Time as an operational definition

Gabriele Carcassi

Physics Department

University of Michigan

Assumptions of Physics

Introduction

- Together with Prof. Christine Aidala, I lead a project called Assumptions of Physics - https://assumptionsofphysics.org – that aims to find a minimal set of starting points from which the laws can be rederived
- It consists of two main efforts:
  - Reverse physics – starts from the laws and finds physical assumptions that provide equivalent formulations
  - Physical mathematics – starts from physical ideas, carefully encodes them in formal definitions, rederives the familiar mathematical structures

Assumptions of Physics

The mathematical structure of time (i.e. real numbers with the standard topology) can be understood as coming from an idealized operational model of clocks

As time resolution increases, this model must fail and needs to be replaced with a more realistic account

Outline

- What is time?
  - What it means to define something; the role of operational definitions in science; the topological structure imposed by experimental verification; the role of clock synchronization in defining time
- A metrological model of time
  - The metrological and logical structure of clocks; the necessary and sufficient conditions for continuous time (i.e. real numbers).
- Inevitable failure of time ordering
  - The untenability of the conditions for continuous time; how time ordering itself must break down.

What is time?

Hard question because time is elusive

What is time?

Then defining anything else should be easy!

table

tā′bəl

noun

1. An article of furniture supported by one or more vertical legs and having a flat horizontal surface.

What is a table?

Hard question because “what is” is elusive

What is time?

Before giving a definition, we need to say:

What is the purpose of the definition?

What are the “primitive elements” that are allowed to be in the definition?

Because that's what all well-defined math objects are!

In **math**, we want a formal definition: some set with some properties

E.g. a variable that can be used as a parameter for the evolution

I really don't know: I still haven't figured out what philosophers consider well-defined

In **philosophy**, we may look for an "ontological definition": some intrinsic feature of reality

E.g. A nonspatial continuum in which events occur

These types of answers do not help us in a lab

In a lab, an operational definition is necessary and sufficient

This is what physicists consider well-defined!

operational: it tells me what to do

Time is what is measured by a clock

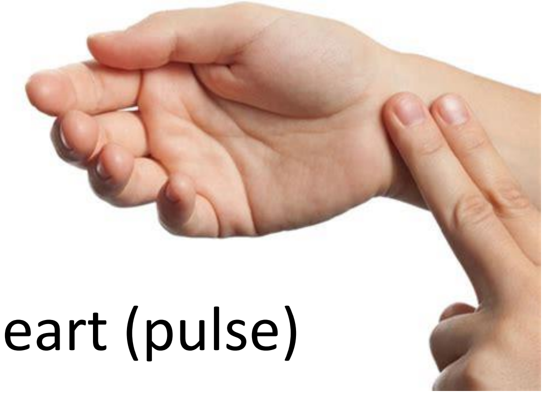
What is a clock?

~~A clock is what measures time~~

not operational: it does not tell me what to do

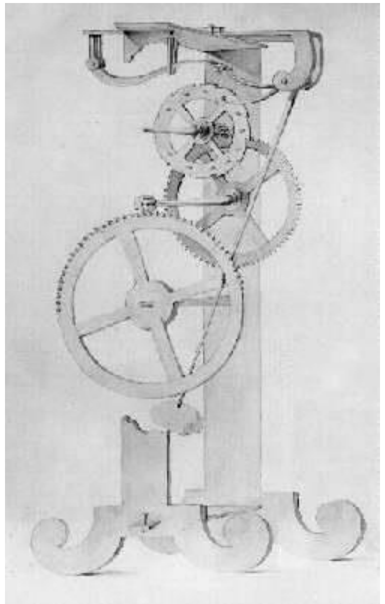
What is a clock?

The sun



The heart (pulse)

The seasons



A pendulum

A bucket of water with a hole

You can get a set of instructions on how to build these: they have operational definitions

But these are just examples of clocks!!!

What is a clock?

