$$

Assume no E dependence

$$E = H$$

$$d_t t = 1 = d_t s \, d_s t = d_t s \, \partial_{-E} \mathcal{H} = d_t s \, \lambda$$

$$d_t s = \frac{1}{\lambda}$$

$$d_t q^i = d_t s \, d_s q^i = d_t s \, \partial_{p_i} \mathcal{H} = \frac{1}{\lambda} \left(\partial_{p_i} \lambda (H - E) + \lambda \partial_{p_i} H \right) = \partial_{p_i} H$$

$$d_t p_i = d_t s \, d_s p_i = -d_t s \, \partial_{q^i} \mathcal{H} = -\frac{1}{\lambda} \left(\partial_{q^i} \lambda (H - E) + \lambda \partial_{q^i} H \right) = -\partial_{q^i} H$$

$$d_t E = d_t s \, d_s E = d_t s \, \partial_t \mathcal{H} = \frac{1}{\lambda} \left(\partial_t \lambda (H - E) + \lambda \partial_t H \right) = \partial_t H$$

Hamiltonian constraint can "hide" multiple Hs

Free particle Hamiltonian constraint

$$\mathcal{H} = \frac{1}{2mc^2}(\sqrt{c^2|p_i|^2 + (mc^2)^2} + E)(\sqrt{c^2|p_i|^2 + (mc^2)^2} - E)$$

Negative energy

Positive energy

$$\mathcal{H} = (H_1 - E)(H_2 - E)\,\frac{-1}{2mc^2}$$

$$d_s t = \frac{1}{2mc^2}(E + E) = \frac{E}{mc^2}$$

Quantum connection

Free particle Hamiltonian constraint

$$\mathcal{H} = \frac{1}{2m}\left(p_\alpha \eta^{\alpha\beta} p_\beta + m^2c^2\right) = \frac{1}{2m}\left(p_i \delta^{ij} p_j - (E/c)^2 + m^2c^2\right)$$

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 + \frac{m^2c^2}{\hbar^2}\right)\psi(t, \mathbf{x}) = 0$$

Density can be non-zero only where $\mathcal{H} = 0$

$$\mathcal{H}\rho = 0$$

Hamiltonian constraint becomes relativistic equations in QM

$$D_\mu \psi = \partial_\mu \psi + ieA_\mu \psi,$$

$$p_\alpha = m g_{\alpha\beta} u^\beta + \mathfrak{q} A_\alpha$$

Gauge covariant derivative related to kinetic momentum

Takeaways

- Relativistic mechanics is recovered without additional assumptions: relativity is needed to make densities/entropy invariant over time transformations
- Mass is a geometric property of the state space when charted by position and velocity
- The basic laws are more constrained than one may think at first
- TODOs:
  - Help popularize these ideas, more pictures/diagrams in the book/etc…
  - Find out what $G_{\alpha\beta\gamma}$ represents
  - Understand whether there is a link between $g_{\alpha\beta}$ and $F_{\alpha\beta}$ already in particle mechanics

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Assumptions of Physics

Reversing phase space

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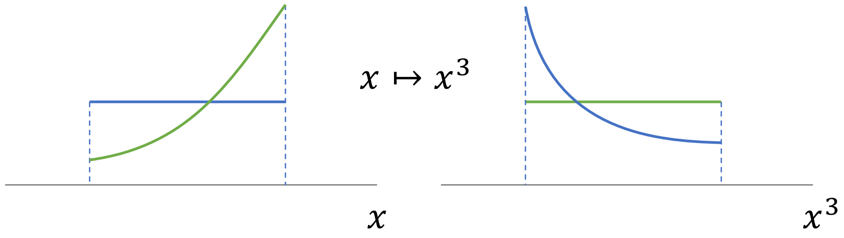
Assumptions of Physics

Hamiltonian Privilege, *Erkenn* (2023)

