Uncovering the Assumptions of Physics

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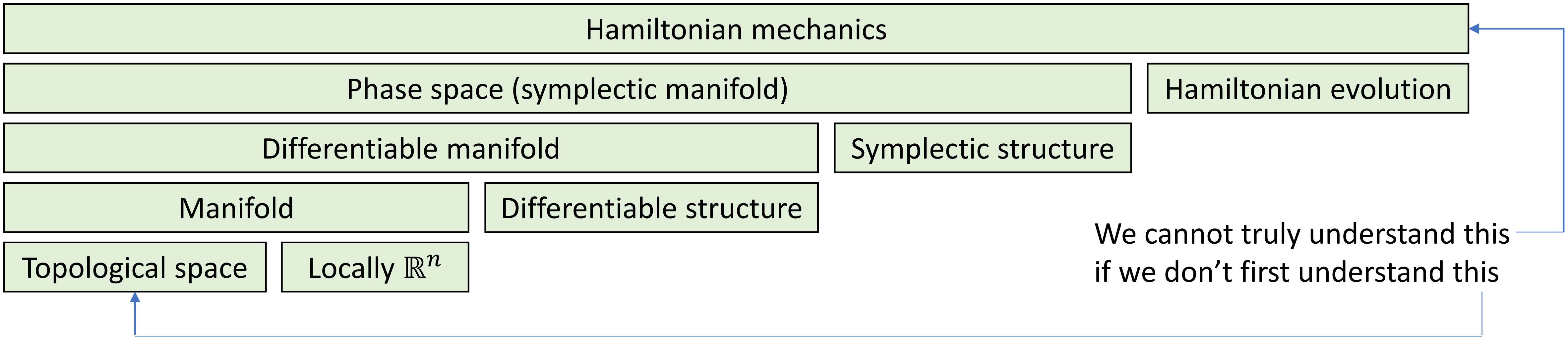
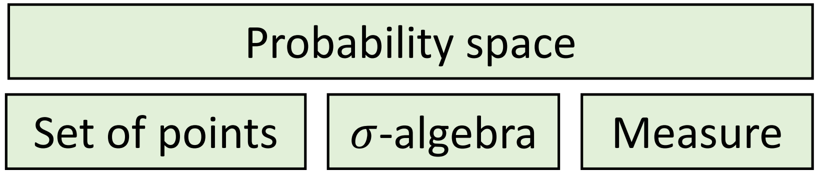
Understanding fundamental structures



Even probability spaces
are not fundamental structures

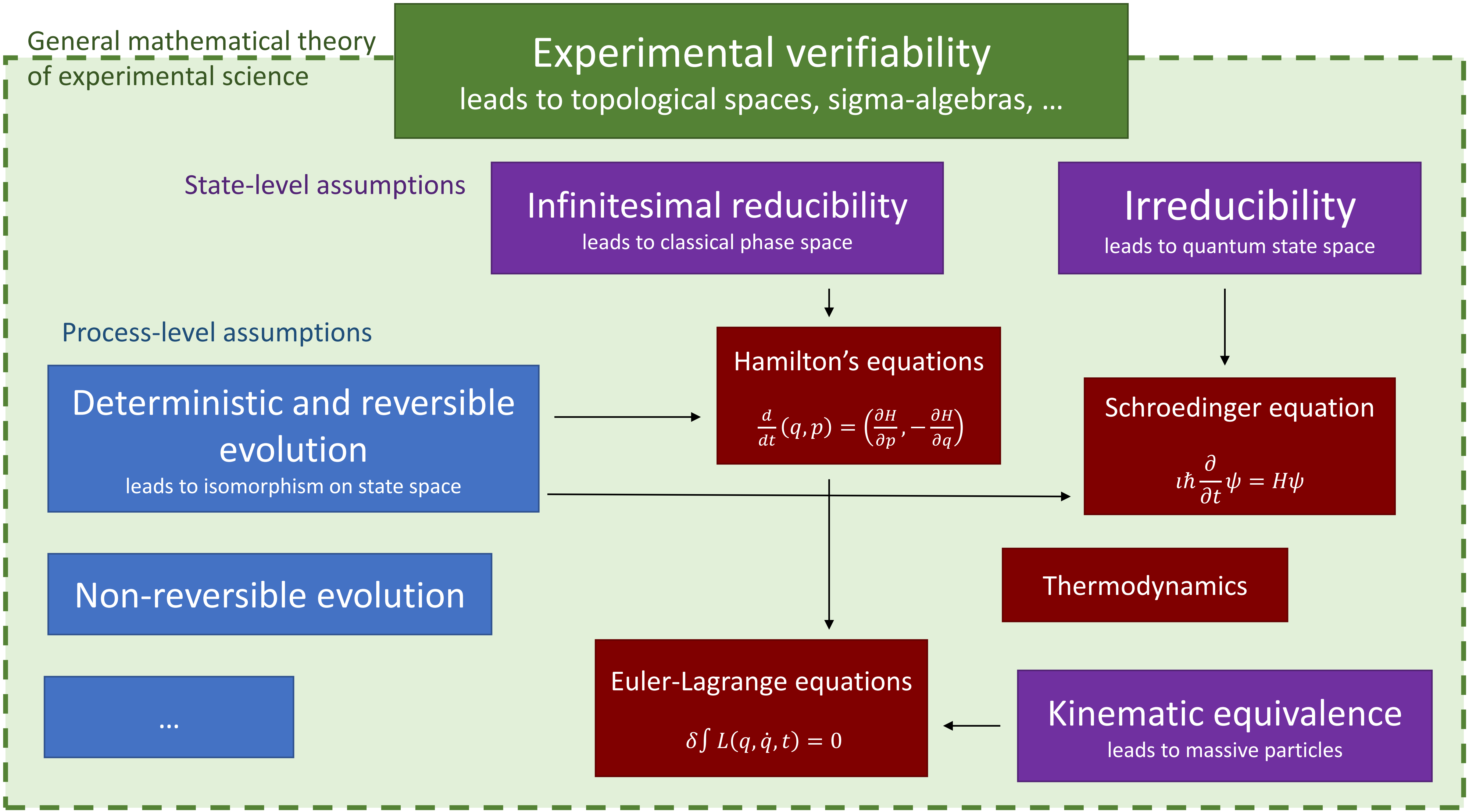


I.e. Before saying "there is a 50% chance to get tails" we need to define what tails, chance and 50% mean

- The desire to understand when and why the higher level structures are needed in science pushed us to backtrack to the most basic ones

Assumptions of Physics

- This is the reason we started a project called Assumptions of Physics (see http://assumptionsofphysics.org/)
- The aim of the project is to find a handful of physical principles and assumptions from which the basic laws of physics can be derived with the following goals in mind:
  - Clarify our assumptions
  - Put physics back at the center of the discussion
    - Derive mathematical structures from physical ideas
  - Give science sturdier mathematical grounds
    - Each mathematical object has a clear physical meaning and no object is unphysical
  - Foster connections between different fields of knowledge
  - Provide a solid basis for new theories



General mathematical theory of experimental science
Experimental verifiability
leads to topological spaces, sigma-algebras, …
State-level assumptions
Infinitesimal reducibility
leads to classical phase space
Irreducibility
leads to quantum state space
Process-level assumptions
Deterministic and reversible evolution
leads to isomorphism on state space
Hamilton's equations
$\frac{d}{dt}(q,p) = \left(\frac{\partial H}{\partial p}, -\frac{\partial H}{\partial q}\right)$
Schroedinger equation
$\imath\hbar\frac{\partial}{\partial t}\psi = H\psi$
Non-reversible evolution
Thermodynamics
…
Euler-Lagrange equations
$\delta\int L(q,\dot{q},t) = 0$
Kinematic equivalence
leads to massive particles

Summary

- First we are going to see how we can construct a formal system that captures some key aspects of experimental verification
  - this leads directly to two fundamental mathematical structures, topologies and σ-algebras, which are the foundations of most of the other mathematical structures used in science
- Then we are going to see three assumptions and their consequences
  - Deterministic and reversible evolution
  - Infinitesimal reducibility, the ability to describe parts of a system
    - which leads to classical phase space and, paired with deterministic and reversible evolution, gives us classical Hamiltonian mechanics
  - Irreducibility, the inability to describe parts of a system
    - which leads to quantum states and, paired with deterministic and reversible evolution, gives us quantum mechanics (i.e. the Schroedinger equation)