A clock is anything that can be synchronized to other clocks

Clock synchronization is operationally defined

The idea of time comes out of our ability to synchronize our clocks; examples serve to "jumpstart" the process

We solved the "purpose" question (i.e. operational definition)
We need to answer the "primitive elements" question

Verifiable statements: assertions that can be experimentally verified in a finite time

T	SUCCESS (in finite time)
F	UNDEFINED
F	FAILURE (in finite time)

The mass of the photon is less than 10^{-13} eV → Verifiable
The mass of the photon is exactly 0 eV → Not verifiable due to infinite precision, but falsifiable

The syntax/semantics/structure of these statements cannot be formally specified further

For example, whether a specific statement is experimentally verifiable or even well-defined may depend on context (e.g. premises, idealization, theory, etc…)

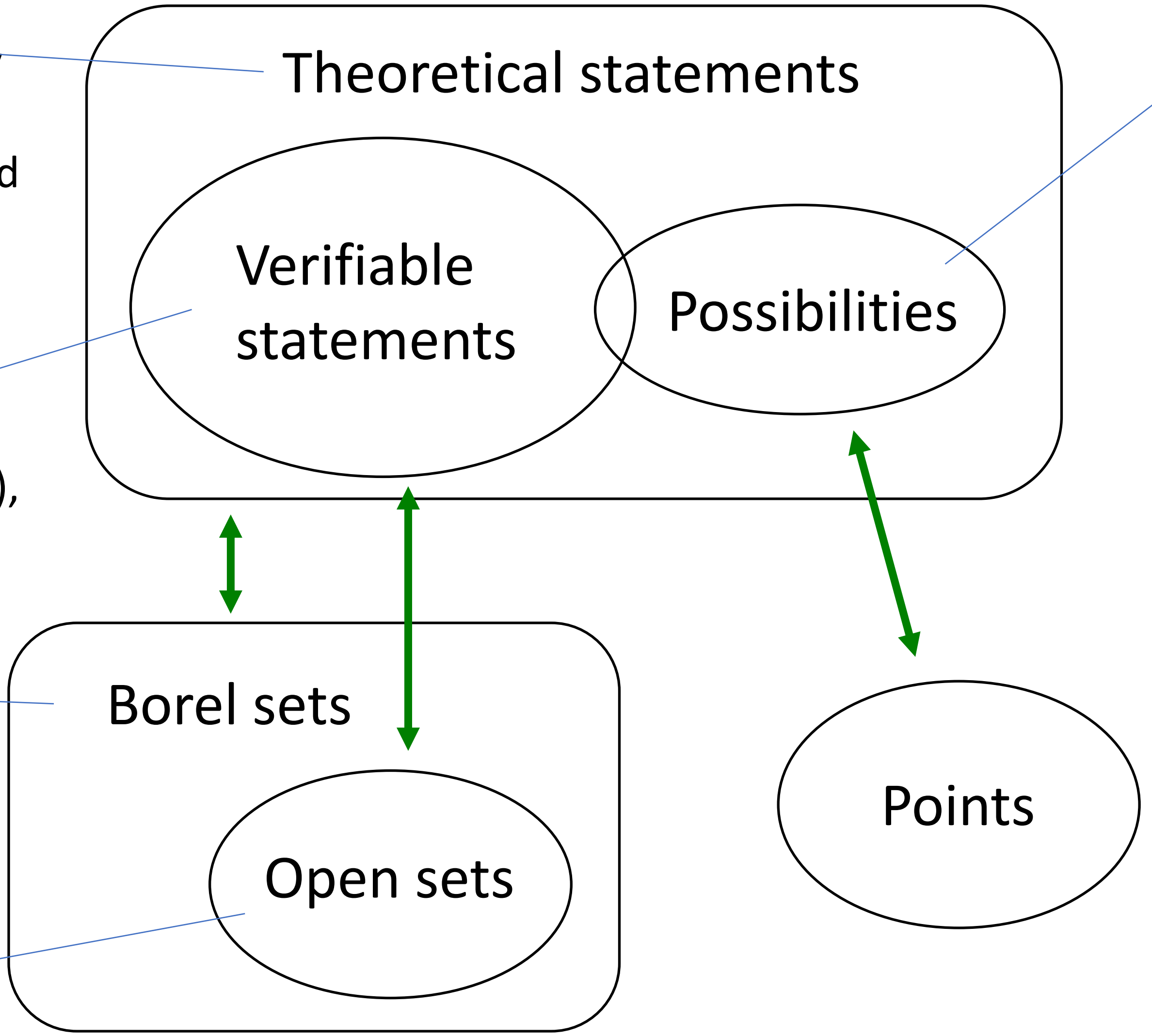
The mass of the electron is 511 ± 0.5 KeV