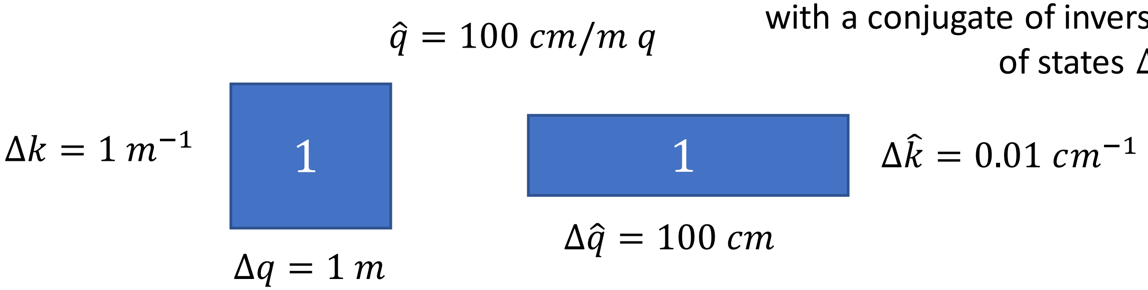
Stud Hist Phil **71**, 082020, 60-71 (2020)

Reversing phase-space



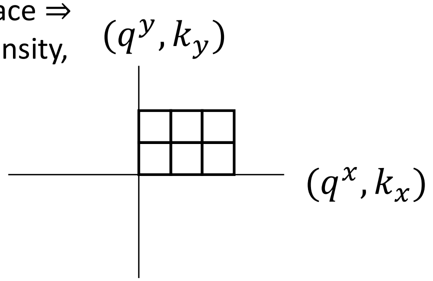
Density, entropy, uniform distributions NOT in general coordinate invariant

Each unit variable (i.e. coordinate) paired with a conjugate of inverse units: number of states $\Delta q \Delta k$ is invariant



Phase space (symplectic) structure is the only one that supports coordinate invariant density, entropy, state count

Independence of DOFs ⇒ independence of units ⇒ orthogonality in phase-space ⇒ invariant marginals (for density, entropy, state count)



Total number of states = product of number of cases in each independent DOF

Hamiltonian mechanics preserves count of states and DOF independence over time

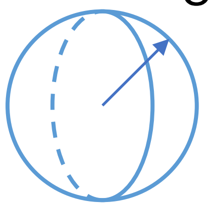
$$\omega_{ab} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \otimes \delta^i_j$$

Symplectic form (geometric structure of phase space)

Areas/possibilities in each DOF

Orthogonality/independence across DOFs

Only 3 spatial dimensions are possible



Directional DOF

2-sphere the only symplectic manifold

Invariance at equal time (relativity) gives us the structure of phase space

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Assumptions of Physics

The following three conditions are equivalent, and link the structure of phase space with equal-time changes of coordinates

Conjugate momentum p_i changes like a covector under changes of coordinates q^i (PS-COV)

The form ω_{ab} is invariant under changes of coordinates q^i (PS-SYMP)

The Poisson brackets are invariant under equal-time coordinate changes (PS-POI)

They can be broken down into two sets of conditions

About orthogonality

The system allows statistically independent distributions over each DOF under any choice of coordinates q^i (PSI-DEN)

The system allows informationally independent distributions over each DOF under any choice of coordinates q^i (PSI-INFO)

The system allows peaked distributions where the uncertainty is the product of the uncertainty on each DOF under any choice of coordinates q^i (PSI-UNC)

About volumes

Phase space volumes are invariant under equal-time changes of coordinates q^i (PSV-VOL)

The Jacobian for the transformation induced by equal-time changes of coordinates q^i is unitary (PSV-JAC)

