Overview and Status of Assumptions of Physics*
(Short version)

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Abstract

This article gives an overview of the project "Assumptions of Physics" which aims to rederive the known laws from a few physically meaningful starting points. It presents the motivations behind the project, a summary of the main findings and current status of the research.

1 Introduction

The aim of this article is to give a broad overview of the project "Assumptions of Physics": its motivations, its status and a list of the main findings. It is intended to be a good start for anyone interested in simply learning more about the project or finding an area for collaboration.

As we plan to update this document, please make sure you have a recent version. Each subsection will be fairly self-contained to allow the reader to skim through the document and focus on the parts of interest. It will contain a summary section at the beginning and a titled paragraph for each of the main idea or finding.

2 Goals and method

A better understanding of physics. The overall goal of the project is to better understand physics: to understand why the laws of quantum mechanics, or classical mechanics, are what they are; to understand the limit of the validity of the different theories; to understand what the Poisson brackets, or any other mathematical operation, represent physically; to understand which mathematical objects correspond to actual physical entities and which are just artifacts; to understand what new theories are possible.

Ideally, we want to achieve a single unified framework for all science that is physically meaningful (i.e. it is clear what each objects represents experimentally and why must it be represented that way), mathematically precise (i.e. the physical assumptions are captured through axioms and definitions and then the arguments are carried out with the rigor of modern mathematics) and philosophically consistent (i.e. the concepts and viewpoints taken fit with the mathematical results and the practice of science).

2.1 Objectives

The overall goal of a better understanding of physics can be broken down into the following five objectives.

Clarify the assumptions. Physics is currently a patchwork of different theories (e.g. classical mechanics, quantum mechanics, thermodynamics, ...) that are used for different systems or for the same system in different contexts. When should we use one instead of another? What are the limits of validity of each theory? We want to understand what are the assumptions one has to make on a system such that it can be studied with a particular theory. The only way to make sure our assumptions are necessary and sufficient is to show the theory can be rederived from them. This, in turn, gives us a better idea of what

*http://assumptionsofphysics.org

each theory describes, how it fails, and what new directions one could take.

Put physics back at the center of the discussion. Physics used to try and identifies basic "laws" or "principles" for itself. Current physical theories, instead, just postulate a mathematical structure relegating physics to a mere after-the-fact interpretation. We want to go back to the old approach: we want to start with the physics and derive the math suitable to capture those physical concepts. Ideally, no mathematically construct should be introduced if it is not physically motivated. Why is phase-space formed with conjugate pairs? What does the commutator represent? If these are really representing physical objects, we should have a better understanding than a vague "you can think of it as..." or a cryptic mathematical "it is the left action of a fiber bundle on a...". Most of our understanding should come from physical intuition and not from the mathematics it is used to represent it.

Give science sturdier mathematical grounds. There is a general sense within physics that, at some point, mathematical details are physically not interesting and should be handled "appropriately" by mathematicians. Therefore even mathematical structures that originally came from physics are now formalized by mathematicians to solve their own needs, and not the physicist's. This, in turns, reinforces the idea that those details are not interesting to the physicists. The reality is that a well-posed physical problem must also be a well-posed mathematical problem, but not the other way around, so leaving the details to the mathematicians means having fundamental structures that are less physical.[1] Mathematicians will come up with definitions so that theorems are easier to prove or calculations are easier to perform. They will not come up with definitions based on whether they are physically meaningful. If we give a theory, as we said before, by simply stating a mathematical structure, then there is no guarantee that what we give is a fully physical theory. Unphysical elements simply means the physical problem was not specified correctly. If we truly are able to give a precise physical meaning to every mathematical object, then our formal structures will map one-to-one to our physical understanding. This means no unphysical mathematical artifacts and a more precise mathematical treatment of physics.

[1]For example, quantum states are modeled with Hilbert spaces even though we know they contain mathematical objects that are unphysical.

Foster connections between different fields of knowledge. Knowledge is increasingly specialized and fewer and fewer people are well versed in more than a couple of subjects. Yet, connections between different fields of science, mathematics and engineering are routinely found to be useful for one simple reason: nature is one and does not care about such divisions. A more holistic vision of scientific knowledge, then, is a natural byproduct of our effort. The notions of state, environment, process and equilibrium, for example, are intertwined and are fundamental to most fields of science and engineering. Proper characterization of such basic concepts will give a common language and mathematical tools that span across different disciplines.