Experimental verifiability

Verifiable statements

The basic objects of our frameork will be **verifiable statements**: objective assertions that can be shown to be true experimentally in a finite time

Examples:

- The mass of the photon is less than 10^{-13} eV
- If the height of the mercury column is between 24 and 25 millimeters then its temperature is between 24 and 25 Celsius
- If I take 2 ± 0.01 Kg of Sodium-24 and wait 15 ± 0.01 hours there will be only 1 ± 0.01 Kg left

Counterexamples:

- Chocolate tastes good (not universal)
- It is immoral to kill one person to save ten (not universal and/or evidence-based)
- The number 4 is prime (not evidence-based)
- This statement is false (not non-contradictory)
- The mass of the photon is exactly 0 eV (not verifiable due to infinite precision)

We have to keep in mind that the meaning of the statements, their relationships and what truth values are allowed depends on context (e.g. premise, theory, etc…)

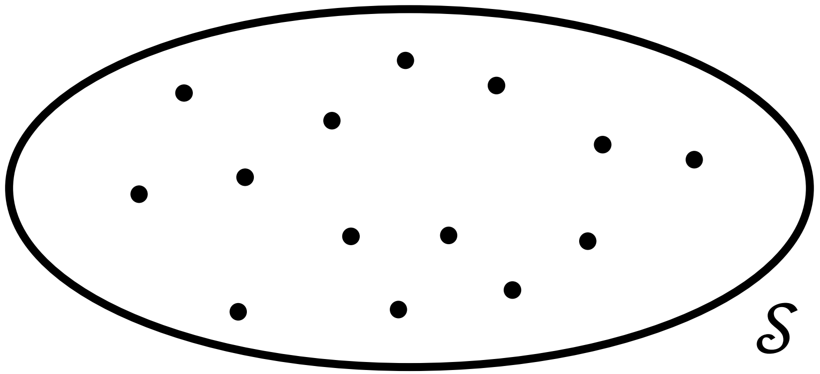
The mass of the electron is 510 ± 0.5 KeV

When measuring the mass is a verifiable hypothesis

Assumed to be true for particle identification

Basic axioms for statements

Axiom 1.2. *A* ***statement*** s *is an assertion that is either true or false. A* ***logical context*** $\mathcal{S}$ *is a collection of statements with well defined logical relationships. Formally, a logical context* $\mathcal{S}$ *is a collection of elements called statements upon which is defined a function* $\text{truth} : \mathcal{S} \to \mathbb{B}$.



	s_1	s_2	s_3	…
a →	T	T	F	…
	T	F	T	…
	T	F	F	…

$\mathcal{A}_{\mathcal{S}}$

Axiom 1.4. *A* ***possible assignment*** *for a logical context* $\mathcal{S}$ *is a map* $a : \mathcal{S} \to \mathbb{B}$ *that assigns a truth value to each statement in a way consistent with the content of the statements. Formally, each logical context comes equipped with a set* $\mathcal{A}_{\mathcal{S}} \subseteq \mathbb{B}^{\mathcal{S}}$ *such that* $\text{truth} \in \mathcal{A}_{\mathcal{S}}$. *A map* $a : \mathcal{S} \to \mathbb{B}$ *is a possible assignment for* $\mathcal{S}$ *if* $a \in \mathcal{A}_{\mathcal{S}}$.

Axiom 1.10. *We can always construct a statement whose truth value arbitrarily depends on an arbitrary set of statements. Formally, let* $S \subseteq \mathcal{S}$ *be a set of statements and* $f_{\mathbb{B}} : \mathbb{B}^{S} \to \mathbb{B}$ *an arbitrary function from an assignment of* S *to a truth value. Then we can always find a statement* $\bar{s} \in \mathcal{S}$ *that depends on* S *through* $f_{\mathbb{B}}$.

s_1	s_2	s_3	…
T	T	F	…
T	F	T	…
T	F	F	…

→ $f_{\mathbb{B}}$ →

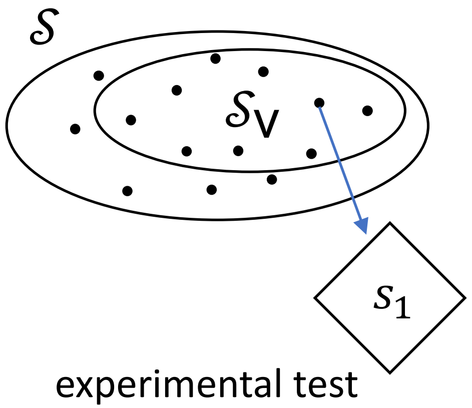
s_2 OR s_3
T
T
F

Basic axioms for verifiable statements

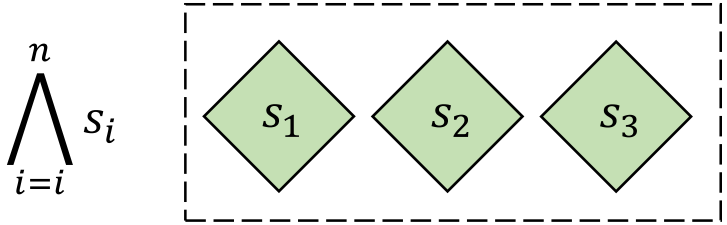
Axiom 1.28. *A* ***verifiable statement*** *is a statement that, if true, can be shown to be so experimentally. Formally, each logical context $\mathcal{S}$ contains a set of statements $\mathcal{S}_v \subseteq \mathcal{S}$ whose elements are said to be verifiable. Moreover, we have the following properties:*

- *every tautology $\top \in \mathcal{S}$ is verifiable*
- *every contradiction $\bot \in \mathcal{S}$ is verifiable*
- *a statement equivalent to a verifiable statement is verifiable*

Remark. The **negation or logical NOT** of a verifiable statement is not necessarily a verifiable statement.



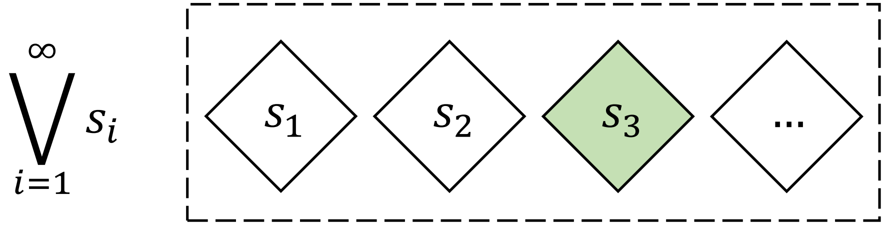
s_1	Test Result
T	SUCCESS (in finite time)
F	FAILURE (in finite time)
	UNDEFINED



All tests must succeed

Axiom 1.32. *The conjunction of a finite collection of verifiable statements is a verifiable statement. Formally, let $\{s_i\}_{i=1}^{n} \subseteq \mathcal{S}_v$ be a finite collection of verifiable statements. Then the conjunction $\bigwedge_{i=1}^{n} s_i \in \mathcal{S}_v$ is a verifiable statement.*

Axiom 1.33. *The disjunction of a countable collection of verifiable statements is a verifiable statement. Formally, let $\{s_i\}_{i=1}^{\infty} \subseteq \mathcal{S}_v$ be a countable collection of verifiable statements. Then the disjunction $\bigvee_{i=1}^{\infty} s_i \in \mathcal{S}_v$ is a verifiable statement.*



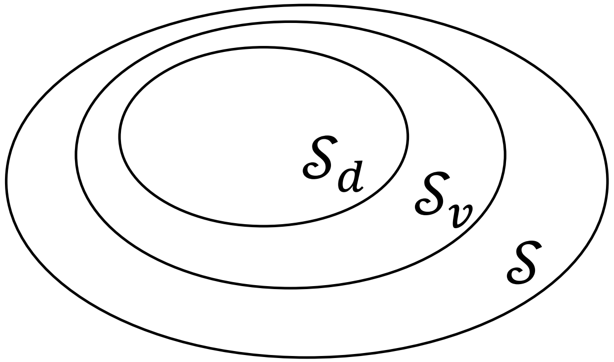
One successful test is sufficient

Properties of the logic system

Different algebras for the different types of statements

Operator	Gate	Statement	Verifiable Statement	Decidable Statement
Negation	NOT	allowed	disallowed	allowed
Conjunction	AND	arbitrary	finite	finite
Disjunction	OR	arbitrary	countable	finite

Table 1.3: Comparing algebras of statements.