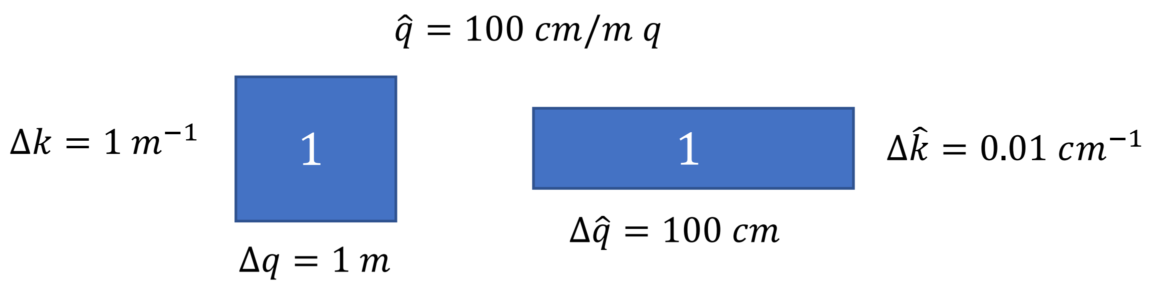
Densities over phase space are invariant under equal-time changes of coordinates q^i (PSV-DEN)

Thermodynamic entropy is invariant under equal-time changes of coordinates q^i (PSV-THER)

Information entropy is invariant under equal-time changes of coordinates q^i (PSV-INFO)

Uncertainty of peaked distributions is invariant under equal-time changes of coordinates q^i (PSV-UNC)

To reconstruct phase space



Space charted by continuous quantities

Some quantities define the unit systems: q^i

Units are independent (change one without affecting the other)

Volume/density/entropy are unit independent

If one unit variable q changes, unit independent variables must stay the same, and unit dependent variable must change, while also keeping the volume the same

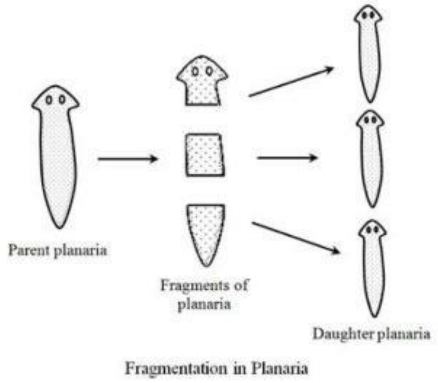
For each variable q there is a variable p whose units are proportional to the inverse units of q, so that the volume stays the same

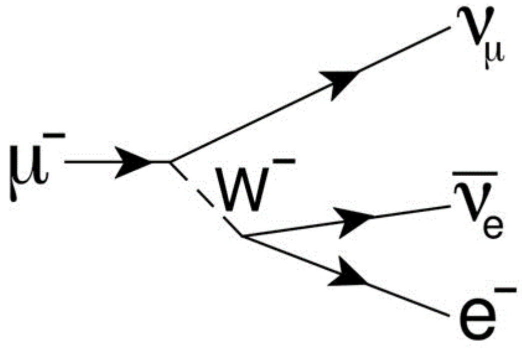
Takeaways

- Structure of phase space is exactly the structure needed to define state densities, thermodynamic entropy, information entropy and statistical uncertainty in a way that satisfies the principle of relativity for equal-time observers
- The relativistic version extends those quantities to all observers
- TODOs:
    - Help popularize these ideas, more pictures/diagrams in the book/etc…