Provide a solid basis for new theories. Most of the attention in fundamental physics is focused on the development of new theory or the search for new effects. A better understanding of the current theories, recast in a single broader and more precise framework, will probably facilitate that search. It makes explicit what ingredients went into the theories, so we know what assumption can fail and what principles cannot be changed.[2] If all physical theories can be seen as instances of a more general structure, they prove that the general structure is sound and provide templates for new theories.

While the objectives cover many areas and are very broad, our experience tells us that they are ultimately connected and cannot be pursued to their fullest independently. Therefore we are always looking for people with diverse background and interest to help us cover the different areas.

[2]For example, we may abandon the assumption that space is measurable at an indefinite precision, leaving the realm of manifold and real numbers; yet we cannot abandon the notion that space is experimentally accessible, staying in the realm of second countable T_0 topological spaces.

2.2 Methodology

As each topic is investigated by the project, it generally goes through three phases. Reverse engineering, where we try to find what assumptions are beneath a theory. Forward engineering, where we try to check whether the theory can be recovered from the assumptions we found. Formalization, where we try to codify in mathematical language the assumptions and the derivation.

2.3 Organization

The overall project is organized into the following macro-categories.

The general theory. This formalizes the basic mathematical framework that is the basis for all science. As every algebraic structure in mathematics is a specialization of the generic structures axiomatized by set theory, all scientific theory are ultimately specialization of the generic structure provided by the general theory. This defines the basic requirements imposed on a theory by logical consistency and experimental verifiability. It defines properties and quantities, accuracy and other concepts of general applicability.

Physics core. This defines the notions that are fundamental to physics specifically, such as states, processes, environment, equilibrium, determinism and so on. The idea is to define these concepts in the most general way possible such that common requirements can be identified. Every physical theory is a further refinement of this structure in that it will choose to study a particular set of states over a particular process.

Physical theories. This studies the different assumptions that are needed to recover the known physical theories. The assumptions typically describe what level of description is available for the system and its parts, how does it change in time and whether trajectories are enough to recover it.

3 The general theory

This part of the project focuses on the development of a rigorous mathematical framework that can provide the building blocks that are common to all scientific theories. As logic and set theory provides the foundation for all other mathematical structures, which are essentially sets with associated operations, the general theory will define common tools (e.g. logic of experimental verification, causal relationships, measurable quantities) that other theories will specialize in different ways.

3.1 The Principle of Scientific Objectivity

Our guiding principle is that "Science is universal, non-contradictory and evidence based." Any scientific theory must be logically consistent, its content be equally true or false for everybody and it must deal with what can be established experimentally. Different branches of science will specialize on different systems and topics, but all the theories and models they develop must satisfy those basic constraints. The general theory deals with the fundamental mathematical structure needed to realize that principle.

3.2 The Logic of Verifiable Statements

The basic element in our framework is the idea of a verifiable statement: an assertion that is either true or false for everybody and for which we have an experimental test that will terminate successfully in finite time if and only if the statement is true. The most basic structures therefore need to provide a logic framework to keep track of what statements are verifiable and their relationships.

3.2.1 Logical contexts

A logical context consists of a set of statements with well defined logical relationships. It is the most fundamental structure and it is the only one that is axiomatically introduced. A scientific theory will consist of a set of statements taken within a logical context with particular logical relationships.[3] The con-

[3] For example, Newtonian mechanics will use statements like "the mass of the object is 1 Kg" and "the acceleration of the object is 1 m/s^2". If both of these are true, then the statement

text defines basic logical relationships (e.g. equivalence, narrowness, compatibility and independence) which form the basis for higher level constructs (e.g. ordering, linear and statistical independence). Every other structure imposed on the statements, to be logically consistent, will need to "play" nice with these fundamental relationships.