(Different) notions of equivalences

Definition 1.16. *Two statements* s_1 *and* s_2 *are* ***equivalent*** $\mathsf{s}_1 \equiv \mathsf{s}_2$ *if they must be equally true or false simply because of their content. Formally,* $\mathsf{s}_1 \equiv \mathsf{s}_2$ *if and only if* $a(\mathsf{s}_1) = a(\mathsf{s}_2)$ *for all possible assignments* $a \in \mathcal{A}_{\mathcal{S}}$.

Are the same statement

"This animal is a bird" = "Questo animale e' un uccello"

Always have the same truth → "This animal is a bird" ≡ "This animal has feathers"

truth("This animal is a bird") = truth("That animal is a mammal")

Happen to have the same truth

Properties of the logic system

Other semantic relationships

Definition 1.21. *Given two statements* s_1 *and* s_2*, we say that:*

- s_1 ***is narrower than*** s_2 *(noted* $s_1 \preccurlyeq s_2$*) if* s_2 *is true whenever* s_1 *is true simply because of their content. That is, for all* $a \in \mathcal{A}_{\mathcal{S}}$ *if* $a(s_1) = \text{TRUE}$ *then* $a(s_2) = \text{TRUE}$.
- s_1 ***is broader than*** s_2 *(noted* $s_1 \succcurlyeq s_2$*) if* $s_2 \preccurlyeq s_1$.
- s_1 ***is compatible to*** s_2 *(noted* $s_1 \asymp s_2$*) if their content allows them to be true at the same time. That is, there exists* $a \in \mathcal{A}_{\mathcal{S}}$ *such that* $a(s_1) = a(s_2) = \text{TRUE}$.

The negation of these properties will be noted by $\npreceq$, $\nsucceq$, $\not\asymp$ *respectively.*

Definition 1.22. *The elements of a set of statements* $S \subseteq \mathcal{S}$ *are said to be* ***independent*** *(noted* $s_1 \perp\!\!\!\perp s_2$ *for a set of two) if the assignment of any subset of statements does not depend on the assignment of the others. That is, a set of statements* $S \subseteq \mathcal{S}$ *is independent if given a family* $\{t_s\}_{s \in S}$ *such that each* $t_s \in \mathbb{B}$ *is a possible assignment for the respective* s *we can always find* $a \in \mathcal{A}_{\mathcal{S}}$ *such that* $a(s) = t_s$ *for all* $s \in S$.

narrower than
"This animal is a cat" $\preccurlyeq$ "This animal is a mammal"

incompatible
"This animal is a cat" $\not\asymp$ "This animal is a dog"

independent
"This animal is a cat" $\perp\!\!\!\perp$ "This animal is black"

- Minimal set of axioms (easy to justify)
- Allows to capture experimental verification
- Allows to describe semantic relationships of the type we have in science
- Gives us a basic structure all other structures have to "play nice" with

Experimental domains (scientific models)

Start with a countable set of verifiable statements

From them generate all verifiable statements (close under finite AND countable OR)

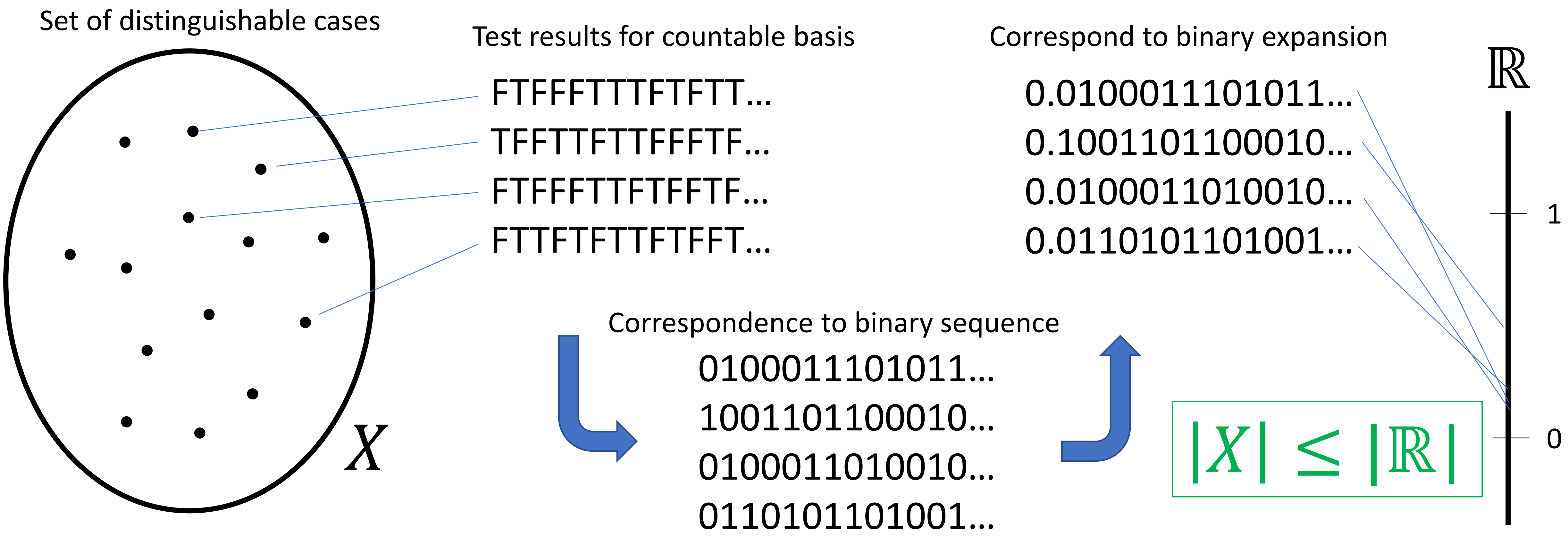
Generate all meaningful predictions (close under negation as well)

Fill in all possible assignments

$x = \neg e_1 \wedge e_2 \wedge \neg e_3 \wedge \cdots$

For each possible assignment we have a theoretical statement that is true only in that case. We call these statements possibilities of the domain.

Maximum cardinality of distinguishable cases



- Sets with greater cardinality (e.g. the set of all discontinuous functions from $\mathbb{R}$ to $\mathbb{R}$) cannot represent physical objects
- Issues about higher infinities (e.g. large cardinals) can be safely ignored

Topologies and σ-algebras

Each column (statement) is also a set of possibilities

$$s = \bigvee_{x \in U} x$$

Topologies (needed for manifold/geometric spaces) and σ-algebra (needed for integration and probability spaces) naturally arise from requiring experimental verifiability

Finite AND and countable OR become finite intersection and countable union

Negation and countable AND become complement and countable union

Basis $\mathcal{B}$				Experimental domain $\mathcal{D}_X$			Theoretical domain $\overline{\mathcal{D}_X}$		
e_1	e_2	e_3	…	$s_1 = e_1 \vee e_2$	$s_2 = e_1 \wedge e_3$	…	$\overline{s_1} = e_1 \vee \neg e_2$	$\overline{s_2} = \neg e_1$	…
F	F	F	…	F	F	…	T	T	…
…	…	…	…	…	…	…	…	…	…
F	T	F	…	T	F	…	F	T	…
~~T~~	~~T~~	~~F~~	~~…~~	~~T~~	~~F~~	~~…~~	~~T~~	~~F~~	~~…~~
…	…	…	…	…	…	…	…	…	…

Possibilities X

The experimental domain $\mathcal{D}_X$ induces a topology on the possibilities X.