When measuring the mass, it is a verifiable hypothesis

When performing particle identification, it is assumed to be true

Close under NOT and countable AND/OR (countably complete Boolean algebra), statements formally associated with an experimental test

Experimentally distinguishable cases



Closed under finite AND and countable OR (Heyting algebra), generated by a countable set

σ-algebra

Topology

Precise map between physical concepts and their mathematical representation

All proofs can be "translated" into physically meaningful language

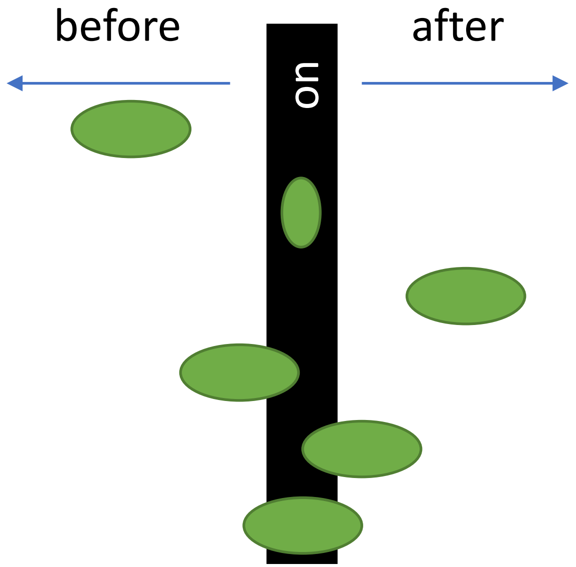
A physical theory is fully specified by a countable set of verifiable statements and their logical relationships

All mathematical objects we use in physics (symplectic manifolds, tensors, Hilbert spaces, Lie groups, …) are ultimately identifying statements and their relationships

A metrological model of time

How do we formally model a clock?

A **reference** (i.e. a tick of a clock) is something that allows us to distinguish between a before and an after



Mathematically, it is a triple (b, o, a) such that:

- b and a are verifiable
- The reference has an extent ($o \not\equiv \bot$)
- If it's not before or after, it is on ($\neg b \wedge \neg a \preccurlyeq o$)
- If it's before and after, it is on ($b \wedge a \preccurlyeq o$)

Before	On	After
T	F	F
F	T	F
F	F	T
T	T	F
F	T	T
T	T	T

A clock is a collection of references

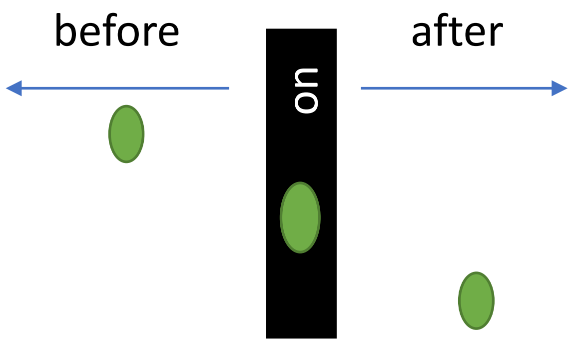
Imagine collecting the references of all possible clocks into a single logical structure. What are the necessary and sufficient conditions such that they identify a point on the real line?

Intuitively, we would need clocks at higher and higher resolutions, all perfectly synchronized, …