3.2.2 Verifiable statements

Logical contexts need to keep track which statements are experimentally verifiable. A statement is experimentally verifiable if we are provided with a test that, if the statement is true, will always terminate successfully in a finite time. The fact that the test has to terminate in a finite time in the positive case and that it may not terminate in the negative case has profound implications. The negation of a verifiable statement is not in, general, a verifiable statement. The finite conjuntion (i.e. logical AND) of verifiable statements is a verifiable statement, but not an infinite one. The countable disjunction (i.e. logical OR) of verifiable statements is a verifiable statement, but not a more then countable infinite one. Therefore verifiable statements are not closed under the standard logical connectors

3.2.3 Experimental and theoretical domains

We define an experimental domain as a set of verifiable statements that can be expressed as the combination of a countable subset (i.e. a countable base). Because of finite time verifiability, this is the biggest space we can experimentally probe given an indefinite amount of time. An experimental domain, then, represents all the information that can be gathered experimentally about a particular subject.

From each experimental domain we construct a theoretical domain by allowing negation as well. This will include all statements for which an experimental test is in principle possible, though there is no guarantee of termination. A theoretical domain, then, represents all statements to which we can attach a prediction.

Within a theoretical domain we define the set of possibilities as those statements that, once known to be true, will set the truth value of all other statements. Each possibility represents a possible case that is distinguishable experimentally and corresponds to a unique possible assignment for the experimental and theoretical domain.[4] Because of the countable base, the set of possibilities can never be greater than the continuum.

3.2.4 Topologies and σ-algebras

An experimental domain provide a natural topology over the possibilities. Each verifiable statements can be identified with the set of possibilities compatible with it.[5] Since verifiable statements are closed under finite conjunction and countable disjunction, the sets corresponding to verifiable statements form a T_0 second countable topology.

On the other hand, theoretical domains provide a natural σ-algebra over the possibilities. Each theorical statement can also be identified with the set of possibilities compatible with it. Since theoretical statements are closed under negation and countable disjunction, the sets corresponding to theoretical statements form a σ-algebra, which is the Borel algebra of the natural topology.

Topologies and σ-algebras provide the foundations for differential geometry, Lie algebras, measure theory, probability theory and many other mathematical tools used in physics and the sciences. As we now have a precise understanding of what they represent, all concepts and proofs in those subject can be understood in terms of experimental verifiability.

"the force on the object is 1 N" is also true.

[4] For example, an experimental domain consists of a set of statements that can be tested experimentally (e.g. "the animal has whiskers", "the mass of the photon is less than $10^{-13} eV$"). The theoretical domains extends to statements that may not be tested experimentally (e.g. "the mass of the photon is exactly 0 eV"). The possibilities consists of all the possible cases (e.g. "the animal is a cat", "the mass of the photon is exactly 0.10210^-34eV).

[5] For example, the statement "the mass of the photon is less than $10^{-13} eV$" is equal to the disjuntion of all statement of the form "the mass of the photon is exactly x eV" with $x < 10^{-13}$

3.2.5 Status and open issues

This part of the work is very well developed. More work could be done in finding meaning for all mathematical concepts (e.g. compact sets, all separability axioms, ...). It would be interesting to reach out to individuals from related fields (i.e. logic, foundations of mathematics, philosophy of science, ...) to see whether any aspect of this section would be novel and interesting to their respective communities.

3.3 Domain relationships

The study of experimental domains gives us basic constructions that investigate the way different domains can be related or combined.

3.3.1 Relationships and equivalence between domains

We define two relationships between domains: inference relationships and causal relationships. An inference relationship establishes that testing a verifiable statement in one domain is the same as testing a verifiable statement in the other. Mathematically it is a map that takes a verifiable statement from one and returns a verifiable statement from the other that is equivalent to the first. A causal relationship establishes that determining which possibility is true in one domain also determines which possibility is true in the second. Mathematically it is a map that given a possibility of the first returns a possibility of the second that is broader than (i.e. less specific, true in more cases) the first.

One result is that causal relationships must be continuous function in terms of the natural topologies, which justifies why functions in science are always assumed to be "well-behaved" (i.e. analytically continuous with at most countable discontinuities). Another results is that there exists an inference implication between two domains if and only if there exists a causal relationship between them. The direction of the inference is the opposite of the causal direction. Therefore we say that a domain depends on another if there exists an inference relationship between the first and the second or if there exists a causal relationship between the second and the first. If the relationships are invertible (i.e. each domain depends on the other) then the domains are equivalent.