The theoretical domain $\overline{\mathcal{D}_X}$ induces a (Borel) σ-algebra

Topologies and σ-algebras

All definitions and all proofs about these structures have precise physical meaning in this context

s_1	Test Result
T	SUCCESS (in finite time)
T	UNDEFINED
F	UNDEFINED
F	FAILURE (in finite time)

$int(U)$ corresponds to the verifiable part of a statement

∂U corresponds to the undecidable part of a statement

$ext(U)$ corresponds to the falsifiable part of a statement

If $U \subseteq X$ is an open set then "x is in U" is a verifiable statement (e.g. "the mass of the electron is 510 $\pm$ 0.5 KeV")

If $\mathrm{V} \subseteq X$ is a closed set then "x is in V" is a falsifiable statement (e.g. "the mass of the electron is exactly 510 KeV")

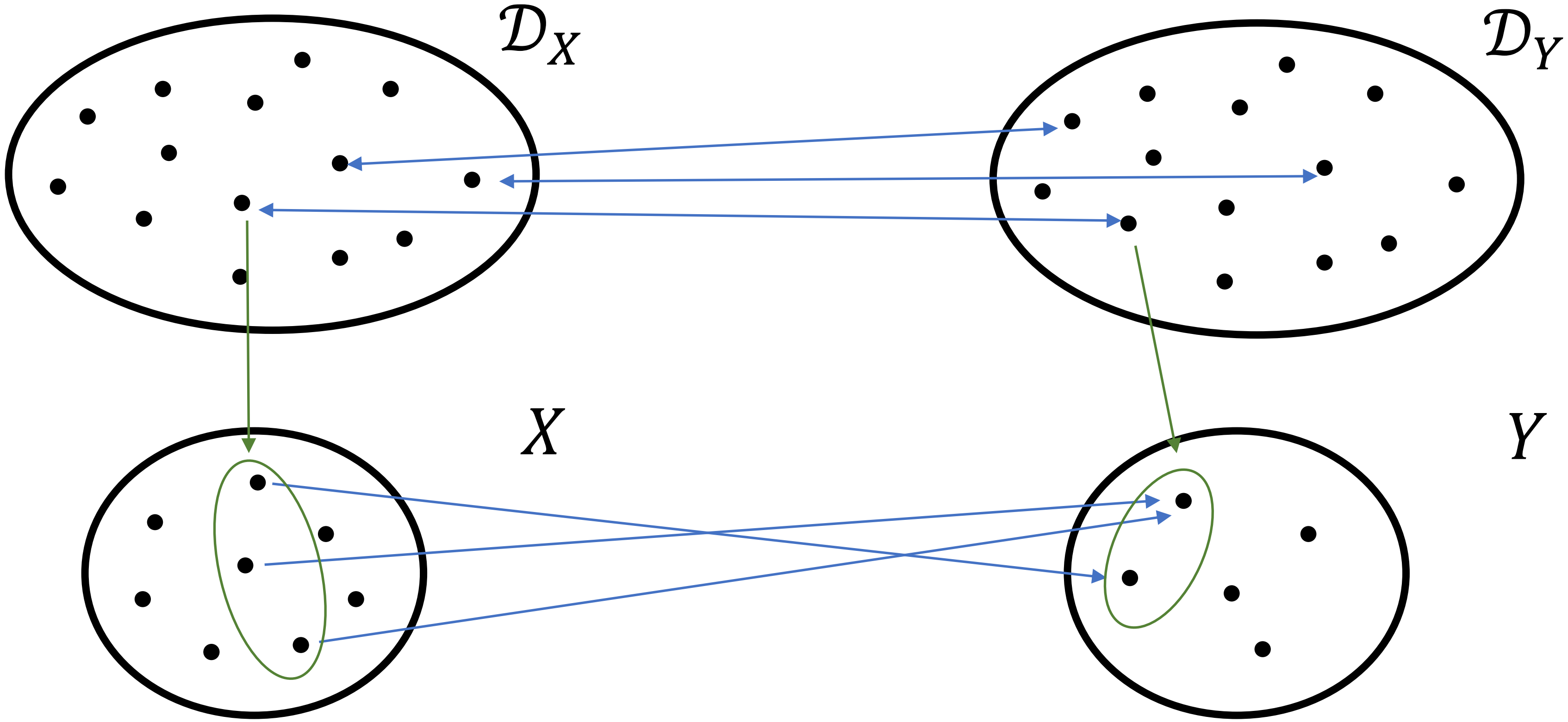
If $A \subseteq X$ is a Borel set then "x is in A" is a theoretical statement: a test can be created, though we have no guarantee of termination (e.g. "the mass of the electron in KeV is a rational number" is undecidable, the test will never terminate)

Topologies and σ-algebras each capture part of the formal structure

For us, they are part of a single unified structure

Inference/causal relationships and continuity

An inference relationship is a map $r: \mathcal{D}_Y \to \mathcal{D}_X$ such that $r(s) \equiv s$



A causal relationship is a map $f: X \to Y$ such that $s \preccurlyeq f(s)$

If two domains admit an inference relationship if and only if they admin a causal relationship.
The causal relationship must be a continuous map in the natural topology.

Takeaway

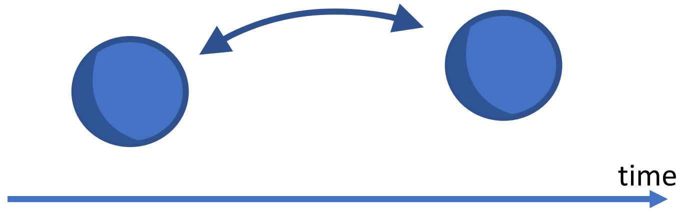
- Logic of verifiable statements -> basic formal structure for scientific theories
- Maximal sets of verifiable statements -> basis for an experimental domain -> topological space over the possibilities (the experimentally identifiable cases)
- Statements associated with a test -> theoretical domain -> σ-algebra over the possibilities

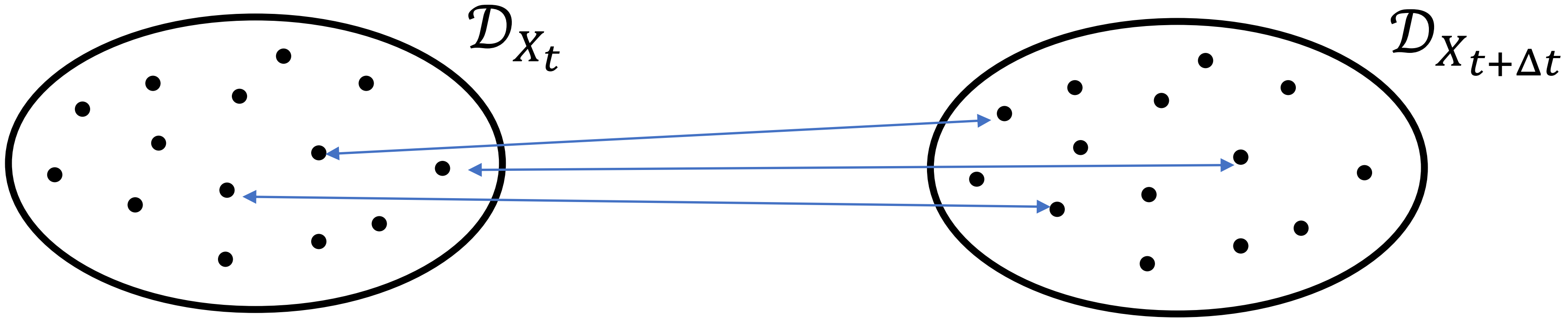
- Experimental verifiability provides the basis for most other mathematical structures used in science (differential geometry, measure theory, probability theory, Lie algebras, …)
  - All scientific models are at least experimental domains (i.e. characterized by a countable collection of verifiable statements)
  - Studying the space of experimental domains means studying the space of possible scientific models
- We can create a “science first” formal structure
  - Better and more precise intuition, better link to mathematical structures, fewer underlying concepts

Determinism and reversibility

Assumption of determinism and reversibility

The system undergoes deterministic and reversible time evolution: given the initial state, we can identify the final state; given the final state, we can reconstruct the initial state