Infinitesimal reducibility

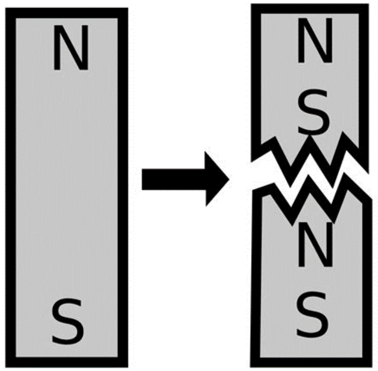
Divisible vs Reducible

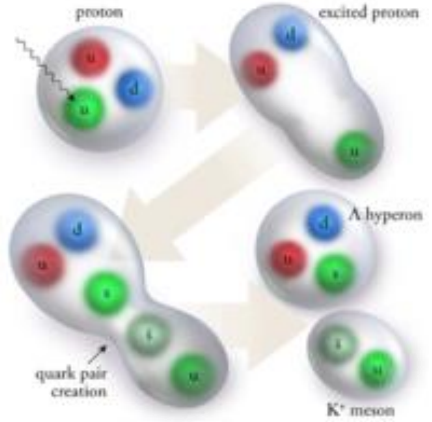




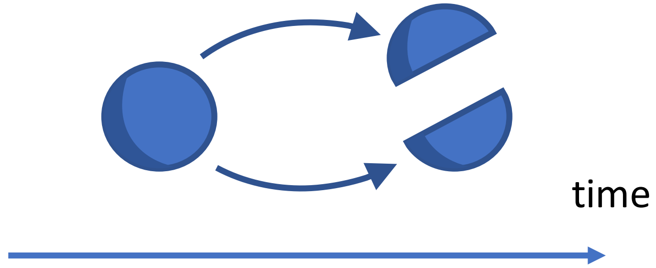
divisible but not reducible

reducible but not divisible

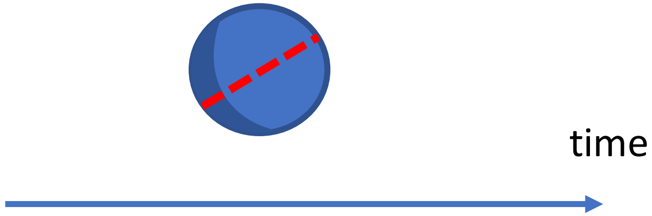




$$\mathcal{P}_t : \mathcal{S} \to \mathcal{S}_1 \times \mathcal{S}_2$$



$$\mathcal{S} \equiv \mathcal{S}_1 \times \mathcal{S}_2$$



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Assumptions of Physics

IR-INF: A classical system can be thought of as being made of infinitesimal parts, called particles