3.3.2 Composite domains

Given two experimental domains, we can create the composite by including all the verifiable statements that can be constructed from them. A possibility of the composite domain will determine the truth assignment for all statements of both original domains. Depending on the logical relationships between the two domain, we have different cases. If the two domains are independent (e.g. position along two different directions), then the possibilities are the scalar product of the possibilities. If one of the domains depends on the other (e.g. the temperature of a mercury column and its height), then the possibilities are the ones of the independent domain. If the domains are incompatible (e.g. plant species and animal species) then the possibilities are the disjoint union.

3.3.3 Relationship domains

Given two experimental domains where one depends on the other, we would like to characterize how the relationship is identified experimentally.[6] The main difficulty is that, within a single context, there can only be one such relationship or we would introduce logical inconsistencies. Choosing an inference relationship, in fact, means fixing the logical relationship between statements[7] which ultimately means fixing a logical context. Identifying the relationship, then, means identifying the right context.

Given a set of contexts, each describing a possible relationship, we can construct an experimental domain for the relationship in which each possibility corresponds to a possible relationship and its context. Whether or not the experimental test will exist in practice, the mathematical construction can always be performed resulting in just another domain. This means that we can construct relationship domains

[6] For example, knowing that there is a causal relationship between the temperature and the height of a mercury column, we would like to measure how the two are related.

[7] For example, "the temperature is 12 C" is incompatible with "the height is 23 mm"

over relationship domains for any arbitrary higher order relationship, meaning that the mathematical framework is closed.

3.3.4 Status and open issues

The overall mechanics of composite domains and relationships domains are fairly well developed and understood. There are details, though, that can be understood better. The connection to category theory can be also better developed. Other common concepts and constructions, like similarities or sequences of domains, could be formulated.

3.4 Properties and Quantities

Another basic tool is the ability identify the possibilities of a domain through the values of its properties, since these are the ones that, in practice, we measure. Part of the general theory is the formalization of these concepts.

3.4.1 Properties and values

A property is defined as a map from the possibilities to the set of values that the property can have. The map has to be topologically continuous as a measurement on the property should correspond to a measurement over the domain. We distinguish between various cases depending on whether the property is defined over all the possibilities and how much is able to distinguish between all the elements

3.4.2 Quantities and ordering

Quantities are defined to be ordered properties, values have magnitudes that can be compared. The objective is to understand how this ordering is fully defined by the logical relationships between verifiable statement. The key insight is that $3 \leq 4$ precisely because "the quantity is less than 3" is narrower (i.e. more specific) than "the quantity is less than 4".

Ordered domains are constructed through references (e.g. ticks of a clock, marks of a ruler or levels of a graded recipient) that allow us to tell whether the quantity is before or after itself. To have an ordered quantity, the set of references have to satisfy a set of necessary and sufficient conditions that are laid out by three ordering theorems.

Once those conditions are satisfied, the real and integer numbers (i.e. continuous and discrete ordering) respectively emerge by requiring either that between two references there is always at least one more or that there are only finitely many.

Of note is that the requirements for ordering itself are quite demanding and are ultimately unlikely to be satisfied for quantities like space and time. Which would mean that not only time (or space) would not be chartable by the real line, but that ordering would not be well defined at the finest scale.

3.4.3 Status and open issues

Properties and quantities are fairly well developed. The extension to manifolds should be straight forward (require independent quantities to identify each possibility within a "small" open set).

3.5 Further work

The general theory at this point covers only the topological aspects and needs to be extended to the geometrical/measure theoretical ones. Our current thinking is that diverse aspects, such as geometry, measure theory, probability theory, information theory, can be recovered by characterizing the granularity (i.e. accuracy) of the statements. Group theoretical aspects may be recovered

4 Physics core

This part has not yet been formally developed, though some common ideas are starting to emerge. The main idea comes from the realization that, when defining a system, the notions of states, processes, time, evolution laws, interaction with the environment and equilibria are all interrelated and can't be given independently. Roughly speaking, to define a system we have to choose a boundary that separates it from the environment. Only the quantities that are unaffected by the interaction with the environment can, in those circumstances, be thought as properties of the system. This in turns is linked to what we can define as state for the system, which is linked to the laws we can write. The processes and the laws are naturally linked to what is assumed about time.