1. Strict references

A reference is strict if before/on/after are mutually exclusive

Before	On	After
T	F	F
F	T	F
F	F	T

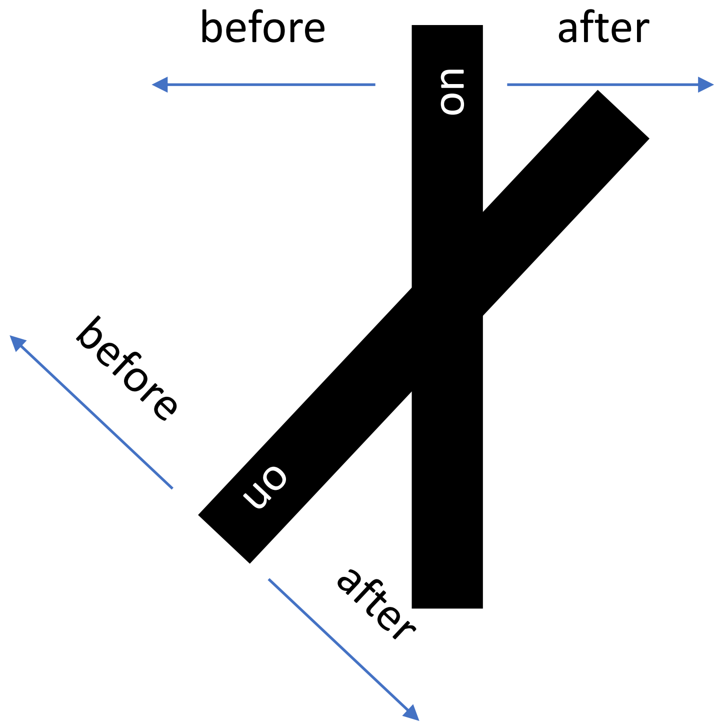


Physically, this means assuming that the extent of what we measure is smaller than the extent of our reference

Multiple references

Without further constraints, references would not lead to a linear order

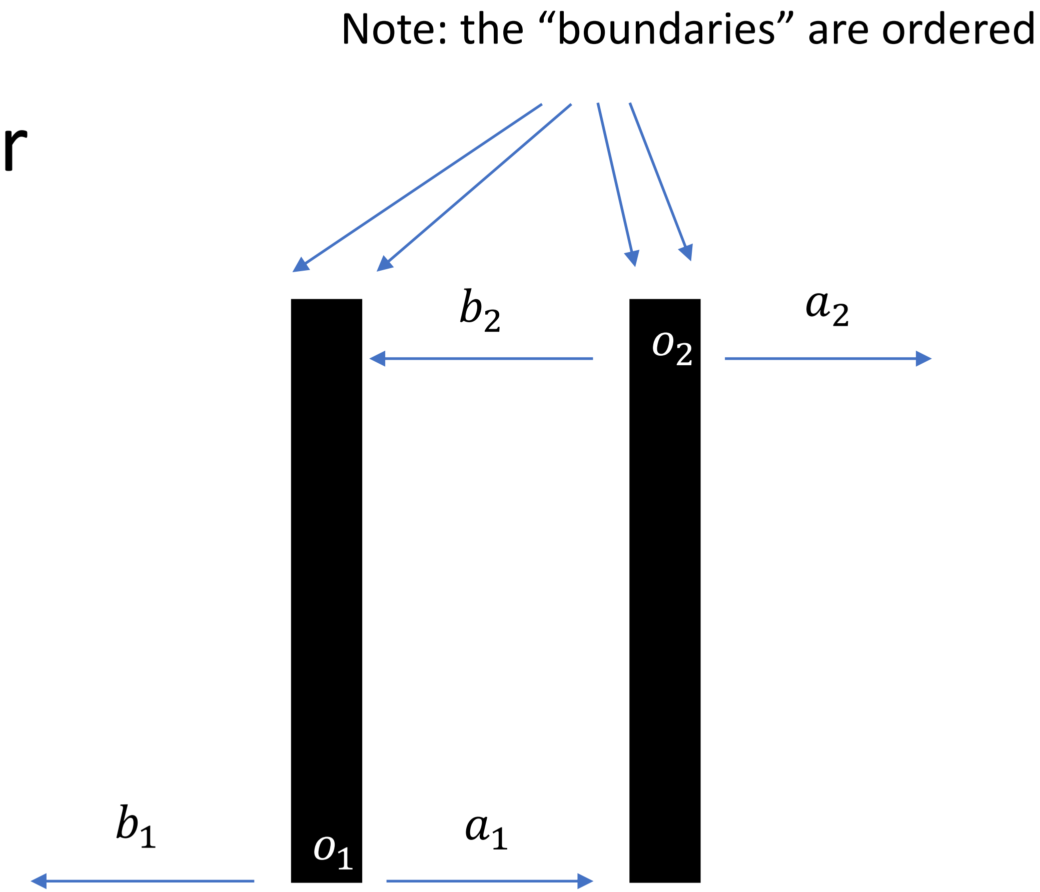
	b_2	o_2	a_2
b_1	✓	✓	✓
o_1	✓	✓	✓
a_1	✓	✓	✓