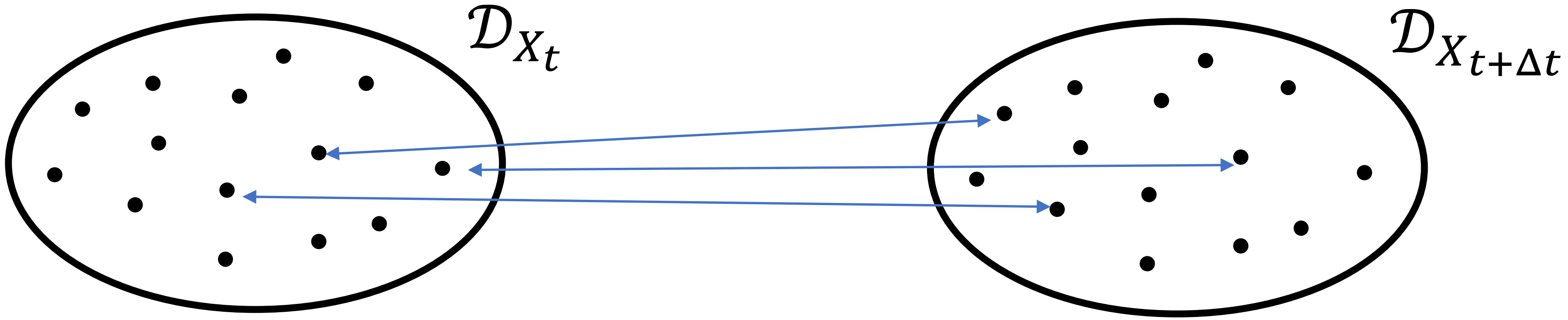
There is an experimental domain $\mathcal{D}_{X_t}$ consisting of verifiable statements on the past state

There is an experimental domain $\mathcal{D}_{X_{t+\Delta t}}$ consisting of verifiable statements on the future state

For each past statement $s_t \in \mathcal{D}_{X_t}$ there is one future statement $s_{t+\Delta t} \in \mathcal{D}_{X_{t+\Delta t}}$ such that $s_t \equiv s_{t+\Delta t}$

Determinism and reversibility means equivalence of experimental domains

Assumption of determinism and reversibility



- Domain equivalence is more than a bijective map: any extra structure that “plays nice” with the fundamental logical structure will be preserved under equivalence
  - Equivalence will, at least, map verifiable statements to verifiable statements -> open set to open set -> homeomorphism
  - If we have a group structure -> group isomorphism
  - If we have a differentiable structure -> diffeomorphism
  - If we have a vector space structure -> invertible linear transformation
- Determinism and reversibility implies an isomorphism in the category

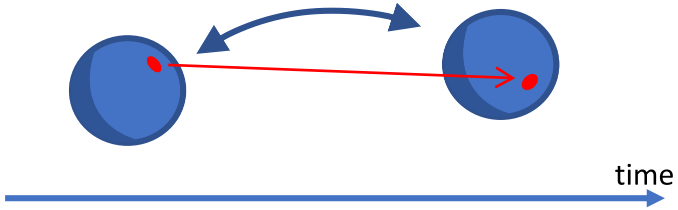
Takeaway

- Determinism and reversibility is more than a one-to-one map: it has to preserve the nature of the system and the type of description
  - Mathematically it will be an isomorphism in the category used to capture states, the associated verifiable statements, and their logical structure
- Determinism and reversibility is an assumption that can be taken to be valid in specific contexts with specific state definitions
  - The state of a balloon is position and velocity or pressure and volume depending whether we study its motion or its expansion
  - If we puncture the balloon, neither of those state definitions is sufficient to predict the future states and the evolution is neither deterministic nor reversible
- But it's an assumption we "need" at some level because it is connected to the ability to define a state
  - We need to prepare states: initial preparation setting must predict the (future) state
  - We need to measure states: final measurement reading must reconstruct the (past) state

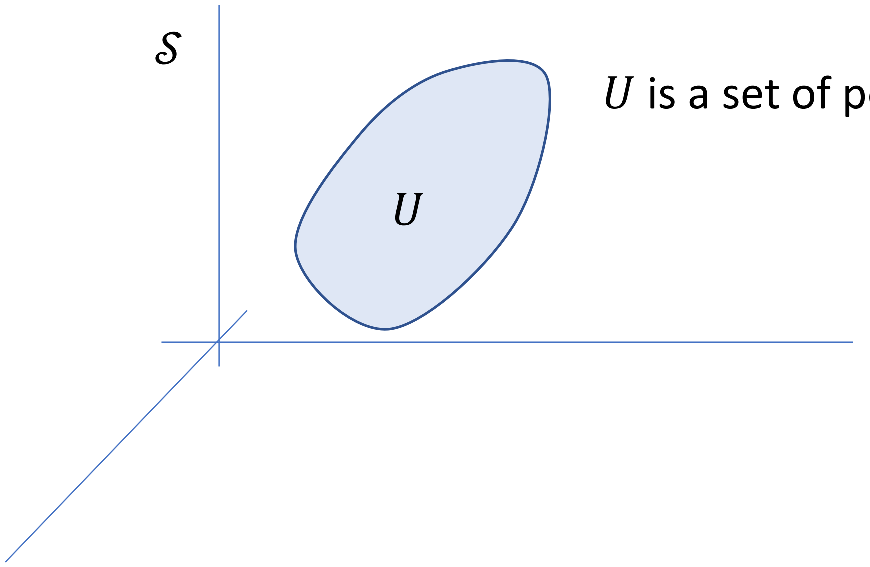
Infinitesimal reducibility

Assumption of infinitesimal reducibility

The system is reducible to its parts: giving the state of the whole is equivalent to giving the state of the parts. The system can be subdivided indefinitely.



$\mathcal{S}$ is the state of the infinitesimal parts (i.e. particles)



U is a set of possible states for the particles

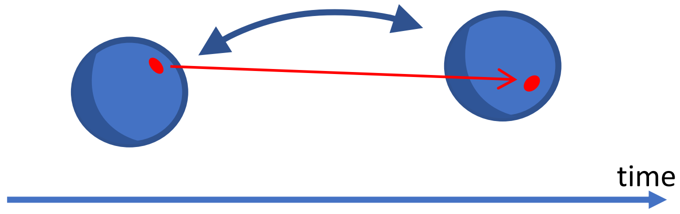
We'll have statements of the form "the amount of material found in U is within the range V"

V is a set of possible amounts of materials

Assuming that the amount over disjoint areas sums, the state of the system can be characterized by a distribution $\rho: \mathcal{S} \to \mathbb{R}$

Assumption of infinitesimal reducibility

The system is reducible to its parts: giving the state of the whole is equivalent to giving the state of the parts. The system can be subdivided indefinitely.



The state of the whole is given by a distribution over the state of the infinitesimal parts (i.e. particles)

$$\rho : \mathcal{S} \to \mathbb{R}$$

$$\rho\left(\mathscr{s}(\xi^a)\right) = \rho(\xi^a)$$

Density expressed in terms of state variables

Under a change of variables $\hat{\xi}^b = \hat{\xi}^b(\xi^a)$ $\quad \mathscr{s}(\xi^a) = \mathscr{s}\left(\hat{\xi}^b\right)$

we have $\rho(\xi^a) = \left|\dfrac{\partial \xi^a}{\partial \hat{\xi}^b}\right| \rho\left(\hat{\xi}^b\right)$ $\quad \rho\left(\mathscr{s}(\xi^a)\right) = \rho\left(\mathscr{s}\left(\hat{\xi}^b\right)\right)$

We need the distribution to change both as a density and be an invariant. How can that work?

Invariant densities

Densities must be defined upon areas that are invariant under coordinate transformations

$\hat{q} = 100\ cm/m\ q$

For each coordinate q that defines a unit there must be a variable k defined on the inverse unit

$\Delta k = 1\ m^{-1}$ 1 $\Delta q = 1\ m$

$\Delta \hat{k} = 0.01\ cm^{-1}$ 1 $\Delta \hat{q} = 100\ cm$

Number of possibilities for the degree of freedom $\Delta q \Delta k$ is invariant

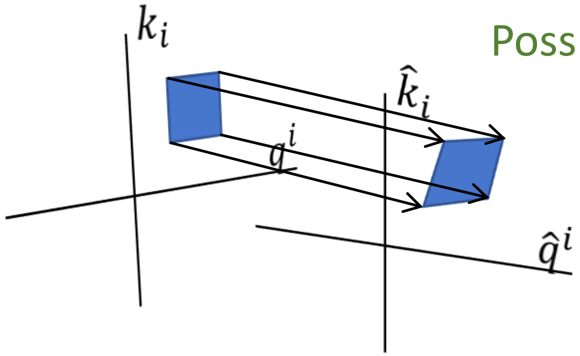
$$\rho(q^i, k_i) = \rho(\hat{q}^j, \hat{k}_j) \qquad \int \rho\left(q^i, k_i\right) dq^n dk^n = \int \rho\left(\hat{q}^j, \hat{k}_j\right) d\hat{q}^n d\hat{k}^n$$