IR-DIST: classical state is given by a distribution over phase space

Suppose IR-INF

S_C state space for the full system

S_P state space for the particles

Technically, from the sigma algebra

$$s \in S_C \iff f(U) \in [0,1] \quad U \subseteq S_P \quad f \text{ additive}$$

Assume finitely many ⇒ manifold

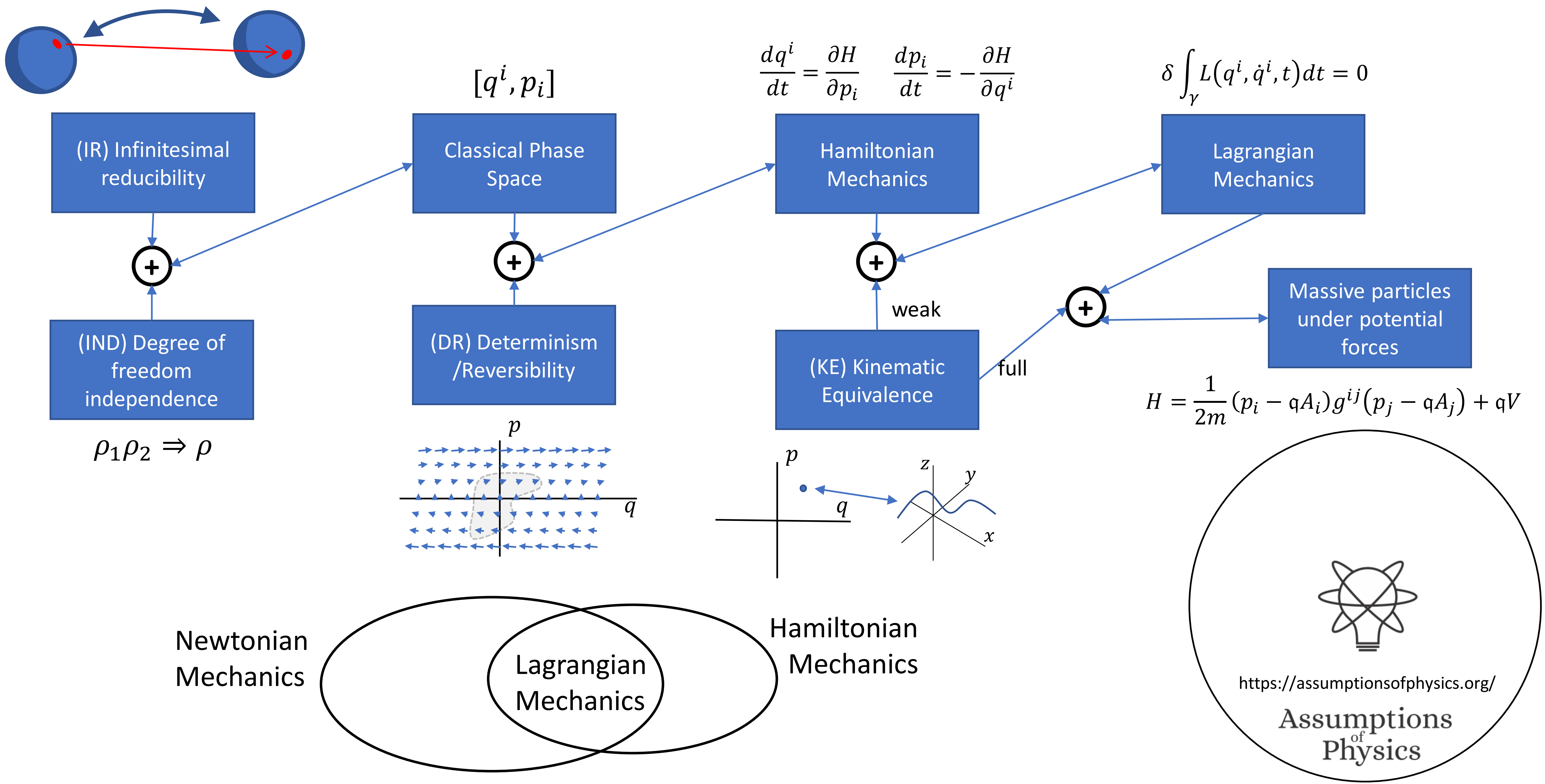
IR-INF ⇒ f real valued, S_P charted by continuous variables

Need to quantify states ⇒ $\mu(U) \in [0,1]$

μ and f should be independent of choice of variables, and so should $\rho = \frac{df}{d\mu}$ ⇒ differentiable structure on the manifold

+ IND ⇒ IR-DIST

Assumptions of classical mechanics



Takeaways

- Infinitesimal reducibility is what requires the space of infinitesimal parts to be a differentiable manifold (assuming finitely many variables)
- Differentiability is exactly the ability to define volumes in arbitrary coordinates
- TODOs:
  - Find a “more mathematical” way to run the argument

Field theories

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Assumptions of Physics

EM field equations

Fields as infinite dimensional systems (countable assuming fields continuous)

Can we formally generalize the symplectic structure?

What are the units of the conjugate field?

Doesn't quite seem right…

$$\xi^a = [q^\alpha \; p_\alpha]$$

$$\xi^i(x) = \left[\phi^i(x) \; \Pi_i(x)\right]$$

$$\omega\left(d\xi^a, d\zeta^b\right) = \sum dq^\alpha dp_\alpha$$

$$\omega\left(d\xi^a, d\zeta^b\right) \; = ? \; \sum\int \delta\phi^i \delta\Pi_i dx$$

Not sure whether the right mathematical formalism exists…

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Assumptions of Physics

General relativity

Need a geometric understanding of Einstein field equations

Note: stress-energy-momentum tensor is the derivative of the (matter) action with respect to the metric tensor

The action is the line integral of the vector potential of the flow of states

The metric tensor defines the count of degrees of freedom (double the space, double the DOFs)

Is the relationship between curvature and energy/momentum a relationship between flow and DOFs?