Multiple references

The fact that a reference is "before" or "after" another is captured by the statements' logical relationship

	b_2	o_2	a_2
b_1	✓	✕	✕
o_1	✓	✕	✕
a_1	✓	✓	✓



But order relationship between references is too restrictive

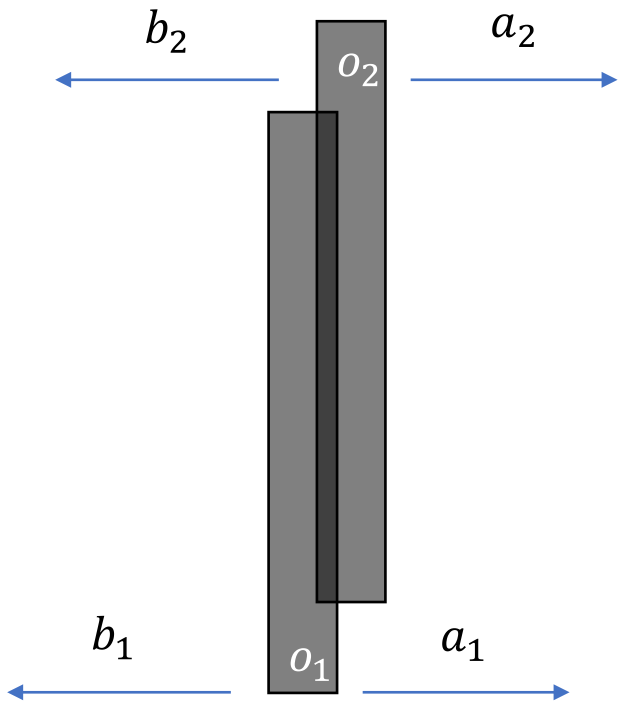
2. Aligned references

Two references are aligned if the before and not-after statement can be ordered by narrowness/implication

For example, $b_1 \preccurlyeq b_2 \preccurlyeq \neg a_1 \preccurlyeq \neg a_2$

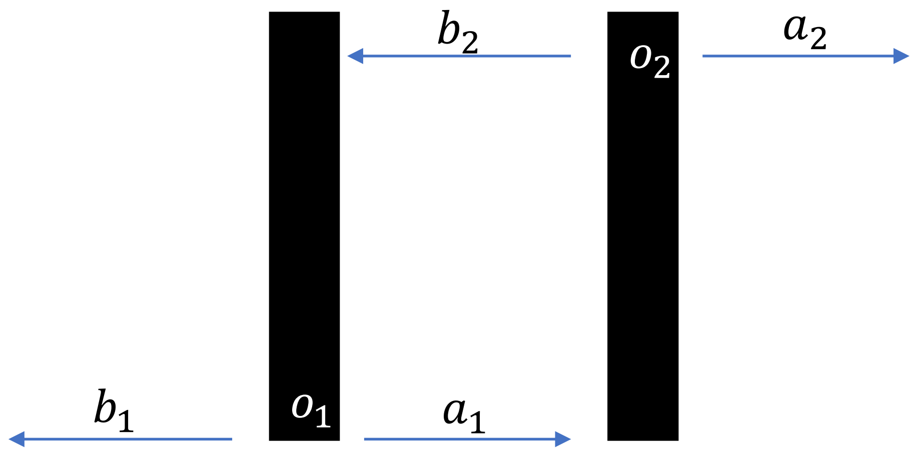
$\preccurlyeq$ Means that if the first statement is true then the second statement will be true as well