- The structure of phase space is needed to define coordinate invariant densities over particle states
  - Without it we couldn’t compare whether we had more particles in one state or another

Hamiltonian evolution

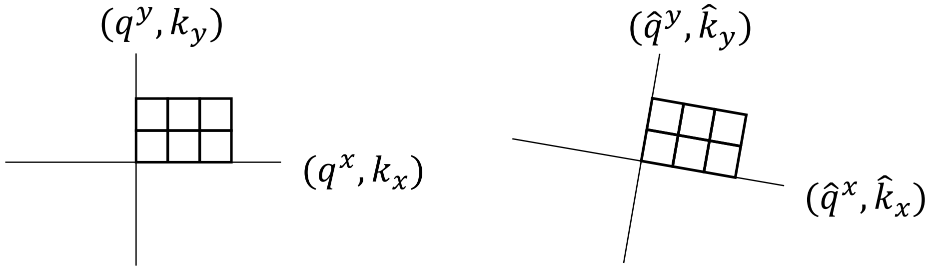
Deterministic and reversible evolution means that densities (including marginals) are mapped exactly from initial to final states

$$\rho\big(s(t)\big) = \hat{\rho}\big(s(t+\Delta t)\big)$$



Possibilities (area) within a d.o.f. conserved

Independent d.o.f. remain independent (orthogonal)



This is exactly what Hamilton's equations do

- Hamiltonian mechanics is just bookkeeping of the count of possibilities for each independent degree of freedom
- Nothing else

For one degree of freedom

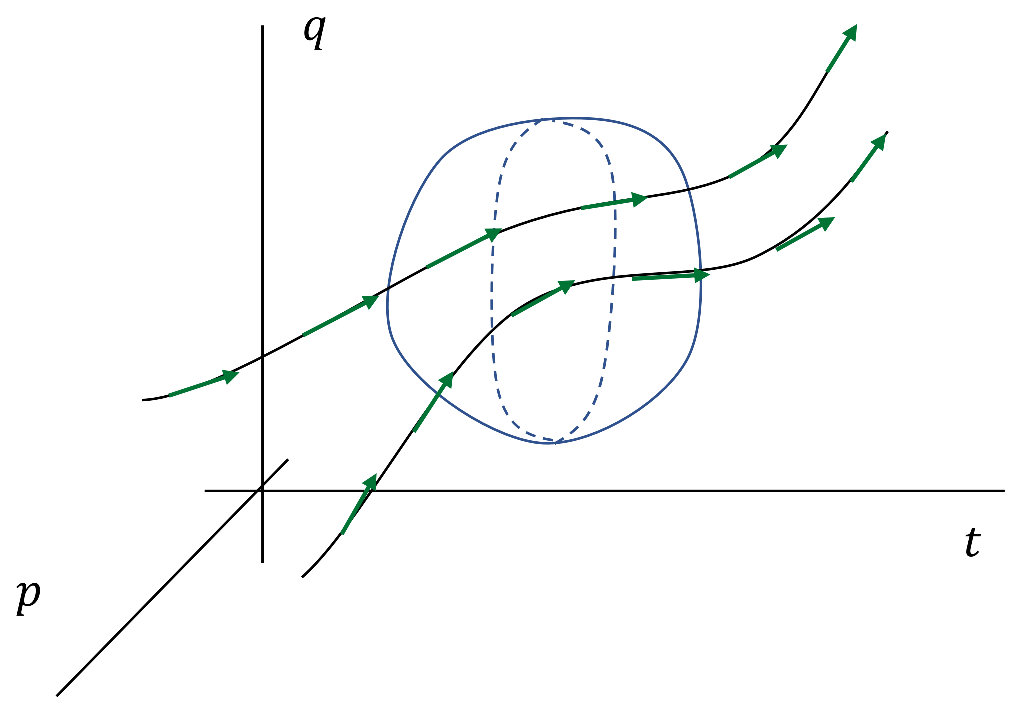
Displacement along the trajectory

$$\vec{S} = \left(\frac{dq}{dt}, \frac{dp}{dt}, \frac{dt}{dt}\right)$$

Deterministic and reversible:
flux over a closed surface is zero

$$div(\vec{S}) = 0$$

$$\vec{S} = -curl(\vec{\theta})$$



Because $\frac{dt}{dt} = 1$ we can choose a gauge such that:

$$\vec{\theta} = (p, 0, -H(q,p))$$

$$\vec{S} = \left(\frac{dq}{dt}, \frac{dp}{dt}, \frac{dt}{dt}\right) = \left(\frac{\partial H}{\partial p}, -\frac{\partial H}{\partial q}, 1\right) = \mathrm{curl}(-\vec{\theta})$$

Conservation of information entropy

Information entropy is invariant over and only over the transformations for which the density is invariant

$$\rho(\xi^a) = \left|\frac{\partial \xi^a}{\partial \hat{\xi}^b}\right| \rho(\hat{\xi}^b)$$

Invariant distributions are precisely the distribution upon which entropy is invariant

$$-\int_S \rho \log(\rho) d\xi^a = -\int_S \rho \log(\rho) d\hat{\xi}^b - \int_S \rho \log\left|\frac{\partial \xi^a}{\partial \hat{\xi}^b}\right| d\hat{\xi}^b$$

$$I[\rho_t] = I_0 = I[\rho_{t+\Delta t}]$$

If the evolution is deterministic and reversible, the information to identify the initial state is the same to identify the final state <-> Hamiltonian mechanics

If we fix the entropy, the gaussian distribution minimizes the spread

$$I_G = \ln(2\pi e\, \sigma_q \sigma_k)$$

During the evolution the spread over phase space is bounded

$$\sigma_q \sigma_k \geq \frac{\exp(I_0)}{2\pi e}$$

Takeaway

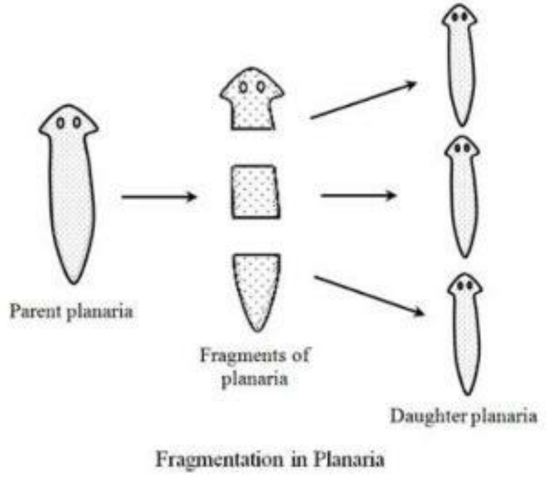
- Infinitesimal reducibility -> distribution over particle states
- Distributions over continuous variables -> differentiability
- Invariant distributions over continuous variables -> phase space (symplectic structure)
- Deterministic and reversible evolution -> Hamilton’s equations (symplectomorphism)

- Hamiltonian mechanics is really just bookkeeping of the count of possibilities for each independent degree of freedom
- Note how much it was implied by a seemingly simple assumption

Divisibility vs Reducibility vs Decomposability

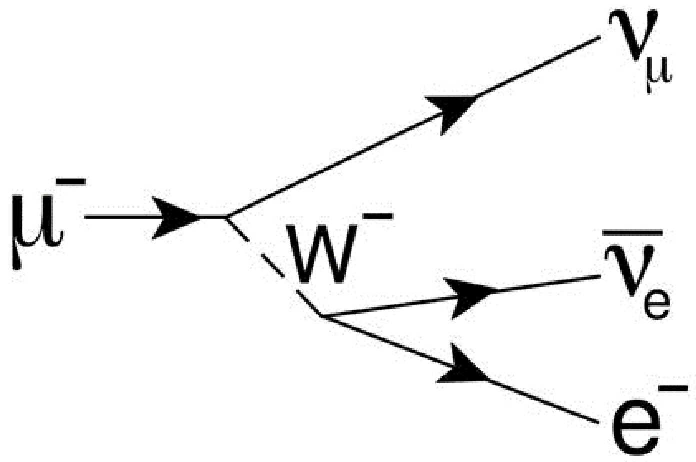
Divisible vs Reducible

There exists a process where the final state consists of two or more independent systems (e.g. $\mathcal{P}_t : \mathcal{S} \rightarrow \mathcal{S}_1 \times \mathcal{S}_2$)