The idea, then, is that giving a set of states is actually giving a much richer structure. It means defining a set of interaction at the boundary, it means defining equilibria with such interactions, it means defining time-scales below which and above which our description fails or is not interesting, it means defining how probing the system may affect those assumptions.[8] The plan is, at some point, to formalize these objects and relationships to provide a basic mathematical structure that all physical theories must adhere to.

[To be completed]

5 Physical theories

5.1 Classical mechanics

Classical mechanics can be derived from three main assumptions: infinitesimal reducibility, determinism and reversible evolution and kinematic equivalence. From the first assumption we can show that the state space of the infinitesimal parts of a system (i.e. particles) has the structure of phase space. From the second we can show that the time evolution follows Hamiltonian mechanics. From the third we can show that the system is Lagranian and is restricted to the case of massive particles under potential forces.

5.1.1 Classical state spaces

Infinitesimal reducibility. A system is said reducible if giving the state of the whole system is equivalent to giving the state of its parts and vice-versa. For example, given a ball, we can throw it and study the motion of the ball. Alternatively, one can take a red marker, make a red dot on the ball and study the motion of the dot. The system is reducible if studying the system is equivalent to studying the motion of all possible red dots. The system is infinitesimally reducible if we can keep reducing the system to smaller and smaller parts. We call particle an infinitesimal part of the system, the limit of recursive subdivision. The system is then described by a distribution of the state of its parts over phase space, which is the only structure that allows coordinate invariant distributions over continuous variables.

5.1.2 Hamiltonian mechanics

Determinism and reversibility. A system undergoes deterministic and reversible evolution if given the initial state one can predict the final state and given the final state one can reconstruct the initial state. In other words, the system is predictable and retrodictable. Densities at each past state must be equal to the density of the corresponding future state, which leads to Hamiltonian dynamics.

5.1.3 Lagrangian mechanics and massive particles

Kinematic equivalence. A system is said to satisfy kinematic equivalence if giving the state of the system is equivalent to giving the trajectory in space and vice-versa. That is, for each possible state there is one and only one trajectory associated with it. Note that this is not true in general: for example, given the trajectory of a photon we cannot reconstruct its momentum since all photons travel at the same speed. Under this assumption the relationship between position/momentum and position/velocity is invertible and the system admits a Lagrangian. Moreover, given the transformation rules of momentum and velocity, we find that there must be a linear relationship between the two, which constrains the dynamics to the one of massive particle under potential forces.

5.1.4 Status and open issues

We consider classical mechanics to be essentially forward engineered. There are some details, though, that could be better understood. The formalization will need a lot more time as it has many mathematical prerequisites.

[8] If we are studying a molecule, we may impose boundaries and processes such that that molecule and its constituent remain stable, which means, for example, interaction energies lower than the chemical bonds.

5.2 Quantum mechanics

Quantum mechanics follows the same strategy as classical mechanics with the difference that it assumes irreducibility instead of infinitesimal reducibility. From this, one can show that the state of the overall system is a complex inner product vector space. The deterministic and reversible evolution corresponds to unitary evolution. From kinematic equivalence one recovers Lagrangian and potential forces as in the classical case

[To be completed]

5.2.1 Quantum state space

Irreducibility. A system is said irreducible if giving the state of the whole system tells us nothing about the state of its parts. For example, given an electron, we can scatter a photon off of it to learn its state. Yet, we cannot scatter with only a part of the electron, we only scatter with the whole electron. The system is irreducible because we cannot study, and assign a state, to its parts.

[To be completed]

5.2.2 Status and open issues

While we consider quantum mechanics to be essentially forward engineered, that is we know the assumptions are necessary and sufficient, there are some conceptual aspects and details that, if clarified, would paint a more satisfactory picture.

5.3 Thermodynamics and statistical mechanics

Reverse engineering work for thermodynamics and statistical mechanics is ongoing.

5.4 Further work

The work still needs to be extended to field theories, classical (electromagnetism and general relativity) and quantum (quantum electrodynamics, quantum chromodynamics, ...). Some ideas have been explored but only at a preliminary stage.

We have not yet attempted to work with field theories, either classical or quantum. On the quantum side, there are two obstacles we would need to solve. The first is how to characterize relativistic spin and the second is the right mathematical tools to use (e.g. algebraic geometry?). On the classical side, the main conceptual issue is that we start with distributions over q and p so it is not clear what these would represent as distributions over field values.