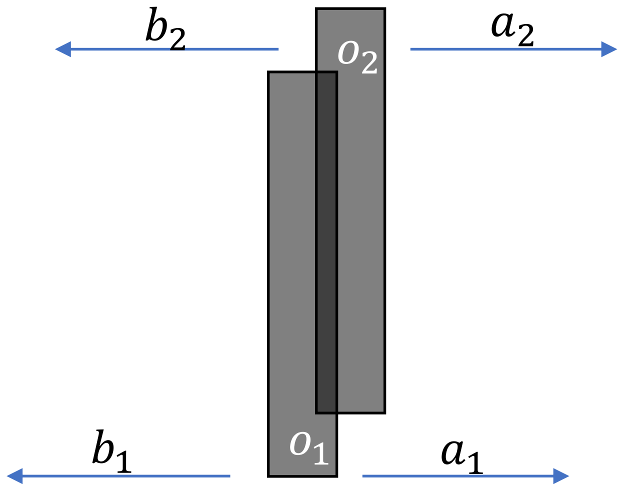
That is, the first statement is narrower, more specific



Filling the whole region

If two different references overlap, we can't say one is before the other: we can't fully resolve the linear order



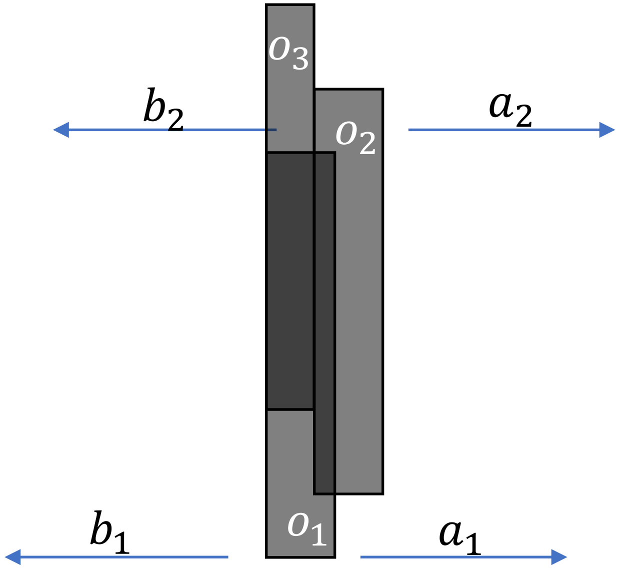


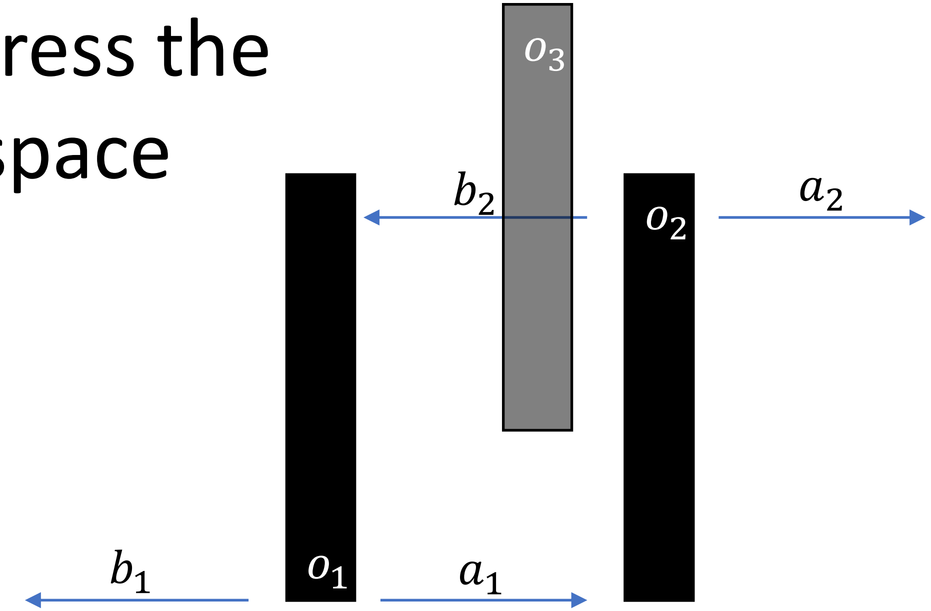
Conversely, if two references don't overlap and there can be something in between, we must be able to put a reference there