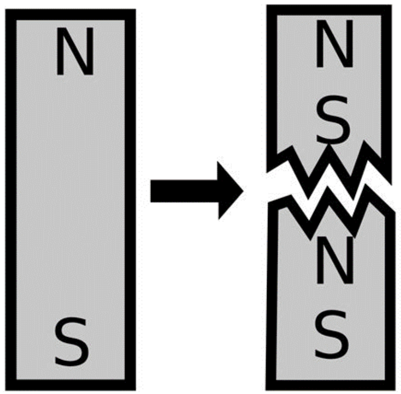
Planarian is divisible into three planaria (but not reducible to them)

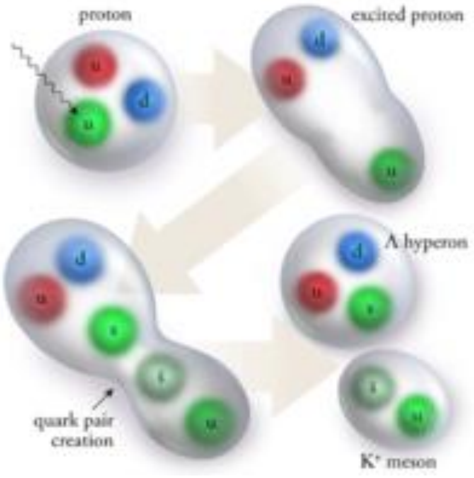
Muon is divisible into an electron and two neutrinos (but not reducible to them)



The state of the whole can be expressed as the state of the parts at the same time (e.g. $\mathcal{S} \equiv \mathcal{S}_1 \times \mathcal{S}_2$)

A magnet is reducible to its north and south pole (but not divisible into them)





A proton is reducible to its quarks (but not divisible into them)

Divisible vs Decomposable

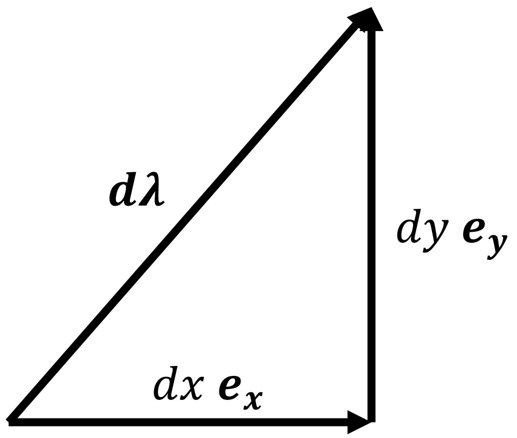
There exists a process where the final state consists of two or more independent systems (e.g. $\mathcal{P}_t : \mathcal{S} \rightarrow \mathcal{S}_1 \times \mathcal{S}_2$)

Reducible

The state of the whole can be expressed as the state of the parts at the same time (e.g. $\mathcal{S} \equiv \mathcal{S}_1 \times \mathcal{S}_2$)

We can calculate a property by combining independent contributions (e.g. $+ : \mathcal{S} \times \mathcal{S} \rightarrow \mathcal{S}$ and $F(s_1 + s_2) = F(s_1) + F(s_2)$)

An infinitesimal line segment is decomposable along coordinates (it is not divisible nor reducible to them)

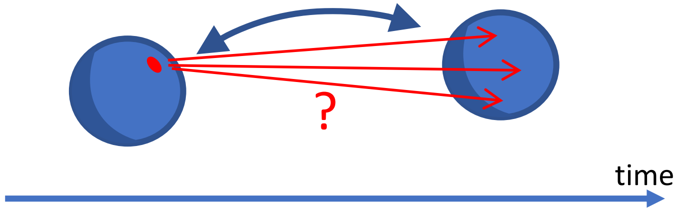


- Divisibility and reducibility are matter-of-fact properties of the system (under given circumstances)
- Decomposability (i.e. linearity) can be defined only over the properties that are linear on that decomposition (if they exist)
  - The work along an infinitesimal line segment is linear: $dW(d\lambda) = F_x dx + F_y dy$
  - The length of the line segment is not linear: $\mathcal{L}(d\lambda) \neq dx + dy$

Irreducibility

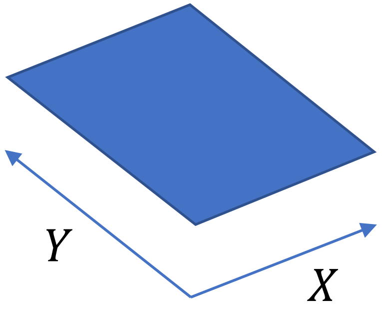
Assumption of irreducibility

The system is irreducible to its parts: giving the state of the whole tells us nothing about the states of the parts.



- Irreducibility does not mean there are no parts and there is no internal dynamics: it means the dynamics is not accessible
  - We do not have a way to interact with each “fragment” independently from the rest of the system, we cannot gather information about them
- Irreducibility is not purely a property of the system: it depends on the process under study and the tools at our disposal
  - We can treat the proton as a single quantum system in many cases
  - We can’t when we perform deep inelastic scattering at suitable wavelength

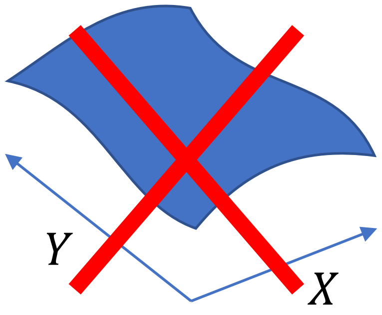
Invariant distribution over random variables



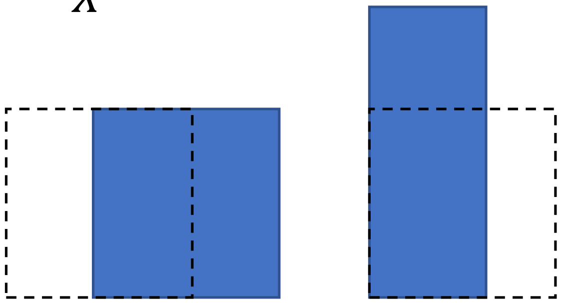
State of the fragments is unknowable

Defined by “internal” random variables

Still needs invariant distributions which we take to be uniform (homogeneity)

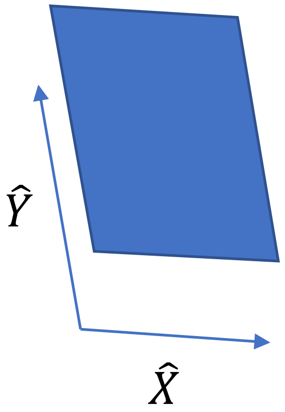


Transformations that break uniformity are not allowed: only linear transformations



Some linear transformations have no effect

Choose X and Y such that $\rho = 1$ and $\sigma_X = \sigma_Y$, which means $\int \rho dxdy \propto \sigma_X \sigma_Y = \sigma_X^2$

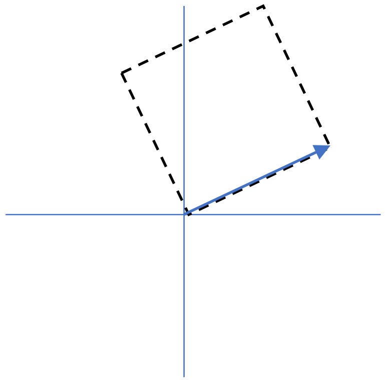


Only significant “internal” changes are those that change the integral (i.e. variance and covariance)

$$\hat{X} = aX + bY$$
$$\hat{Y} = -bX + aY$$

- Internal transformations are identified by a complex number $\mathcal{T}(a + \imath b): \mathcal{S} \rightarrow \mathcal{S}$
  - The square modulus $|a + \imath b|^2 = a^2 + b^2$ represents the change in variance and the phase represents the change in correlation as the arccosine of the Pearson correlation coefficient $\rho_{X,\mathcal{T}(a+\imath b)(X)} = \cos(\arg(a + \imath b))$

Invariant distribution over random variables



The complex plane represents the space of pairs of random variables

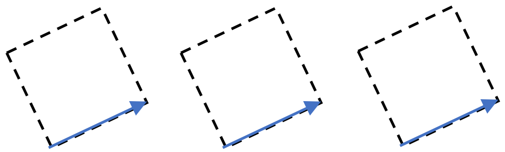
A complex number represents one variable

The second one is implied by the symplectic structure (closing the square)

$$\sigma^2_{X+Y} = \sigma^2_X + \sigma^2_Y + 2\sigma_X\sigma_Y\rho_{X,Y}$$
$$|c_1 + c_2|^2 = |c_1|^2 + |c_2|^2 + 2|c_1||c_2|\cos(\Delta\theta)$$

Linear composition obeys addition of random variables, variance adds linearly only if variables are uncorrelated

Homogeneity → different d.o.f. carry the same spread and correlation
A single complex plane for all d.o.f.