$$S = \int \left[\frac{1}{2\kappa} R + \mathcal{L}_m\right] \sqrt{-g}\, d^4x$$

$$T_{\mu\nu} = \frac{-2}{\sqrt{-g}} \frac{\delta\left(\sqrt{-g}\mathcal{L}_m\right)}{\delta g^{\mu\nu}}$$

$$\Rightarrow R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T_{\mu\nu}$$

Takeaways

- In principle, the same assumptions could be applied to field theories
- TODOs:
  - Find or develop the right mathematical framework to do the extension

Wrapping it up

- Classical mechanics has been given the full Reverse Physics treatment, and shows that there is a lot of physics hidden within the equations
- Still some areas where things can be understood better
- Ideally, we want to apply the same approach to all other physical theories

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Assumptions of Physics

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Assumptions of Physics

Metric curvature in phase space?

Open problem 3
Details here

$$q^\alpha = [\, t, q^i \,] \quad p_\alpha = [\, -E, p_i \,] \quad x^\alpha = [\, t, x^i \,] \quad \Pi_\alpha = m g_{\alpha\beta} u^\beta \quad u^\alpha = d_s x^a$$

Canonical variables

$$\xi^a = [\, q^\alpha, p_\alpha \,] \quad \omega_{ab} = \begin{bmatrix} 0 & I_n \\ -I_n & 0 \end{bmatrix}$$

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Assumptions of Physics

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Particle under EM forces

$$\mathcal{H} = \frac{1}{2m}(p_\alpha - qA_\alpha) g^{\alpha\beta} (p_\beta - qA_\beta) = \frac{1}{2} m u^\alpha g_{\alpha\beta} u^\beta \qquad p_\alpha = \Pi_\alpha - qA_\alpha$$

Non-canonical variables

$$\xi^a = [\, q^\alpha, \Pi_\alpha \,] \qquad \omega_{ab} = \begin{bmatrix} -qF^{\alpha\beta} & I_n \\ -I_n & 0 \end{bmatrix} \qquad \{\Pi_\alpha, \Pi_\beta\} = -qF_{\alpha\beta}$$

Geometry of EM forces?

$$\xi^a = [\, q^\alpha, u^\alpha \,] \qquad \partial_\alpha g_{\beta\gamma} - \partial_\beta g_{\alpha\gamma}$$

???

$$\omega_{ab} = \begin{bmatrix} -mG_{\alpha\beta\gamma} u^\gamma - qF^{\alpha\beta} & m g_{\alpha\beta} \\ -m g_{\alpha\beta} & 0 \end{bmatrix}$$

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Assumptions of Physics

Metric curvature in phase space?

Open problem 3
Details here

$$d_s x^\alpha = \{x^\alpha, \mathcal{H}\} = u^\alpha \qquad d_s u^\alpha = \{u^\alpha, \mathcal{H}\} = -g^{\alpha\delta}\Gamma_{\delta\beta\gamma}u^\beta u^\gamma + \frac{q}{m}F^{\alpha\gamma}g_{\gamma\beta}u^\beta$$

$$D_s u^\alpha = \frac{q}{m}F^{\alpha\gamma}g_{\gamma\beta}u^\beta$$

Equation of the geodesics

Must be encoding some geometrical information!

$$\nabla_\alpha A_\beta - \nabla_\beta A_\alpha = \partial_\alpha A_\beta - \partial_\beta A_\alpha$$

$$\nabla^\alpha A^\beta - \nabla^\beta A^\alpha = \partial^\alpha A^\beta - \partial^\beta A^\alpha + G^{\alpha\beta\gamma}A_\gamma$$

$$\omega_{ab} = \begin{bmatrix} -mG_{\alpha\beta\gamma}u^\gamma - qF^{\alpha\beta} & mg_{\alpha\beta} \\ -mg_{\alpha\beta} & 0 \end{bmatrix}$$

What is the geometry on a hypersurface with $\mathcal{H} = \frac{1}{2}mc^2$?

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Assumptions of Physics