3. Refinable references

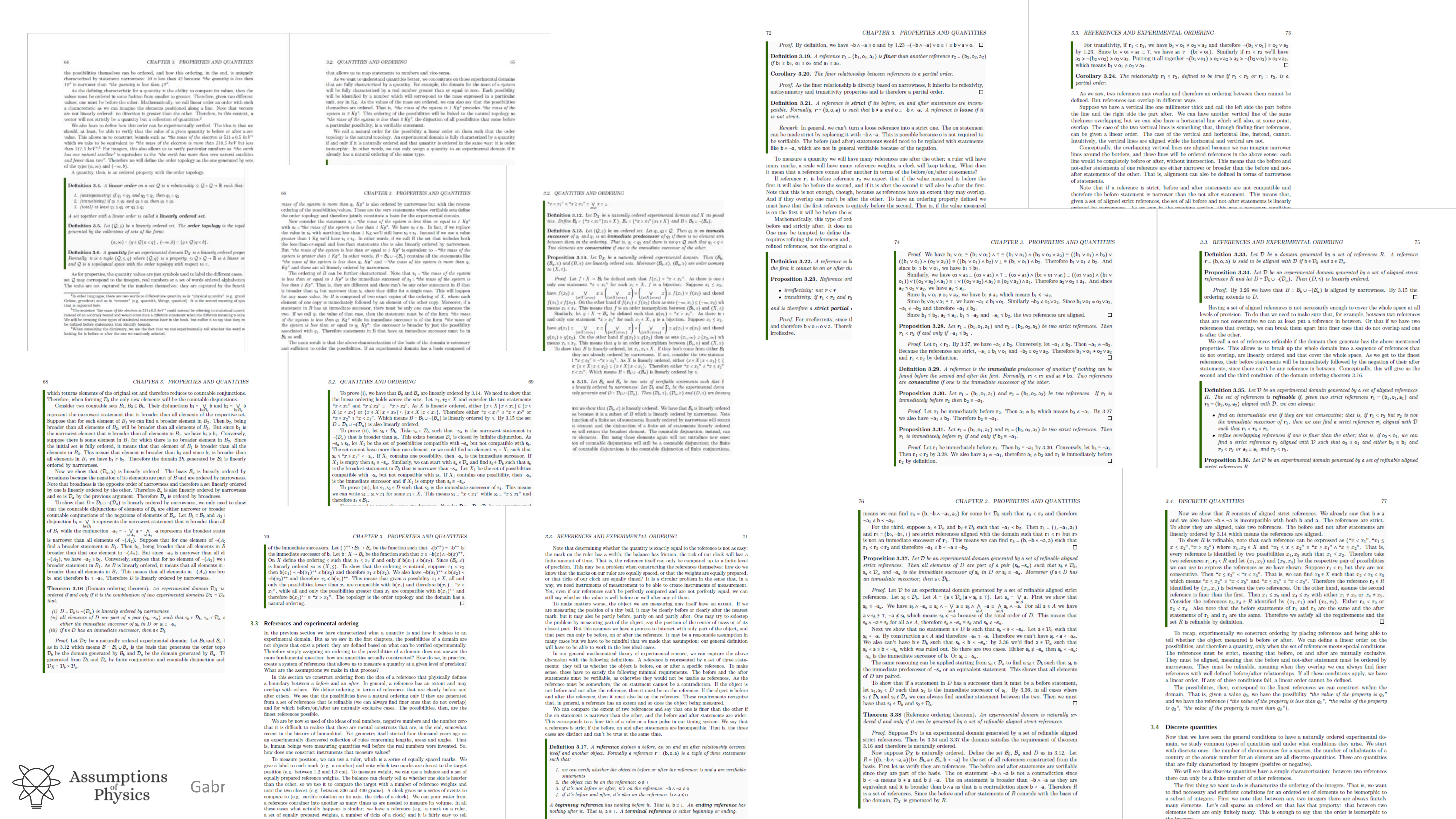
A set of references is refinable if we can address the previous two problems and resolve the full space

If two references overlap, we can find a references that refines the overlap





If something can be found between two references, then there must be another reference in between



Assumptions of Physics
Gabr

Reference ordering theorem

To define an **ordered** sequence of “instants”, the references must be (nec/suff conditions):

- Strict – an event is strictly before/on/after the reference (doesn’t extend over the tick)
- Aligned – shared notion of before and after (logical relationship between statements)
- Refinable – overlaps can always be resolved

Additionally:

Between any two references we can always have another reference ⇒ **real numbers**

Only finitely many references between any two references ⇒ **integers**

For time, these conditions are idealizations

Inevitable failure of time ordering

How does this model of time break down?