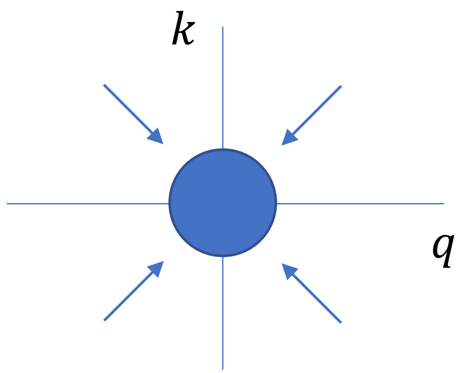
Pearson coefficient

- The state space is a complex vector space
  - Complex inner product $\langle X|Y\rangle = \sigma_X\sigma_Y e^{\imath \arccos(\rho_{X,Y})}$ gives variance and correlation information
- The rest is mostly standard arguments
  - Measurable quantities must have a real valued average over all states → Hermitian operators
  - Deterministic and reversible evolution must preserve the inner product → unitary evolution → Schroedinger equation
- No real justification for Hilbert space, though
  - Cauchy limits are not necessarily physical
  - Assuming the expectations for all polynomials of Q and P are finite gives us Schwartz space

Effects due to irreducibility

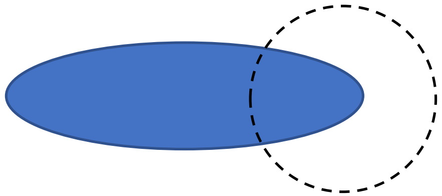
Minimum uncertainty

Suppose arbitrarily small spread -> internal dynamics more and more constrained -> we can know more about it -> contradiction



In fact, all states must be associated with the same information entropy (which is set to zero)

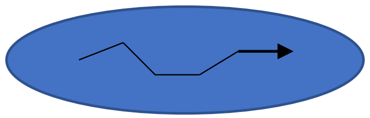
Non-locality



System spread in space -> interact within a region -> but can't interact with only a part-> must be interacting with the rest of the system as well

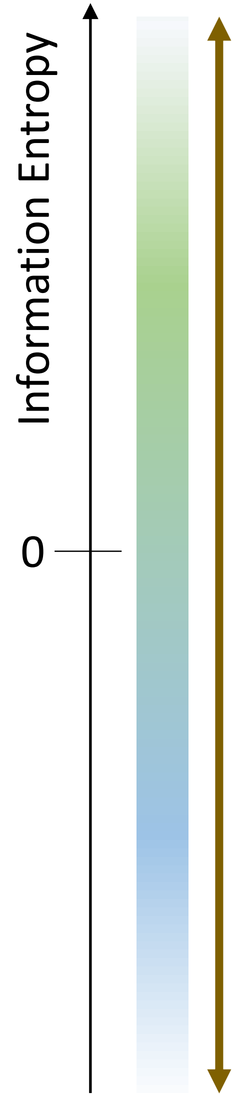
Superluminar effects …

Suppose the internal dynamics is constrained by the speed of light -> we know something about the internal dynamics -> contradiction



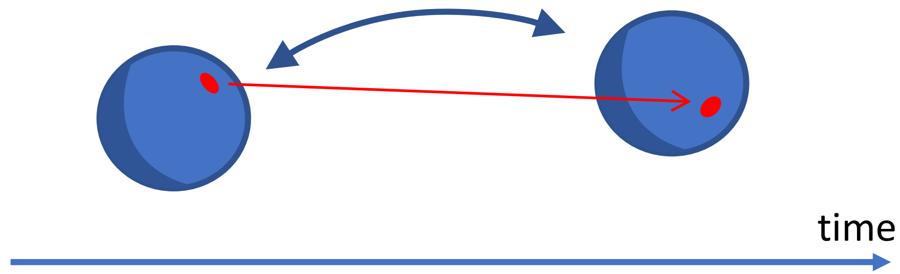
… that can't carry information

Suppose we can use the internal dynamics to carry a message -> we have a way to manipulate the internal dynamics -> contradiction



Classical state

$$\rho(q^i, k_i)$$

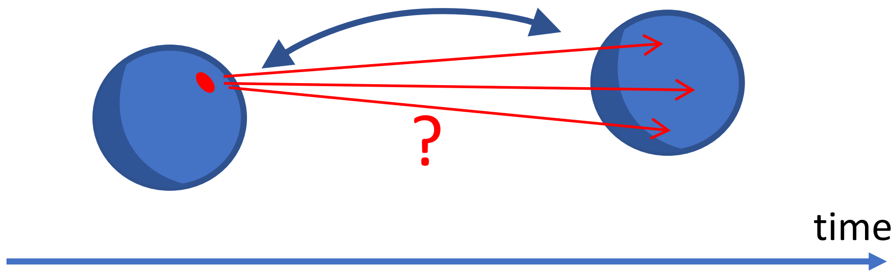


We always have access to the internal dynamics

Any initial value for information entropy is allowed: we can study arbitrarily small parts

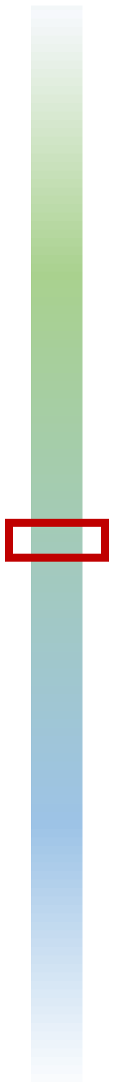
Quantum state

$$\psi(q^i)$$



We have no access to the internal dynamics

All pure quantum states have the same information entropy (i.e. zero): no description for parts



Takeaway

- Irreducibility -> invariant uniform distribution over random variables -> space of random variables represented by a complex plane
- Linear properties of random variables -> vector space over said complex plane
- Statistical relationships between random variables -> inner product
- Determinism and reversibility -> unitary evolution

- The difference between quantum and classical systems is not size, is reducibility

…

Other results

- With an additional assumption one can recover massive particles under potential forces
  - Kinematic assumption: giving the state is equivalent to giving the space-time trajectory
- We have identified a set of necessary and sufficient conditions under which the possibilities of a domain can be identified by numbers
  - That is, how **quantities** are constructed through experimental verifiability
  - This gives us insights on what assumption are required by the continuum and how they would fail
- We have started working on thermodynamics and statistical mechanics
  - The idea is to characterize evolution that is not reversible
  - Working on extending our logic structure to integrate additional concepts (e.g. measures, probability, entropy)

Project status

Status – future activities

- Consolidate and expand the mathematical framework
  - We suspect that extended the framework with an extra axiom that allows to compare the level of description (i.e. granularity) of statements we can recover concepts from measure theory, probability theory, geometry and information theory
  - We also want to recover ideas from differential topology and geometric measure theory from finite value functions of finite valued shapes that are linear (i.e. value for the whole is the sum of the value for the parts)
- Extend to other areas
  - Currently working on thermodynamic and statistical mechanics
- Always looking for individuals that can provide insights/discussions/…
  - Collaborating with Julian Barbour on a better characterization of Shannon entropy
  - Collaborating with Lorenzo Maccone (Quantum Information) on system composition in quantum mechanics

Conclusions

- Only a handful of physical assumptions are required to rederive classical/quantum particle mechanics
  - They capture idealizations for a system under certain conditions
  - They can be seen as characterizing information availability
    - (Reducibility) How much information is accessible? (Determinism/Reversibility) How is it mapped through time? (Kinematic equivalence) Are trajectories enough to gather that information?
- We can create a physically meaningful formal structure for science that starts from simple well-motivated axioms (not abstract mathematical structures)
  - Maps very well to established mathematical structures
  - Objects are both physically and mathematically well defined (no “interpretation” needed, no unphysical objects)
- A comprehensive reorganization of the current theories can lead to better understanding and new ideas