The ticks of a clock have an extent and so do the events (references not strict)

If clocks have jitter, they cannot achieve perfect synchronization (references not aligned)

We cannot make clock ticks as narrow as we want (references not refinable)

No consistent **ordering**: no "objective" "before" and "after"

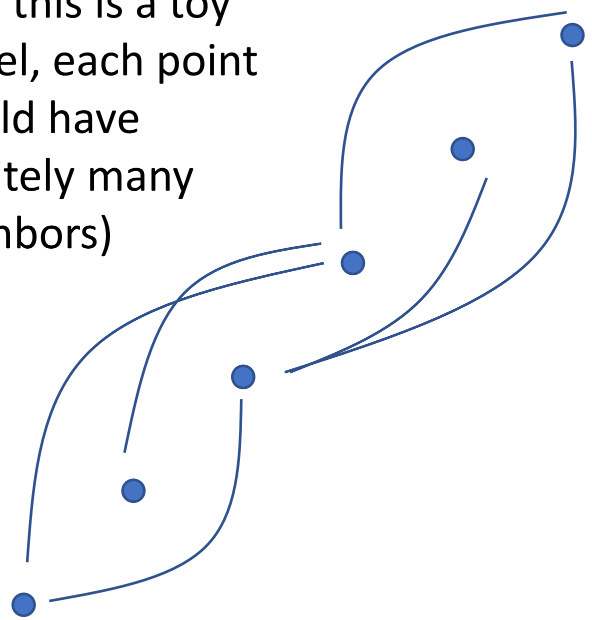
In relativity, different observers measure time differently, but the order is the same. We should expect this to fail at "small" scales.

A better understanding of space-time means creating a more realistic formal model that accounts for those failures

What type of models should we use?

Hard to say, but we can argue from necessity

(N.B. this is a toy model, each point should have infinitely many neighbors)



Lack of order at small scales, order at large enough scale

What we can distinguish experimentally (i.e. topology) seems to be linked to how precisely we want to distinguish (i.e. geometry)

Current mathematical tools have a hard division between topology and geometry

Likely need new math

Our reasoning contradicts the expectations of many that time is simply "discrete" at the smallest scale

This intuition is based on the idea that the continuum is like the discrete but "with more points"

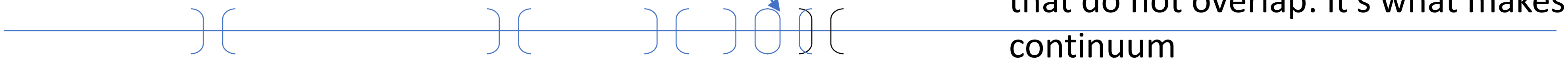
This idea (though extremely common in physics) is flawed

Topology of the reals

Connected: cannot be divided into two disjoint open sets

Open set – open interval

Always space between two references that do not overlap: it's what makes it a continuum



Reference

No contingent decidable statements

Topology of the integers

Disconnected: can be divided into two disjoint open sets

No space between two consecutive references

Reference

All contingent statements are decidable

Conclusion

- Physically well-defined objects must be in terms of operational definitions
  - The primitive elements are the verifiable statements, which are typically idealized and left formally undefined
  - A physical theory is fully characterized by the logical relationships of countably many verifiable statements
- Time is what is measured by a clock
  - The main feature of clocks is that they can be synchronized with each other, which is operationally defined
- Clocks can be formally modeled by a set of ticks that experimentally define a before and after
  - We recover time as a continuum under suitable idealized conditions (i.e. all clocks can be perfectly synchronized, reach arbitrary resolution, …)
- We should not expect the assumptions required by ordered (and continuous) time to hold at the smallest scale
  - A better understanding of space-time means creating a more realistic formal model that takes into account those failures

Supplemental